A PHYSICIST'S APPROACH TO PUBLIC TRANSPORTATION NETWORKS

Yurij Holovatch – DOI: https://doi.org/10.1051/epn/2024207

Institute for Condensed Matter Physics of the National Acad. Sci. of Ukraine, Lviv, Ukraine, & L4 Collaboration and International Doctoral College "Statistical Physics of Complex Systems", Leipzig-Lorraine-Lviv-Coventry, Europe & Complexity Science Hub Vienna, Austria & Centre for Fluid and Complex Systems, Coventry University, UK

The above picture shows public transportation on the Halyts'ka Square in my home city of Lviv, Ukraine. One can date the photo simply by having a closer look at the trams with the knowledge that the electric tram appeared in Lviv during 1894 and the horse tram ceased to exist by 1908.

▲ **FIG. 1:** Public transportation hub of two tram lines in Lviv at the turn of century: too early for complex networks.

If physicists contributed to public transportation in those days, it was only as passengers or directly or indirectly related to the design of electric or mechanical parts of the transport vehicles. Daily use of public transport is an old practice that has shifted today from the cabriolets and horse-drawn trams to subways, buses, modern trolleybuses, trams, *etc.* As such the contribution of physics and physicists has changed to analyzing inherent statistical features and collective processes in public transportation networks and this new phenomenon will be discussed in this article.

Complex systems and complex networks

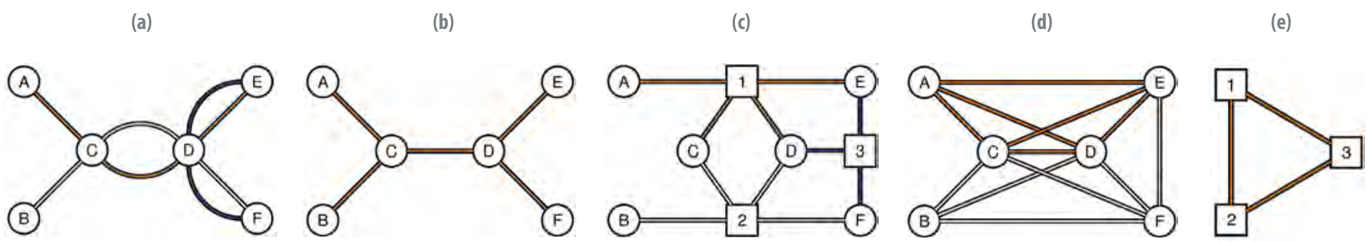
The research we will talk about is entirely due to the intensification of our daily lives and new technologies. Evidence of this is not only the object of analysis itself - the emergence of large public transportation networks (PTNs) - but also the creation of computer technologies, which, on the one hand, allow collecting and storing data about these networks, and, on the other hand, enable big data analysis. Typical analysis of a large amount of data results in measuring statistical properties of a PTN as a whole as well as in quantitative characterization of various processes going on such networks. Very soon it became clear that PTN collective properties do not follow trivially from the behaviors of the network constituents. In turn, it called for theoretical insights aimed at explaining emergent properties of complex transportation networks. In this sense, the study of the properties of large PTNs is a part of a larger process taking place in modern science and, more generally, in modern culture - awareness and conceptualization of the notion of complexity and attempts at its quantitative analysis [1].

Just as mathematics is the language of physics as well as of many other natural sciences (and now, increasingly, of social sciences and even of the humanities), the lingua franca of complexity science is an emerging science of complex networks, *i.e.* of graphs with non-trivial topological features [2]. In the formal description of a complex system of interacting agents, each agent is assigned a vertex (node) of such a graph-network, while the edges (links) of the graph reflect different types of interactions between the agents. It would seem obvious that when constructing such a complex network to describe the PTN, its nodes should be matched with PTN stations, and links between nodes-stations would mean the availability of transport connections between them. Such links can be multiple, if the stations are connected by several routes, or directed, indicating the direction of movement. However, such an interpretation of a PTN in the form of a so-called L-space graph [3] is not the only possible one. Fig. 2 shows several other possible graph interpretations of a PTN. Obviously, only the graphs of Figs. 2a, 2b are embedded in a 2D space, the rest do not share properties of the spatial networks [4]. Moreover, sometimes weight is introduced to network edges (*e.g.* to express the traffic load) or multilayer network interpretations are used to describe situations when it is important to discriminate between different types of transportation. All in all, the journey that began with the application of complex-network tools to analyze the Boston subway and Vienna U-Bahn with $N = 124$ and $N=76$ stations [5], continues today, when entire megalopolis public transportation systems are considered (see Fig. 3b) and it becomes difficult to find a city on the world map where such tools have not yet been applied to analyze its PTN. Leaving aside the purely technological and applied aspects of such research, let's focus on the application of the methods and concepts of physics here.

Statistical physics

Whereas complex network science serves as an universal language that allows for the description of various complex systems, PTNs being one of them, the methodological and conceptual framework of such analysis originates from many traditional disciplines, statistical physics being probably one of the most important ingredients. It equips such analysis with an arsenal of tools and concepts traditionally used in physics to describe collective phenomena. Below we will aim to show how the physical perspective enriches statistical analysis and data processing when dealing with a complex system of many interacting agents "non-physical" in nature. Among many examples, let us concentrate on universality and scaling, robustness and percolation, fractality and self-avoiding walk statistics as PTN inherent features.

Traditionally, the physical approach consists in distinguishing certain universal features, common governing laws, among the diversity of surrounding phenomena. Quests for such universal characteristics of PTNs for cities that differ in their history, geographical location, culture, and economy has been carried out too. It has become clear that despite the obvious diversity of such networks, being characterized as a whole they share a number of common features. In particular, it concerns their node degree distributions $P(k)$ - this function gives the probability that an arbitrary chosen node of the network has k links. It turns out that not only do such distributions have a common shape, but also in certain intervals they are characterized by a power-law decay. Due to obvious ●●●



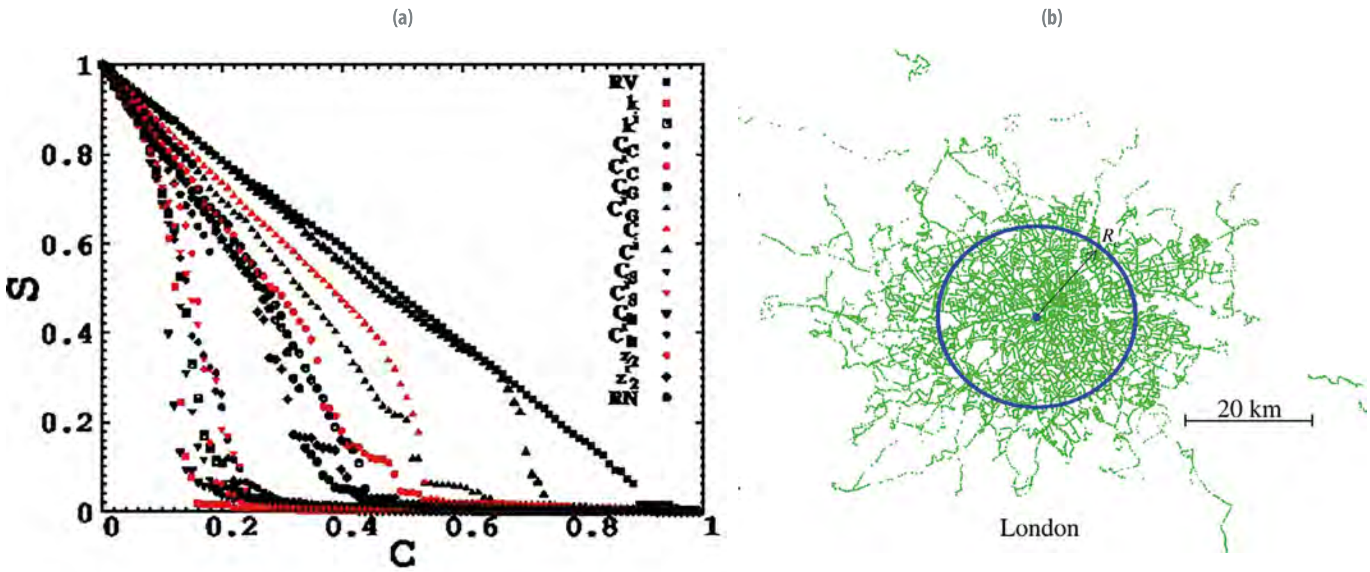
▼ **FIG. 2:** **(a)** a simple public transport map. Stations A-F are serviced by routes No 1 (orange), No 2 (white), and No 3 (blue). **(b)** L-space graph. **(c)** B-space bipartite graph. Route nodes are shown as squares. **(d)** P-space graph, the complete subgraph corresponding to route No 1 is highlighted (shaded orange). **(e)** C-space graph of routes [3].

spatial constraints, power-law behavior can be observed in the L-space (see Fig. 2b) for rather low values of k. In other representations (*e.g.* in P-space, where all stations that belong to a given route are represented by a complete graph, Fig. 2d, or in the coarse-grained L-space representation) the construction of a network enables much higher node degrees and the power-law dependency has been observed for a wider region of k. Complex networks that obey power-law $P(k)$ dependency are *scale-free* [2], they characterize the structure of many natural and man-made systems and have a number of unusual properties. In particular, they are robust to accidental (random) damage to their structure but are vulnerable to targeted attacks.

Speaking about PTN reaction on random failures and targeted attacks it is to place mention of the percolation phenomenon, as far as both phenomena have a lot in common. In the problem of (lattice) percolation one considers the possibility of the existence of a spanning percolation cluster, provided that lattice components, nodes or links, are occupied with a certain probability. When parts of the network, such as its nodes or links, cease to function, an analogue of the spanning cluster is network Giant Connected Component (GCC). For networks of finite size, such as PTNs, the role of a GCC is played by the network largest connected part. Unlike a uniform lattice, usually PTNs are inhomogeneous, some of their constituents are more important than others, therefore there are many ways to model removal of network constituents. Such removals, usually called attacks, simulate node malfunction and can be caused by various factors.

Fig. 3a shows how the size of the largest connected component of the PTN of Paris S changes when the share of its station nodes c are removed following different scenarios (both values are normalized so that the values S=1, c=0 correspond to the entire network). The uppermost curve in Fig. 3a shows the change in S when nodes are removed randomly, this simulates random failures in the functioning of stations. As one can see from the plot, Paris PTN remains robust to such attacks: the size of its largest connected component decreases smoothly with c. However, the behavior of S changes when the attacks become directed at the most important nodes of the network. The most important nodes do not necessarily have to be hubs with the largest node degree k. Curves of Fig. 3a corresponds to various attack scenarios, when the importance of a node was determined not only by its degree, but also by its other characteristics such as the so-called centralities, number of next nearest neighbors, clustering, *etc.* The value of S drops sharply at some limiting c, that depends on the attack scenario. This resembles the behavior in the vicinity of the percolation threshold and indicates *collective effects* in the structure of the PTN. Analytical methods allow *a priori* identification of such Achilles' heels in the network structure and comparison of different PTNs of different cities resistance to random failures and targeted attacks.

Although PTNs are embedded in 2D space, empirical analysis shows that very often the whole networks as well as their components are characterized by fractal behavior. One of the examples is shown in Fig. 3b by the PTN network in Greater London. While the central part of the network is densely filled and the number of stations increases with radius as $N{\sim}R^2$, with further growth of $R{>}R_c$ the dependence changes to $N{\sim}N^{d_f}$ with the fractal dimension $d_f{<}2$. Fractal behavior is often also characteristic of the PTN constituents. So, the distance r between initial and final stations for a passenger's journey traveling for l stops on a single route scales as $r \sim l^\nu$ with an exponent ν that is rather close 3/4, which is the well known self-avoiding walk exponent in two dimensions [3]. There are ongoing attempts to describe emerging PTN structure in terms of evolutionary growth models and to describe their



▼ FIG. 3: (a) Largest component size of the PTN of Paris as function of the fraction of removed station nodes for different attack scenarios. Each curve corresponds to a different scenario as indicated in the legend [6]. (b) Public transportation network of Greater London. Each of 16397 stations is represented as a network node. The radius R_c ~15.4 km corresponds to the transition from the compact central area to the rarefied space with d_f < 2 [7].

evolution using the statistics of *interacting self-avoiding walks* in two dimensions. In turn, the observation of fractal structures evokes an analogy with the physical models of the fractal growths at diffusion limited aggregation or at percolation.

Physics is what physicists do

I gave only a few examples of the application of physical concepts in the analysis of PTNs. Many other examples and even whole areas such as dynamics, cascading failures or traffic jams were left out. Going back to the title of this article, it is a good place to cite Giorgio Parisi who started one of his articles (entitled "Complex systems: a physicist's viewpoint" [8]) with the words: "In recent years physicists have been deeply interested in studying the behavior of complex systems. The result of this effort has been a conceptual revolution, a paradigmatic shift that has far reaching consequences for the very definition of physics.". Of course, if physics is defined as the science of four fundamental interactions, the science of matter and energy, it has nothing to do with public transportation in the sense discussed. However, a physical conceptual framework, physicist's approach to PTN analysis serves in favour of another opinion [9]. ■

About the Author

Yurij Holovatch is a chief researcher at the Institute for Condensed Matter Physics of the National Academy of Sciences of Ukraine and a full member of the Academy. He is a co-founder and a co-director of the L4 Collaboration and International Doctoral College "Statistical Physics of Complex Systems". He is also an external faculty member at the Complexity Science Hub Vienna (Austria) and a visiting professor in Coventry university (UK). His principal research fields are phase transitions and critical phenomena, complex systems, and the history of science.

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