A HISTORY OF QUANTUM ENTANGLEMENT AND BELL'S INEQUALITY

THEORETICAL FOUNDATIONS FOR OPTICAL QUANTUM EXPERIMENTS WITH ENTANGLED PHOTONS

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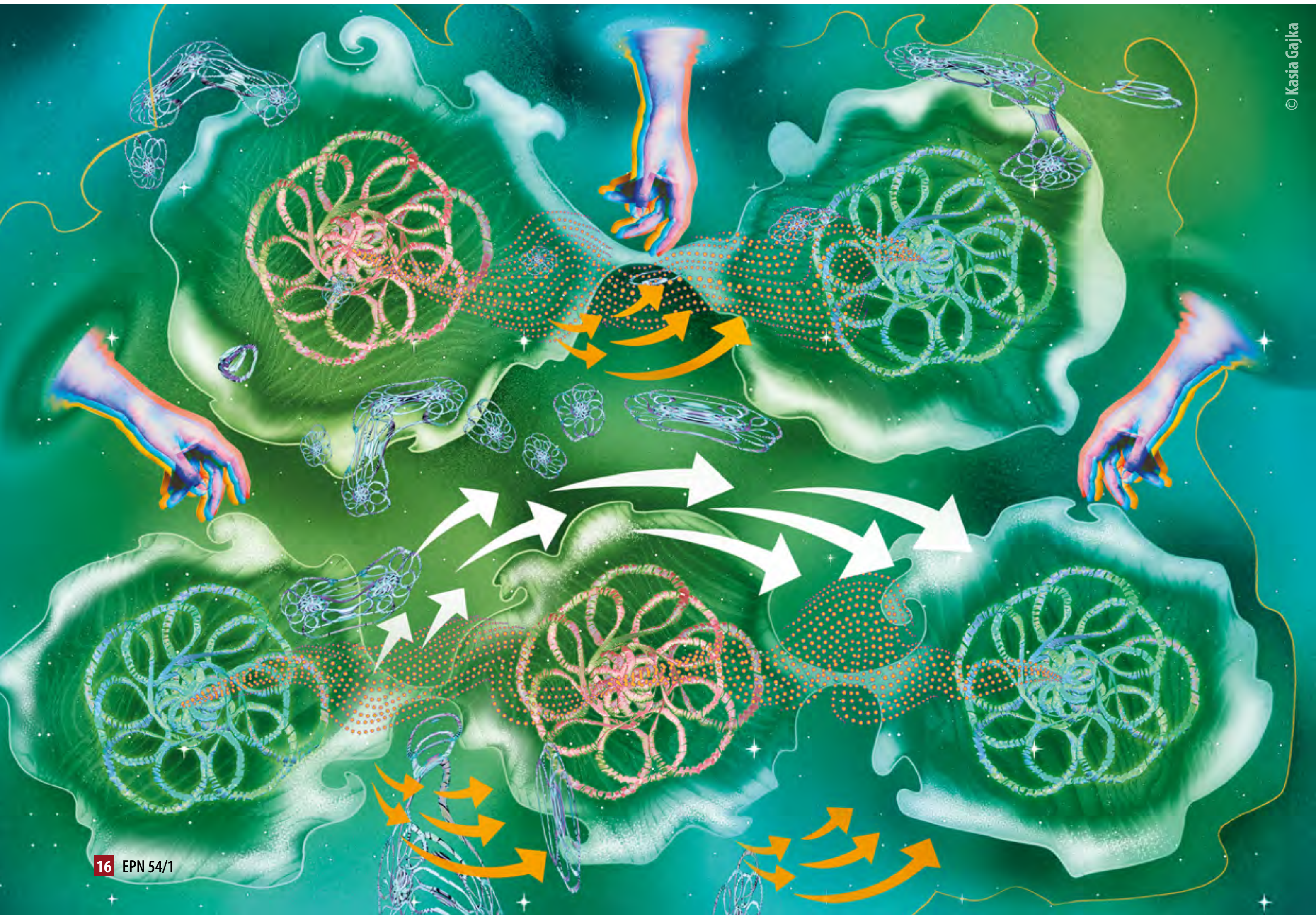
Quantum mechanics gives probabilistic predictions, it is very abstract, and leads to paradoxes. It was discovered in 1925 in two acts of ingenious creativity. By Heisenberg, at the Helgoland island, and by Schrödinger half a year later in Alps. It allows to explain and predict an enormous range of phenomena, but almost immediately met an opposition. Einstein fully recognized it as a practical tool, but criticized its non-deterministic nature. The Einstein-Bohr debate began. Clauser, Aspect and Zeilinger ended it.



Einstein challenged the completeness of the theory in the EPR (1935) paper: ``elements of reality" can be defined via perfect correlations, and they are missing in quantum mechanics.

EPR noticed that perfect correlations are predicted by quantum mechanics for pairs of systems in “entangled states”, a name introduced by Schrödinger for pure two-particle states which do not factorize into a product of states for each particle.

Perfect correlation. Imagine two observers Alice and Bob, who respectively measure some observables A, and B, of values a, $b = \pm 1$. We have a perfect correlation, if *e.g.* the conditional probability of Alice getting 1 and when Bob gets -1, equals one, $P(a = 1|b = -1) = 1$. EPR used a different wording. I follow a simplified argument by David Bohm.

The prediction does not depend on the distance of the objects and their observers and applies to simultaneous detection events. Such events cannot influence each other, as influences propagate no faster than light. Thus, the value $a = 1$ for Alice's particle must have been a predetermined "element of reality". As Alice and Bob can be far from each other and from the source, what observable they individually measure can be set also in a way that cannot influence the free choice of the other partner and his/her result and the operation of the source. The possibility of choice of observables, meant for EPR that a possibility of an alternative choice by Bob to measure a complementary observable B' means that the value of a certain complementary observable A' of Alice can be potentially predicted with certainty. Thus, it must also preexist, if one has $P(a' = 1|b' = -1) = P(a' = -1|b' = 1) = 1$. As Bob's choices and result cannot affect or disturb "the reality" at Alice's location, her particle must carry predetermined values a and a'. They showed an *example* of an entangled state which had such correlation properties.

Bohr's quick response: Bob can measure either B or B', choice of one precluded measurement of the other, and this makes the argument void. Physicists reacted with relief, "shut up and calculated". Only some worked further. David Bohm found that the above properties seems to have the singlet state, $|singlet\rangle$, of two spins 1/2:

$$\frac{1}{\sqrt{2}}\left(|1/2_z\rangle_A\,|-1/2_z\rangle_B - |-1/2_z\rangle_A\,|+1/2_z\rangle_B\right)$$

Here, states of individual spins which are related with spin up or down results, if we measure components "z". Singlet is invariant with respect to any rotation of the full system, and thus has the same form if we use "y" components, or whichever identical ones for both spins. Singlet has the correlation properties mentioned earlier, if one re-scales the eigenvalues to ∓ 1, and treats measurement of z components as A and B, and measurement of "y" components as A' and B'.

29 years later Bell commented on EPR work. He derived his inequality, which applied to systems possessing perfect correlations, and for any theory allowing EPR elements of reality. Using $|singlet\rangle_{AB}$, he showed that the inequality does not hold in quantum theory: the EPR completion destroys quantum mechanics. Almost nobody cared.

In 1969 John Clauser with Mike Horne, Abner Shimony and George Holt (CHSH) derived the first experimentally testable Bell inequality which holds for any classical like theory satisfying Einstein's locality (not only for EPR elements of reality) and showed that it is violated by quantum predictions for $|singlet\rangle_{AB}$. They proposed a test! Specific two-photon cascades in Calcium possess polarization entanglement, which can be one-to one mapped with the singlet. In 1972 he and Freedman performed the experiment. The results followed quantum predictions. *En passant*, in another experiment based onthe same contraption he falsified the semi-classical theoryof light (Phys. Rev. A **6**, 49 (1972)). In 1974 he and Horne derived another inequality, much better for analysis of experimental imperfections.

In his 1978 review with Shimony coined the term "local realism", for general theories of the Einstein type attempting to model quantum mechanics. Such theories are sometimes called "local hidden variable theories". Hidden variables, say λ, are anything that is outside of quantum formalism, which is used to explain the stochasticity of quantum predictions. One assumes a stochastic distribution of these, $\rho_\psi(\lambda)$, related with the quantum state, ψ. For a measurement of A the probability of result a to happen is $P(a|A,\psi) = \int d\lambda\rho_\psi(\lambda)p(a|A,\lambda)$. Locality implies that $P(a\&b|\psi) = \int d\lambda\rho_\psi(\lambda)p(a|A,\lambda)p(b|B,\lambda)$. The Clauser-Horne inequality for this reads in a simplified notation $P(a\&b) + P(a\&b') + P(a'\&b) - P(a'\&b') - P(a) - P(b) \le 0$. The maximal quantum value of that for $|singlet\rangle_{AB}$ is $QM_{max} = \frac{\sqrt{2}-1}{2}$.

The inequality can be also derived by postulating the existence of proper probabilistic distributions of values of all observables is the formulas, ϱ(a, a', b, b') or even just postulating counterfactuality. The latter allows us to use in a theory values of measurements which were not performed, as unknown but defined. Hidden variables λ are sometimes called "causes", or “situations”.

Alain Aspect proposed in 1976 and realized in 1980-1982 several experiments which could be called *ultimate versions of Clauser experiments*. In the last one observation stations (Alice and Bob) were switching their settings quasi-periodically using standing-wave acousto-optical switches in such a way that upon emissions no information was at the source on the settings of polarization analyzers which the photons were to face. The stations were 12 meters away from the source, which was a beam of Calcium atoms excited by a laser passing via the focal points of collection lenses. As not all pairs • • •

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of photons were collected, and not all collected detected, the test had a "detection loophole". One had to assume that the sample of detected photons fairly reproduces the original emitted ensemble. The experiments were thought by some as conclusive, by some as non-conclusive, also because non-randomness of the switching, an effective "locality loophole".

The subject was studied mainly by researchers skeptical about quantum theory. Bell's view: "Quantum Mechanics is rotten". However, in 1970's-1980's experimental physics with single quantum objects emerged. E.g., single neutron interferometry in the case of which Zeilinger had outstanding achievements, atomic interferometry, atomic traps. "Gedanken" experiments of the Bohr-Einstein debate were becoming feasible.

Progress in non-linear quantum optics gave the spontaneous parametric downconversion (PDC) effect. Cascade emissions from atoms do not have directional correlations, thus only a fraction of photons is collected. The downcoversion gives almost perfectly directionally correlated pairs of photons, called signal and idler, with their frequencies adding up to the frequency of the strong laser pumping field propagating in a "non-linear" birefringent crystal, *e.g.* BBO or KDP, and inducing a nonlinear response of its electric polarization. From 1986 downconversion became the work horse in Bell experiments, the first ones by Mandel's group, and Shih and Alley. In 1994 Zeilinger and Shih groups showed that PDC of type II is a fantastic source of polarization entangled photons. A direct consequence of that was the Bell experiment of Weihs *et al.* of Zeilinger's group in Innsbruck which convincingly closed the "locality loophole". Alice and Bob stations were 800 meters apart, and their polarization analyzers' settings were decided by local quantum random number generators.

Before we discuss the 2015-2017 fully "loophole free" experiments, let me turn back to theoretical advances allowing observations of three-or-more photon interference. In 1989 Daniel Greenberger, Horne and Zeilinger (GHZ) showed that three or more photon interference effects lead to a direct falsification of the EPR approach. This signaled that a plethora of new quantum paradoxes was waiting to be discovered. Earlier, a rule of thumb, more particles, less quantumness dominated.

The simplest case. Take a state $|GHZ(3)\rangle$ which reads

$$\frac{1}{\sqrt{2}}\left(|a\rangle_A\,|b\rangle_B\,|c\rangle_C + e^{i(\varphi_A+\varphi_B+\varphi_C)}\,|a'\rangle_A\,|b'\rangle_B\,|c'\rangle_C\right).$$

It is describing entangled properties of three particles. Each particle goes to a different of three spatially separated observation stations of Alice, Bob and Cecile who observe time coincident detections. They are imposing local phase shifts, respectively ϕ_A, ϕ_B, ϕ_C, which transform the state, see the relative phase factor $e^{i(\phi_A+\phi_B+\phi_C)}$.

Essentials of calculation. If their detectors are measuring identical observables with eigenstates $|\mp\rangle_X = \left(\frac{1}{\sqrt{2}}\right)\left(|x\rangle_X \mp |x'\rangle_X\right)$, where $x = a,b,c$ and $X = A,B,C$, and we have $\langle x|x'\rangle = 0$, the overall probability amplitudes of coincident detections are proportional to $(1 \mp e^{i(\phi_A+\phi_B+\phi_C)})$ Following Bell, one can associate with a detection event described by the final state $|\mp\rangle_X$ values $\mp 1 = X(\phi_x)$. This leads to the following average of the products of values of the coincident events: $\langle A(\phi_A)B(\phi_B)C(\phi_C)\rangle = \cos(\phi_A + \phi_B + \phi_C)$.

Whenever $\langle A(\phi_A)B(\phi_B)C(\phi_C)\rangle = \pm 1$, see the box, one has "perfect GHZ correlations", a version of the EPR ones, defining elements of reality. Such elements of reality $X(\phi_x) = \mp 1$ must satisfy

$$A(\phi_a = 0)B(\phi_b = 0)C(\phi_c = 0) = \cos 0 = -1,$$
$$A(\phi_a = 0)B(\phi_b = \pi/2)C(\phi_c = \pi/2) = \cos\pi = -1,$$
$$A(\phi_a = \pi/2)B(\phi_b = \pi/2)C(\phi_c = 0) = \cos\pi = -1,$$
$$A(\phi_a = \pi/2)B(\phi_b = 0)C(\phi_c = 0) = \cos\pi = -1,$$

With these relations, as $X(\varphi_x)^2$, via a primary school algebra we get 1=–1. The method of EPR makes no sense.

How to observe such correlations? A solution appropriate for the times, was found in ZZHE paper, and improved in two subsequent Zeilinger *et al.* theoretical papers. One can use three pairs of entangled systems, independently emitted form three different sources, such as PDC crystals, and swap their entanglement to get $|GHZ(3)\rangle$.

Take a simpler case of two different entangled pairs with no entanglement between the pairs, say two signal and idler pairs, each pair from a different PDC source. By an arrangement for "Bells-state-measurement" which detects together both idlers, in such a way that the information on the origin of the idlers is completely erased, one can entangle the two remaining signal photons which never interacted, and can be very far away. The indistinguishability of the idlers can be reached by narrow filtering of these, while the pumping fields must be sharply pulsed, as this allows for the frequency spectral widths $\Delta\omega$ to satisfy, $\Delta\omega_{idlers=filters}/\Delta\omega_{pump} < 0.1$. In such a case the original tight frequency correlation of signal and idler from each pair, $\omega_{signal} + \omega_{idler} = \omega_{pump}$, which betrays their source, is effectively erased. Importantly, only idlers are subject to filtering, signals propagate to remote observation stations untouched, but now entangled.

As it was noticed in the last page of ZZHE variations on the theme of entanglement swapping allow one to make experimental "quantum teleportation", observe GHZ correlations, and allow for an "event-ready" Bell test, heralding the signal-events by detection of the idlers, and thus operationally defining detection failures of these.

Meanwhile theoretical quantum information science emerged. This can be traced back to Feynman suggesting quantum simulations and computing (1982), Wiesner's quantum money (1983), and Bennett's and Brassard's quantum cryptography based on Bohr's complementarity, or EPR correlations (1984). Theoretical quantum teleportation, that is a transmission of an unknown state form one system in say Alice's lab, to another one at Bob's lab, was announced in 1993.

The teleportation is a protocol involving a three-particle interference, with one particle in a state, ξ, to be teleported, and the other two in a singlet state shared by Alice and Bob. In the case of teleportation of photon polarization states, a classical transmission of just two bits of information to Bob is needed, who knowing these can unitarily transform the state of his particle from the EPR pair to ξ. Alice performs a "Bell state" measurement (jointly on the particle in state ξ and her particle of the singlet), of four equally possible results numbed by the bits 00, 01, 10, 11, which totally hides the origin of the particles and is erasing ξ. Successful teleportations were reported by Zeilinger's experimental group on 11 December1997 (in *Nature*), two-pair entanglement swapping in 1998, and observation of GHZ correlations in 1999.

In 2015-2017 four loophole free Bell experiments were announced, therefore closing the debate of the two Nobel laureates who *received* the Prize in… 1922. A Delft group of Hanson, who performed an entanglement swapping Bell experiment between separated by 1300 meters NV centers in diamonds, observed 250 such events, excluding any local realistic interpretation of them with probability of 96%. Two other experiments with a high detection efficiency observation of photon correlations, one of them by Zeilinger's group, were also announced in 2015, giving confidence of practically 100%. Weinfurter's group in 2017 reached this confidence level with entanglement swapping entangling single atoms in two traps separated by 400 meters. ■

In [PCYWZZ] review article one can find a detailed description of the story up to 2012. MZ is supported by FNP IRAP project ICTQT co-financed by EU Smart Growth programme.

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Additional reading

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