THE STERN-GERLACH EXPERIMENT RE-EXAMINED BY AN EXPERIMENTER

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The historic Stern-Gerlach experiment (SGE), which was performed in 1922 in Frankfurt, is reviewed from an experimental point of view. It is shown that the SGE apparatus is a purely classical momentum spectrometer, in which the trajectories of particles are measured. With modern detection devices the passage of each single atom can be identified and its trajectory in the magnetic field precisely determined. At the time of their experiment Stern and Gerlach achieved a hitherto unprecedented momentum resolution corresponding to an energy resolution of one μeV.

Otto Stern – a cross-thinker

Otto Stern was a lateral thinker, someone who favoured carrying out experiments in unexplored areas of physics. Two years working closely together with Albert Einstein (Charles University Prague 1912 and ETH Zürich 1912-1914) strongly influenced him to examine scientific questions very carefully [1]. Thus it was typical of him to oppose the hypothesis of Pieter Debye and Arnold Sommerfeld concerning the " Richtungsquantelung" (space quantization, but the German word Richtungsquantelung means "directional quantization") of internal atomic magnetic momenta in the presence of an external magnetic field [2]. For him this hypothesis seemed to completely contradict common sense, as he said in his 1961 Zürich interview [3]. Using the Molecular

Beam Method (MBM) that he invented in 1919 in Frankfurt, he was convinced that he could test this hypothesis experimentally [4]. When he conceived the now-famous Stern-Gerlach experiment (SGE) in 1919, he designed an ingenious apparatus which could measure the tiny momentum transfer between a single atomic particle and a classical detection device with very high resolution. This enabled him to investigate internal atomic properties of atoms in their ground state in an unprecedented manner.

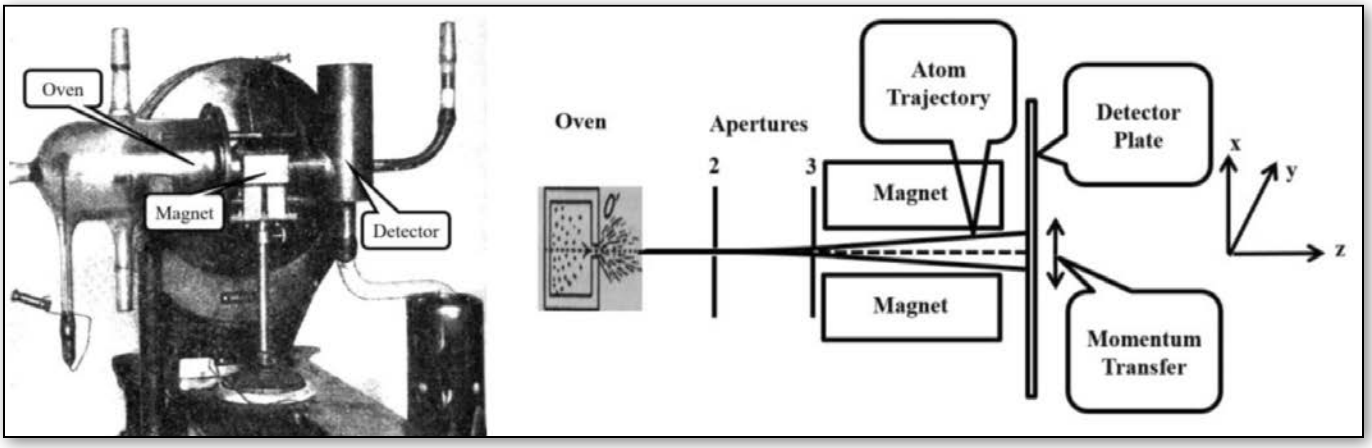
Invention of the Molecular Beam Method

In February 1919 after World War I, Stern returned to the Institute of Theoretical Physics at the University of Frankfurt, which was directed by Max von Laue. A few months later von Laue moved to the University of Berlin, alongside Max Planck, to work with Albert Einstein, and Max Born became von Laue's successor. With the help of Adolf Schmidt, a young mechanic, Stern invented the MBM at the theoretical institute. This enabled him to create in vacuum a beam of atoms (or molecules) all moving in the same direction with about the same velocity, that is all atoms in the beam were prepared in a well-defined momentum state. He used an effusive beam and thus the atoms had a Maxwell-Boltzmann velocity distribution. By deflecting these single atoms by an external force (*e.g.* a magnetic field), he was able to probe their internal magnetic properties and achieve a very high momentum resolution in the transverse direction. In order to deduce absolute values of these quantities by this kinematic method he had to know the absolute velocity of the atoms in the direction of the moving beam (z direction). In spite of the very difficult financial situation so soon after the war, he was successful in measuring the Maxwell-Boltzmann velocity distribution of silver atoms evaporated from a solid at the boiling point of silver. He invented a rotating streak camera and measured the beam trajectories when the system was rotating and non-rotating. From the two different strips he deduced the atomic velocities and obtained good agreement with the Maxwell theory [4].

Design of the Stern-Gerlach experiment

Knowing the beam velocity and using the molecular beam deflection method, he was aware that he could test the hypothesis of "Richtungsquantelung" (space quantization), and even measure the ground state magnetic moment of the silver atom, which was not possible by spectroscopic means. In 1921 he published, as sole author, the proposal for such an experiment [5]. As was typical of all experiments he performed, he carefully calculated the required conditions (the beam collimation parameters, the strength of the magnetic field, *etc.*) in order to be able to resolve, from the deflected beam, the tiny transverse momentum transfer due to the existence of an internal atomic magnetic moment (fig.1). For the silver beam used, which had a velocity of about 540 m/s, the average momentum in the z-direction was 49 a.u. (an electron with 13.6 eV kinetic energy has a momentum of 1 a.u.). To be able to test "Richtungsquantelung" he needed a momentum resolution in the transverse direction of about 0.1 a.u. which seemed hardly achievable at the time. This momentum resolution corresponds to a kinetic energy resolution in the transverse direction of about 2 μeV, about 25 times smaller than the internal atomic Zeeman splitting of the silver ground state in a magnetic field of 20 kGauss, the field strength required to perform the SGE. The prepared silver atom momentum vector $\boldsymbol{p}$ finally reached a quality of $\Delta\boldsymbol{p} = (\Delta p_x, \Delta p_y, \Delta p_z)$ = (± 0.1 a.u., ± 0.1 a.u., 49 ± 5 a.u.). Stern and Gerlach used mercury diffusion pumps, a glass Gäde pump for a rough vacuum and a one-stage glass Volmer pump for high vacuum (about 10-5 torr). Both pumps had a rather low pumping speed.

▾ **FIG. 1: left: photograph of the Stern-Gerlach apparatus [6-8]; right: scheme of the apparatus designed by Stern, with four sections; 1: the oven, heated to a temperature of about 1050° C, from which the silver vapor effused through a tiny aperture; 2: the collimator, in which at a distance of about 2.5 cm from the oven a second aperture and, 3.3 cm beyond that, a third aperture, were carefully positioned; 3: the magnet, 3.5 cm long, providing an inhomogeneous magnetic field; 4: the detector, a cooled glass plate at the freezing point of carbonic acid, where the atoms were collected, the silver being made visible as silver sulphide (AgS).**



The role of Walther Gerlach

The theoretically-trained physical chemist Otto Stern was fortunate in finding in Walther Gerlach a collaborator who was an excellent experimental physicist. Gerlach had left his job in industry and in 1919 had joined the group of Richard Wachsmuth, who was the director of the Institute for Experimental Physics at Frankfurt. Gerlach was willing to help Stern in this really difficult project. Following Stern's proposal and design, it was Gerlach, together with the mechanic Adolf Schmidt, who enabled the experiment to finally succeed.

In October 1921 Stern left Frankfurt for a professorship at the University of Rostock, but he continued to collaborate with Gerlach, though the latter had to carry out the SGE alone. On November 4th Gerlach succeeded in observing for the first time a broadening of the silver spot when the magnetic field was switched on. But the momentum resolution in the transverse direction was still too poor, so that only a rough estimate of the magnetic moment could be made, and no conclusion about "Richtungsquantelung" (space quantization) could be drawn. The main problem was avoiding slit-scattering due to the passage of the beam through the three very tiny apertures. At a meeting in Göttingen in early February 1922, Gerlach and Stern discussed this problem and decided to replace the final circular aperture by a longer rectangular slit. This modification achieved the breakthrough required and during the night of 7th-8th February, Gerlach succeeded in observing the splitting of the beam into two components (fig.3) [7].

The Interpretation of the SGE doublet splitting

The SGE results clearly showed a doublet splitting. Gerlach was convinced that the prediction of Niels Bohr was correct, proving the classical expectation that the electron can run clockwise or anti-clockwise with magnetic projections m = ± 1. Based on the analysis of the normal Zeeman effect , Arnold Sommerfeld had however predicted a triplet splitting (angular momentum components m = +1, 0, -1). In 1922 a third explanation also existed: according to Alfred Landé the doublet splitting was due to the magnetic moment of a single electron in a 1s state moving around a core [9]. Landé recognized from the analysis of Zeeman multiplet structures that k = ½ with a g-factor of 2. In 1923 he published this interpretation of the SGE, implying the existence of a half integral quantum number m = ± ½ with g-factor of 2 [10]. However, he did not then explicitly attribute this quantum number to the electron internal angular momentum, namely its spin of ½.

▲ FIG. 2: Members of the Frankfurt Physics faculty in 1920: seated, right to left (excluding ladies): Otto Stern, Max Born, Richard Wachsmuth; standing, 3rd from the right Alfred Landé, 4th Walther Gerlach [photograph: Nachlass Otto Stern, Bancroft Library, Berkeley].

The role of Albert Einstein in the SGE

Einstein followed the SGE with great interest and he even supported the experiment by providing money from the Kaiser Wilhelm Gesellschaft for the magnet. When the SGE finally provided evidence for the existence of "Richtungsquantelung" (space quantization), Einstein and Ehrenfest immediately tried to find a theoretical explanation of how this process of rotating magnetic momenta in well-defined directions could occur [11]. They expected,

THE IMPACT OF THE SGE

1. The SGE was the first application of the MBM as a dynamic measuring approach yielding excellent subatomic momentum resolution, *i.e.* micro eV energy resolution. It provided a new breakthrough for the momentum measurement of atomic particles.
2. The SGE in 1922 was the first measurement where a ground-state quantum property of an atom could directly be determined.
3. The SGE measured for the first time in a direct way the magnetic moment of an atom, *i.e.* Ag atoms.
4. The SGE verified Debye's and Sommerfeld's hypothesis of RQ (directional quantization) of inner magnetic moments in outer magnetic fields (Zeeman effect).
5. The SGE presented the first direct experimental evidence that the inner-atomic angular momenta are quantized in units of $\hbar = h/2\pi$.
6. The SGE showed for Silver atoms the doublet splitting, *i.e.* it provided the first direct observation of the electron spin. As we know today this splitting is due to the inner magnetic moment of the electron of about one Bohr magneton resulting from the electron spin = $1/2\hbar$ with a g-factor of about two.
7. The SGE delivered an atomic beam in a well-defined quantum state yielding the basis for population inversion and the maser development.
8. The SGE produced the first fully spin-polarized atomic beam.

instead of discrete space quantization, a continuous classical Larmor precession and no change of angle between internal momenta and the external magnetic field. According to them an adiabatic rotation process could only be induced by another (unknown) force. Without such a force in the SGE, the rotation would take thousands of years. From the Schrödinger equation it also follows that the Larmor rotations must be quantized (for integer quantum numbers one obtains m = 0, ± 1, ± 2, … and for half-integer quantum numbers m = ± 1/2, ± 3/2, ...). The only allowed lowest Larmor quantum states are those two states which were observed in the SGE. Space quantization is a purely quantum mechanical process and is thus not to be understood within classical physics. On entering the magnetic field in the SGE, each atom is immediately directionally quantized with 100% probability, independent of its velocity and its time duration in the field. Much later, Frisch and Segrè performed a three-stage SGE in Stern's institute in Hamburg, and showed that, if in stage one a pure spin state is selected and injected into the second stage with a parallel magnetic field, all atomic momenta remain in the same orientation and no flipping of the magnetic moment occurs [12].

LAUDATION BY ERIK HULTHÉN

I shall start, then, with a reference to an experiment which for the first time revealed this remarkable so-called directional or space-quantization effect. The experiment was carried out in Frankfurt in 1920 by Otto Stern and Walter Gerlach, and was arranged as follows: In a small electrically heated furnace, was bored a tiny hole, through which the vapor flowed into a high vacuum so as to form thereby an extremely thin beam of vapor. The molecules in this so-called atomic or molecular beam all fly forwards in the same direction without any appreciable collisions with one another, and they were registered by means of a detector, the design of which there is unfortunately no time to describe here. On its way between the furnace and the detector the beam is affected by a non-homogeneous magnetic field, so that the atoms - if they really are magnetic - become unlinked in one direction or another, according to the position which their magnetic axes may assume in relation to the field. The classical conception was that the thin and clear-cut beam would consequently expand into a diffuse beam, but in actual fact the opposite proved to be the case. The two experimenters found that the beam divided up into a number of relatively still sharply defined beams, each corresponding to one of the just mentioned discrete positional directions of the atoms in relation to the field. This confirmed the space-quantization hypothesis. Moreover, the experiment rendered it possible to estimate the magnetic factors of the electron, which proved to be in close accord with the universal magnetic unit, the so-called "Bohr's magneton".

Particle or Wave?

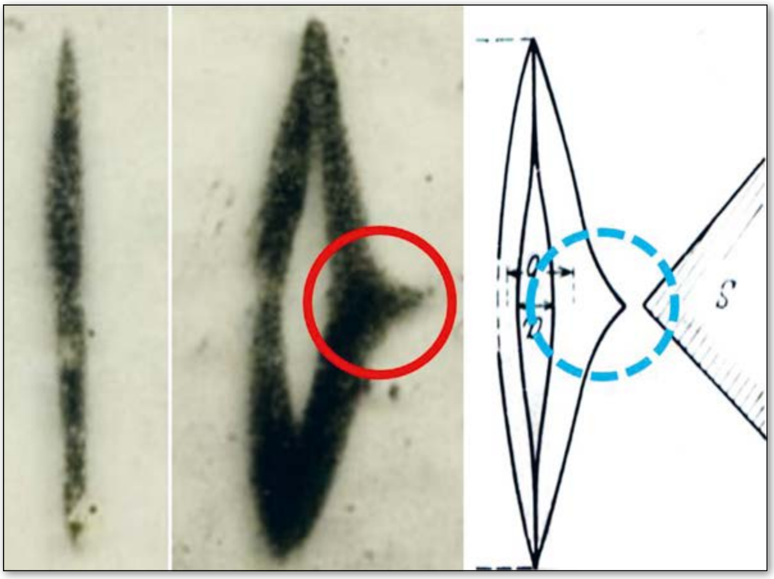
Another fundamental question often raised in the theoretical interpretation of the SGE is: when an atom passes through the apparatus, is it to be treated as a quantum mechanical wave or as a classical particle [13]. In many theoretical articles the atomic beam motion is treated as a wave so that the partial waves of the two spin directions can interfere, since the experimenter does not know in which spin orientation the atom passes the magnet.

To refute this argument one can show that the experimenter does in fact know the orientation of the magnetic moment from the measured deflection, that is, from the curvature of the trajectory. One must consider what the possible origins of the deflection could be. The atoms can either be transversely deflected due to slit-scattering (diffraction), or due to transverse momentum exchange with the magnet via the magnetic force. To answer this question one must consider what is the possible origin of the measured deflection of a few millirad? Since the aperture widths in the SGE are about 100 μm and the de Broglie wave length of silver atoms is only about 0.1 Ångström, diffraction structures will be observed only for deflection angles below a few μrad. In the SGE therefore, the observed deflection results only from the magnetic force between the atom and the magnet. From the impact position on the detector one knows the direction of the magnetic force and thus the orientation of the magnetic moment. If the magnetic moment would have oscillated between both directions and then, at impact, would have been fixed where the wave collapses on the detector, the atom would never impact with the observed maximal deflection. Furthermore the asymmetric silver density distribution for both magnetic components in the historic SGE (fig.3, red circle) provides further evidence that the trajectories of the atoms are influenced by the inhomogeneity of the magnetic field (fig.3, blue circle) and by the tiny misalignment of the beam trajectory with respect to the edge of the magnet. Classical trajectory effects are thus clearly visible in the detection pattern. As Gerlach and Stern pointed out, these effects clearly tell the experimenter on which trajectory each atom passed the magnet.

The paradigm experiment of electron scattering on a double slit is often used to justify the necessity of describing the SGE in the wave approach. One should however note, that the deflection of the atom in the SGE and in electron scattering on a double slit have nothing in common. In the double-slit experiment the de Broglie wave length of the electron and the slit width are of similar magnitude; thus one cannot decide with which slit the electron interacted. Beyond the slits the scattered electron wave moves in a straight line; so that one can never decide from the shape of the trajectories at which slit the electron was scattered. However the atoms in the SGE move on clearly distinguishable curved trajectories.

Nobel Prize history concerning Stern and Gerlach

For their contributions to modern quantum physics, Stern and Gerlach were jointly nominated for the Nobel Prize in Physics on 31 occasions, the first being in 1924 by Albert Einstein and the last in 1944 by Manne Siegbahn [14]. For his invention of the MBM (in Frankfurt) and his measurements of the magnetic moments of the proton and deuton (now deuteron), as well as for his helium beam interference experiments with the direct measurement of the de Broglie wave length of helium atoms (in Hamburg), Stern received 52 additional individual nominations. Finally, Stern was awarded the Nobel Prize in Physics for 1943 alone (the official appreciation on the Nobel certificate was for the 'Invention of the MBM and measuring the proton magnetic moment'). Gerlach was not considered for the Prize since at the time when this award was made he was head of the German 'Uran Verein' ('Uranium Club'). Nevertheless, the laudation speech for Stern given by Erik Hulthén on December 10th 1944 did actually mostly celebrate the SGE. ■



▲ FIG. 3: The AgS distribution obtained in the SGE, the photographs are from the private slide collection of Otto Stern. Left: the Ag beam spot without magnetic field (spot length about 1.1 mm, spot width about 0.06 to 0.1 mm). Centre: the beam spot with magnetic field, showing a splitting into two components with broadening of the silver condensate due to the breadth of the velocity distribution. The asymmetry of the magnetic field strength between the two magnetic poles is clearly visible in the shape of the condensate. Atoms passing near the tip S of the pole are more strongly deflected.

About the Author

Horst Schmidt-Böcking was from 1982 till 2004 Professor for experimental atomic and molecular physics at the University Frankfurt. He retired in 2004. Together with his group and collaborators he has developed the COLTRIMS reaction microscope a multi-coincidence momentum imaging device which can visualize inner atomic and molecular dynamics on the one atto-second scale. For his contributions to experimental quantum physics he has received in 1991 the "Max Planck Forschungspreis, in 2008 the Davisson-Germer award of the American Physical Society and in 2010 the Stern-Gerlach medal of the German Physical Society. In recent years he has published several books on the life and work of Otto Stern.

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