MECHANICS-BASED MATHEMATICAL STUDIES PROVING SPONTANEITY OF POST-IMPACT WTC TOWERS COLLAPSE

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The cause of collapse of the World Trade Center (WTC) in New York on 9/11/2001, clarified mathematically by mechanical analysis, has been questioned by some lay critics without any meaningful calculations. They blame the collapse on controlled demolition, implying some sort of conspiracy. The present article summarizes the reasons why the collapse must have been spontaneous and an inevitable result of the aircraft impact damage and the subsequent fire, and how the collapse is explained by mathematical analysis based on mechanics and confirmed by all the available observations.

Previous Mechanics-Based Analytical Studies

The WTC towers were designed not to fail under the impact of an aircraft of almost the same size, and they did not fail. Their maximum horizontal deflections did not exceed 0.4 m, which is about one half of the deflection expected in a hurricane. The actual cause of collapse was an enormous fire that simultaneously engulfed three stories and caused viscoplastic buckling of steel columns, the thermal insulation of which was stripped during the impact and initial explosion of fuel [1].

The first simple analysis [2, 3], with a simple mathematical proof of the inevitability of collapse based on an approximate comparison between the kinetic and dissipated energies, was published soon after the collapse. This analysis was quite straightforward.

A detailed and more tedious mechanical model, published several years later, dealt with the entire collapse process [4, 5, 6, 7]. The results agreed with all the observations and showed that the progressive collapse must have been spontaneous, gravity-driven, and that after impact, no external weakening of the structure was needed to explain the collapse. These results passed standard anonymous reviews in top journals and are generally accepted by the mechanics experts in ASCE, ASME, SES, Royal Society of London, IUTAM and in other reputable professional and scientific societies, as well as by the mechanics experts at the leading research universities.

Mechanism of Spontaneous Collapse Driven by Self-Weight

The main cause of the total collapse of the towers damaged by impact was a fire of enormous proportions (Fig. 1a and b). The simultaneous ignition of three floors was atypical. It differed from normal fires, which gradually spread from one place to the next and, when the next place is burning, the previous one has already burned out. This resulted in a slower release of heat and higher temperatures because the volume-to-surface ratio of fire zone was much greater than normal [1].

Furthermore, this atypical fire caused that most columns reached high temperatures almost simultaneously, rather than one column cooling down when another

one is being heated. Although temperatures must have reached much higher, a mere half-hour of heating above 150 °C would have sufficed to cause marked viscoplastic creep of the structural steel used. This led to slowly growing lateral bowing of columns, which was documented photographically [1].

Meanwhile, the heating also caused a large thermal sagging of the steel trusses supporting concrete floors [1]. In consequence, many steel truss girders likely separated from the columns and beams, especially during the cooling phase. This is evidenced by photos of outward bowing of external columns [1]. It must have led to doubling, or even tripling, of the effective buckling length of some columns.

The buckling was aggravated by eccentricity of aircraft impact, which severed or damaged columns mainly on one side of tower [1]. With fewer columns functioning on one side, the individual columns on that side had to carry more than the average load of the remaining columns (and on the other side less than the average). After these columns buckled, the load on each of the remaining columns increased and caused them to buckle, too. Because of greater eccentricity of aircraft impact into the South tower, the resulting column overloads on one side were greater than those of the North tower. This agrees with the fact that the South tower collapsed earlier. The one-sided column overload is also confirmed by the observed tilt of the upper part of tower. That the observed tilt was mild is no surprise because calculations showed it could not have exceeded about 2.8° in the direction of impact eccentricity; see Eq. 7 in [3].

Compared to the conservative simplifying assumptions of the analysis that sufficed to prove the necessity of collapse, there were further aggravating factors:

1. initial impact damage to surviving (non-severed) columns,
2. stripping of column insulation,
3. the creep bowing of columns under prolonged heating,
4. increase of the effective length of some columns due to multi-story buckling,
5. the aforementioned one-sided column overloads,
6. creep growth of lateral deflection of bowing heated columns, and
7. loss of lateral supports of column ends by disconnected sagging floor trusses.

Because of insufficient quantitative information on these aggravating factors, they were conservatively omitted from the analysis of collapse trigger. But it is clear that they significantly enhanced the likelihood of reaching the stability limit and of triggering the vertical fall of the top part of building (Fig. 1c). Their consideration was unnecessary because even the minimalist assumptions sufficed to demonstrate the order-of-magnitude excess of the kinetic energy of falling mass over the energy dissipation capacity of the columns.

At the moment of downward impact of the top part of tower onto the undamaged cold story below the fire zone, the kinetic energy of the top part exceeded by an order of magnitude the energy required for complete buckling of all the columns of the cold story calculated under very optimistic assumptions, especially: 1) no fracturing of steel, and 2) fall of the top part through the height of only one story. If one takes into account the aforementioned aggravating factors, particularly the fact that the steel must have fractured, and that the initial fall was likely through the height of not one but two or even three stories weakened by fire, the excess of kinetic energy must have been even much higher.
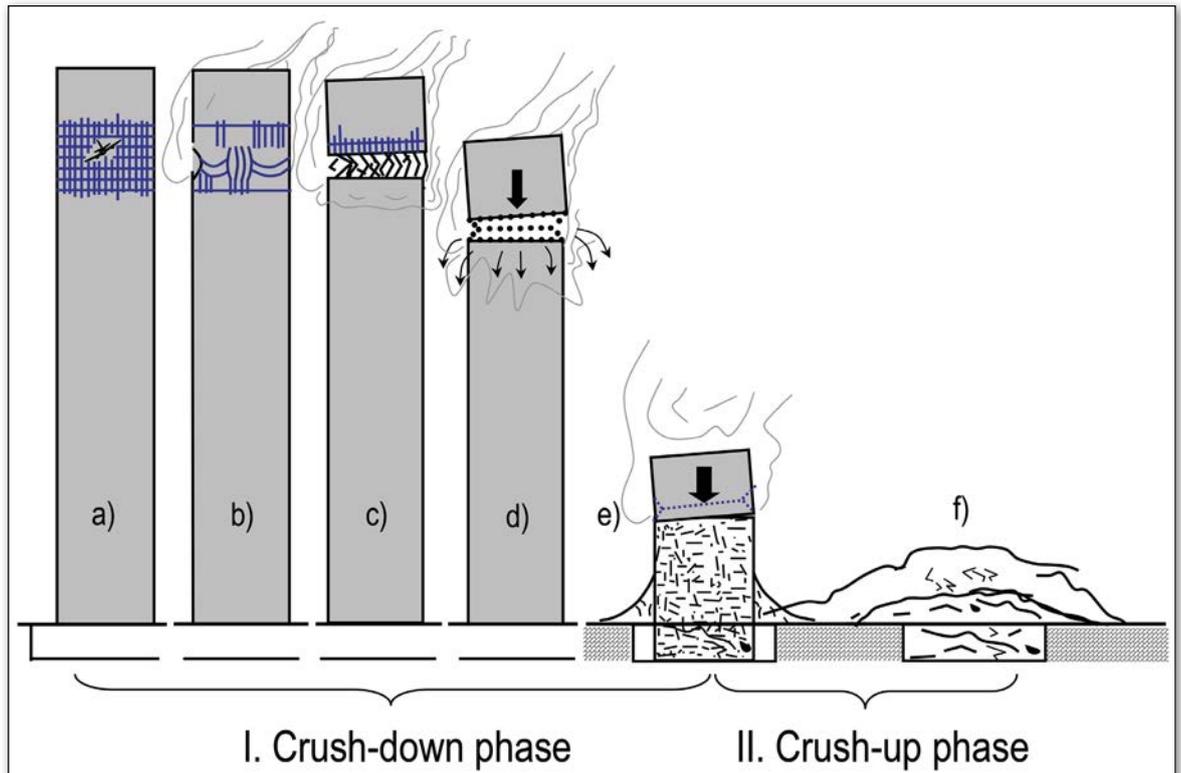
During the progressive collapse of subsequent stories, the velocity of the crush front and the kinetic energy excess grew rapidly. The duration of fall up to the moment at which the crushed part of tower hit the ground was calculated to be 12.81 s for the North tower, and 10.47 s for the South tower [5]. These durations agreed with the seismic record at Columbia University and were about 60% longer than the duration of free fall. After making corrections for the small tilt of the falling top part, it was also demonstrated that the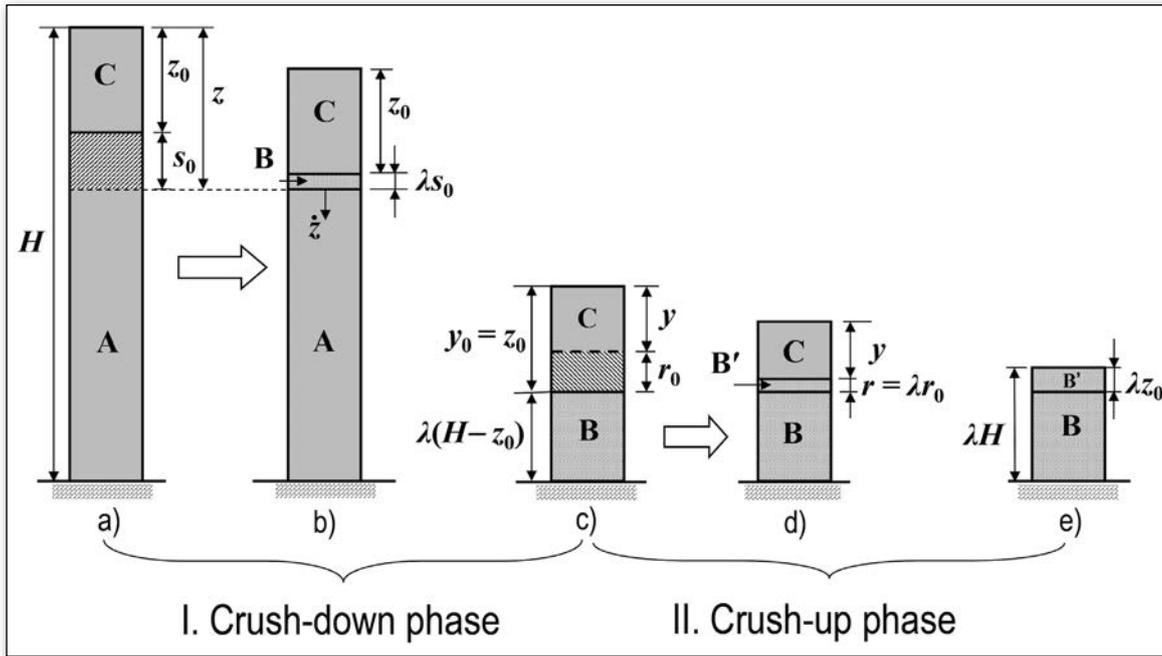 calculated motion closely agreed with the video record of the motion of the topmost corner of the tower top during the first 2 or 3 seconds, before the top disappeared in smoke.

Recently, an objection was raised on the basis of reduced scale buckling tests of aluminum columns reported in [8], which indicated that the total energy dissipation during near-180° rotation in the aluminum plastic hinge was about the triple of the value calculated from the standard beam-theory expression for the yield moment at small rotations. However, in the interpretation of these experimental results, three crucial differences were overlooked (in detail, see [9]):

1. The ends of the tested columns were allowed to rotate freely about the edge of contact with the loading platens, while in WTC the column ends were elastically restrained by the adjacent structure.

2. The photo of the buckled test column revealed that the extensions of aluminum flanges exceeded 100% without any ruptures. However, the structural steel in WTC columns had a limited ductility, maximally 35% [10]. So, at larger deflections, the actual steel columns must have fractured and thus lost their resistance. That such fractures did occur is verified by photos of flying column segments.

3. The energy dissipation measured in these tests corresponded to the relative rotation of nearly 180° at the mid-height plastic hinge. But the rotation at the ends of WTC steel columns could not exceed 90° (see the sketch in [3]) and probably was much smaller because the end joints were also rotating. It follows that, even if the fracturing of steel before reaching the maximum possible rotation is discounted, the energy dissipation in each WTC column must have been still even smaller, by far, than the value estimated in [8].

▶ **FIG. 1:** Schematic of different stages of the collapse process.

Critics now come up with the assertion that the initial kinetic energy excess must have been much smaller than the value calculated from the free fall over the height of one story. This assertion is incorrect because of the aforementioned aggravating factors, and especially for three reasons:

1. The columns of that floor were hot and partially broken, and so their resistance could not have been significant.
2. Many of these columns were destroyed already during the aircraft impact and explosion.
3. The excess of energy was likely much greater than the value calculated for the fall through one story height because the aircraft impact and subsequent fire afflicted three stories simultaneously.

Therefore, the collapse most likely began by a fall of the top part through the height of two to three floors. Indeed, photographs show perimeter columns with a lateral deflection over three floors exceeding 1 m [1]. This also indicates that at least some of the truss girders supporting the concrete floor slabs were detached from column joints before the collapse began. Thus the resistance against the fall of the top part was diminished further.

In the lower floors, one story got squashed within mere 0.07 s. Calculations showed that the air ejected from the story must have reached the velocity of sound, *i.e.*, Mach 1. So the sonic booms heard, the rapidly expanding dust clouds and the wide ejection of debris are no surprise (while some critics erroneously claimed that the booms could have been caused only by explosives). The size distribution of concrete particles, calculated from the energy of impact on floor slabs [5], matches the distribution of the particle sizes seen on the ground, which ranged from 0.01 mm up.

The critics claimed that such small particles could be produced only by explosives. Yet the experience from mining and tunneling (*e.g.*, [11, 12]) shows that such small particles

could be produced by explosives only if the alleged authors installed about 150 tons of TNT into small holes drilled into all concrete floor slabs of each tower. The critics do not explain how such a massive operation, requiring many workers, could have been carried out in secrecy, no one leaking it later to the public. Many workers would have been needed even for the usual demolition by explosives installed on the columns of one story, and, to match the collapse progression as seen, the aircraft would have had to impact each tower just above the story wired by explosives.

Succinct description of the mathematical model

In [4], two phases of collapse were distinguished and their differential equations were derived. In the first phase, called crush-down, the stories are getting crushed at the lower margin of the crushed zone (see Fig. 1a-e and Fig. 2a-c). A simplified model treats each tower as a one-dimensional continuum of mass density $\mu(z)$ where z is the vertical coordinate measured down from the tower top in the original state. The upper part of tower, of height z_0 (labeled as C in Fig. 2), begins to fall at time t_0 and then progressively crushes the underlying stories. The coordinate of the crushing front at time t is denoted as $z(t)$, while $z(t_0) = z_0$.

In the crushed zone, the material is compressed to mass density μ_c (per unit height). The density ratio $\lambda = (1 - \kappa_{out})\mu/\mu_c$ is equal to the ratio of the height of compressed material to the original height (κ_{out} = mass shedding fraction = fraction of the mass that is ejected outside the tower perimeter before the end of crush-down phase). The crushing process may be idealized as fully localized into the crushing front, which is moving down at velocity $\dot{z}$. During time interval dt, a layer of original height $\dot{z}dt$ gets compressed to height $\lambda\dot{z}dt$ and so the rigid compressed
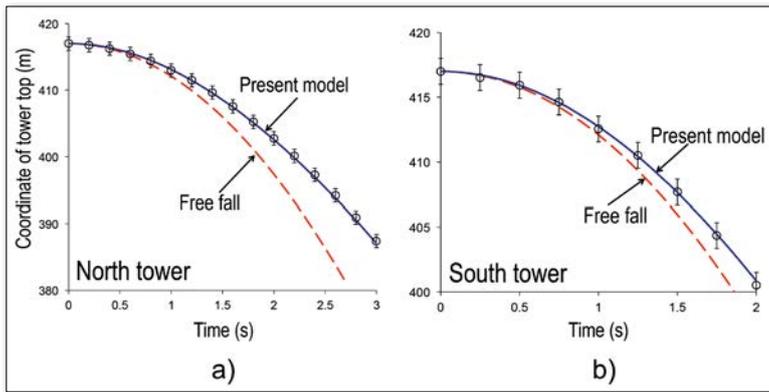
▲ **FIG. 3:** Comparison between the predicted motion of tower top and the video record: a) North tower and b) South tower.

block above this layer moves vertically by $(1-\lambda)\dot{z}dt$. Therefore, the downward velocity of this block is $(1-\lambda)\dot{z}$.

The mass of the part of tower above the crushing front can be expressed as $m(z)=\int_0^z \mu(\zeta)d\zeta$. The momentum of this part is $(1-\lambda)m(z)\dot{z}$. The advance of the crushing front is resisted by force F_c needed for the destructive process below. At the same time, gravitational force $m(z)g$ acts on the tower part above the crushing front. In this manner, Bažant and Verdure [4] obtained the following differential equation:

$$\frac{d}{dt}\left([1-\lambda(z)]m(z)\frac{dz}{dt}\right)-m(z)g = -F_c(z,\dot{z}) \qquad (1)$$

A slightly refined equation, with variable distribution of λ over the crushed zone, was used by Bažant *et al.* [5]. It should be pointed out that Eq. 1 can also be derived rigorously from an extended Lagrangian formulation for dynamic systems with a moving mass varying as a function its spatial coordinate rather than the time [13].

At the beginning, the front of crushing is at level z_0 and its velocity is $\dot{z}_0$. The crushing process will begin if $m(z_0)g>F_c(z_0, 0)$. This condition, of course, is not satisfied in the undamaged state of tower, and not even immediately after the aircraft impact. As a consequence of the fire, the resisting force F_c in the afflicted three floors gets gradually reduced. The contributions to this force are expressed by the sum $F_c = F_b + F_s + F_a + F_e$ where F_b is the force needed for buckling and fracture of steel columns, F_s is the force needed to provide the work of comminution of concrete floor slabs into small particles, F_a is the force needed for fast ejection of air as the floor is getting squashed, and F_e is the force needed for lateral ejection of some of the

▼ **FIG. 4:** Calculated motion of tower top for the entire collapse process: a) North tower and b) South tower.
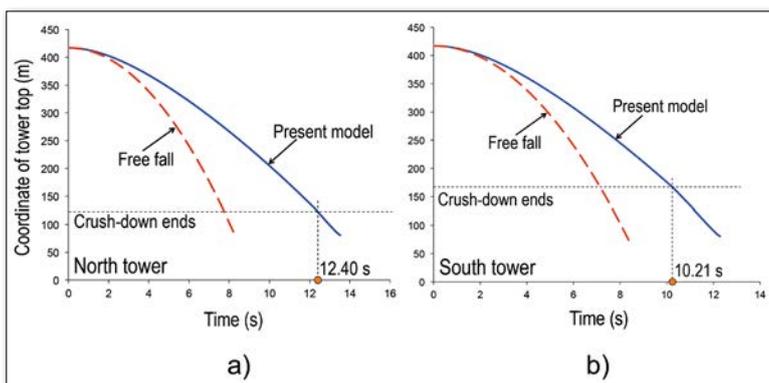
crushed debris. In the present continuum approximation, all the forces should be understood as averaged quantities which, when multiplied by the story height, are equal to the dissipated energy per unit height. Eq. 1 also indicates that, in addition to the aforementioned resisting forces, another major resistance is derived from the fact that the accreted mass at the crush front must be accelerated from rest to the velocity of the top part.

In [4], the analysis considered only force F_b, which is dominant in comparison with the resisting forces due to concrete comminution and air and mass ejection. In view of the order of magnitude difference between the energies of motion and of resistance, the analysis in [4] sufficed to reconfirm the inevitability of spontaneous progressive collapse shown in [3], to elucidate the role of various parameters and to show that spontaneous collapse had to occur for a broad range of input data. However, to clarify and match various observations, such as the duration of collapse, particle size distribution of comminuted concrete, video record of initial motion, speed of air jetting out, *etc.*, a more refined analysis was necessary [5].

At the moment the crush front hit the ground, the second phase of collapse, called crush-up, got under way. Here, as the top part continued to fall, the crush front propagated upward (Fig. 2c-e). Under the same simplifying assumptions as mentioned before, the following crush-up differential equations was derived [4]:

$$m(y)\left[\frac{d}{dt}\left([1-\lambda(y)]\frac{dy}{dt}\right)+g\right] = -F_c(y,\dot{y}) \qquad (2)$$

Here y is the vertical coordinate of the upper boundary of the crushed zone measured from the top of tower in the initial undamaged state (Fig. 2d) [3, 4]. Based on the difference between the crush-down and crush-up motions, it has been shown that both phases of collapse could not proceed simultaneously [5].

Eqs. 1 and 2 have been solved numerically using the 4th-order Runge-Kutta method. The same input values were used as in the previous analysis [5], except that, based on recent information, the reduction factor β of the column resisting force was increased from 2/3 to 0.73 for normal-strength columns, and from 0.24 to 0.26 for high-strength columns (here $F_b = \beta F_p$, where $F_p =$ column resisting force calculated based on standard three-hinge column buckling analysis [3,5]). The solution led to graphs describing the time evolution of motion of the topmost rim corners of the North and South towers. Fig. 3a-b compares the predicted motion with the available video record. Good agreement is found. Fig. 4a-b shows the calculated motion of the tower top during the entire collapse. The model (Eq. 1) predicted the durations of the crush-down phase for the North and South towers to be 12.40 s and 10.21 s, respectively. These predictions matched well the observed crush-down durations based on the seismic record (12.59 ± 0.5 s for the North tower and 10.05 ± 0.5 s for

the South tower) [5]. These analyses clearly demonstrated that a mechanism of spontaneous collapse driven purely by gravity must inevitably have developed.

Spontaneity of the collapse of WTC7: Explicable in retrospect

Lay critics also questioned the collapse of the WTC tall Building 7. Unlike the North and South WTC towers, Building 7 did not suffer from airplane impact. NIST's detailed analysis [14] of the reasons for collapse is worth summarizing here.

The burning debris ejected from the North and South towers hit Building 7 and ignited fires on multiple stories simultaneously. Meanwhile, the collapses of the North and South towers also damaged the city's water main. As a result, the sprinkler system on the lower stories (below 20th story) of Building 7 was not functional. The uncontrolled fire engulfing many floors caused a large thermal expansion of floor girders. Calculations showed that this expansion, occurring simultaneously on many floors, caused failures of the bolts attaching the ends of girders to the supporting beams.

Because of the loss of support by the floor systems, perimeter columns buckled over multiple stories [14] and caused the overlying floor systems to fail. The chain of structural failures that initiated the overall collapse of Building 7 was computationally simulated by a high-fidelity finite element code at NIST [14]. In Building 7, the collapse front propagated not only vertically but also laterally. In that case, the simple one-dimensional model of overall collapse formulated for the twin towers is insufficient.

In summary, the initiation of collapse of WTC Building 7 is explained by the uncontrolled fires burning for a long time simultaneously on many stories. These fires first undermined the floor systems, which led to progressive loss of columns and thus initiated a gravity driven total collapse.

Conclusion

From the viewpoint of physics, and structural mechanics in particular, it is perfectly clear that no WTC demolition took place. The collapse was triggered by an atypical fire ignited simultaneously in a large volume. It was driven by gravity, and was spontaneous. In hindsight, it was, under the given circumstances, inevitable.

For further information click 'WTC collapse' on the website http://www.civil.northwestern.edu/people/bazant, and download the articles [2, 3, 4, 5, 6] as well as various discussions with replies.

Editors note

This manuscript represents a response to the view presented in the article on the WTC collapse, published in EPN 47/4. In line with the new criteria for publication in EPN as outlined on the EPN webpage and in EPN 47/5&6, it was decided to publish this manuscript in order to conclude the discussion on this subject.

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