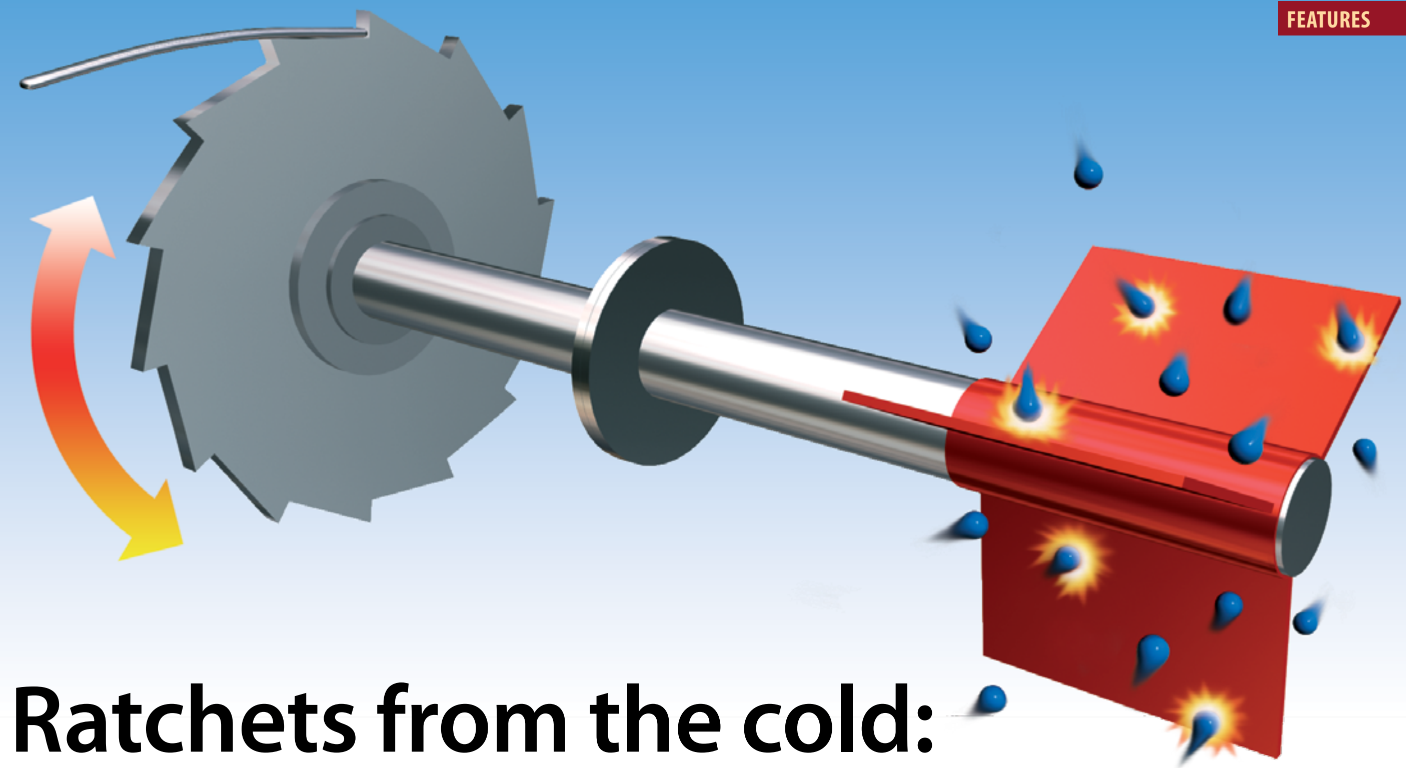

Ratchets from the cold:

Brownian motors with cold atoms in optical lattices

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Brownian motors, or ratchets, are devices which 'rectify' Brownian motion, that is, they can generate a current of particles out of unbiased fluctuations. The present article reviews recent experimental realizations of ratchets employing cold atoms in optical lattices. This is quite an unusual system for a Brownian motor as there is not a real thermal bath, and both the periodic potential for the atoms and the fluctuations are determined by laser fields. Such a system has allowed us to explore a number of fundamental features of ratchets.

▲ Artist impression of the ratchet. © Antoine Tisserand

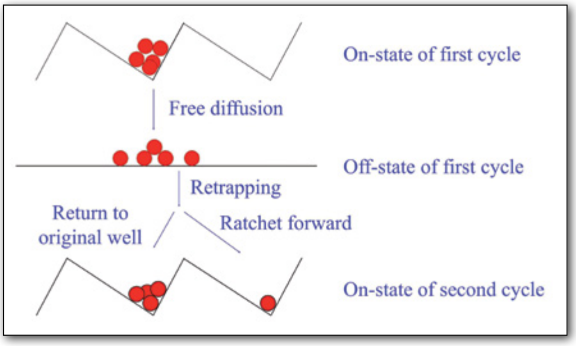
Brownian motors, or ratchets, are devices which "rectify" fluctuations, turning unbiased Brownian motion into directed diffusion, in the absence of any net applied bias forces [1]. These unusual devices have been attracting growing attention in different communities involving a number of applications: including, for example, particle separation, the modelling of molecular motors, and the realization of novel types of electron pumps, just to name a few.

A Brownian motor generates a current from unbiased fluctuations. Strict limitations on the operation of a ratchet are imposed by the second law of thermodynamics, which rules out the possibility of producing a current at thermodynamic equilibrium [2]. Thus, the effective generation of a current requires the system to be driven away from equilibrium. How this can be implemented is best demonstrated by considering two specific examples of ratchet devices: the flashing and the rocking ratchets.

The flashing ratchet

Consider a sample of Brownian particles in a (static) asymmetric periodic potential. The second principle of thermodynamics rules out the possibility of directed motion. However, things are very different if the potential is "flashed", *i.e.* if it is turned on and off repeatedly, either periodically or randomly [3]. This is sufficient to set the Brownian particles into directed motion, due to the mechanism illustrated in Fig. 1.

Consider an initial situation with the potential turned on and the Brownian particles localized at the bottom of a given well. Then the potential is turned off, and the Brownian particles will symmetrically diffuse in space. Then the potential is turned on again, and the Brownian particles are retrapped in both the original well and in a few neighbouring ones. However, as the potential is asymmetric the retrapping will lead to an asymmetric situation, with the number of particles trapped in the



▸ **FIG. 1:** Working principle of the flashing ratchet

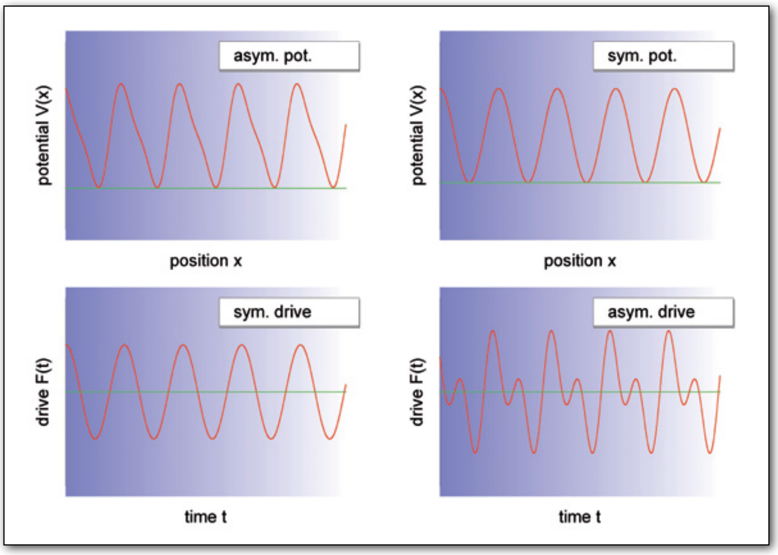
wells to the left of the original well different from the number of particles trapped in the wells to the right of the starting location. Indeed it is clear from Fig. 1 that the wells located at the right of the starting well will collect more particles during the retrapping phase. This because these wells are closer to the "steep wall" of the original well, therefore the atoms have to travel less to be retrapped there. In this way the centre of mass of the particle cloud will move, and directed motion is thus obtained.

It is important to point out why the operation of the flashing ratchet does not violate the second law of thermodynamics. This is because work is done on the system while turning on the potential. Thus, although fluctuations are rectified and a current is generated, this does not imply that work has been extracted out of just one heat source as some additional work was necessary to turn on the potential. Therefore the second law is not broken.

The rocking ratchet

In the *rocking* ratchet [4], particles in a periodic asymmetric potential also experience an applied ac (alternating) force. The applied force, which is zero-average and time-symmetric, drives the system out of equilibrium (see Fig. 2, left column). As a result of the asymmetry of the potential, a net current of particles can be generated.

▾ **FIG. 2:** Different choices of potential V(x) and ac drive F(t) for the realization of a rocking ratchet. In the left column, V(x) is spatially asymmetric and F(t) temporally symmetric. In the right column, V(x) is spatially symmetric and F(t) temporally asymmetric.



The same effect can be obtained for a spatially symmetric potential and a temporally asymmetric drive (see Fig. 2, right column). A bi-harmonic force is a popular choice for a time-asymmetric drive, with the time-symmetry of the drive controlled by the relative phase between harmonics (details in Box).

In general, the operation of a ratchet requires two main elements: an out-of-equilibrium set-up, so as to overcome the limitations imposed by the second law of thermodynamics, and the breaking of the symmetries which would otherwise prevent directed motion. Box 1 reviews the symmetry analysis for the specific case of ac driven ratchets [5].

Rocking ratchets for cold atoms

Optical lattices are periodic potentials for atoms created by the interference of two or more laser fields [6]. In dissipative optical lattices a set of laser fields – see Fig. 3 - produce at once the periodic potential acting on the atoms and a dissipative friction mechanism. In these lattices the depth of the optical potential can be varied at will by properly choosing the laser parameters. Likewise it is possible to vary the damping rate of the atomic velocity, by varying the scattering rate of the photons. The possibility of varying the damping rate is essential to investigate the phenomenon of dissipation-induced symmetry breaking which will be described in the following.

Dissipative optical lattices thus offer two essential elements for the realization of rocking ratchets: the periodic potential, and a fluctuating environment which results in a friction and in a fluctuating force. The last element necessary to implement a rocking ratchet is the oscillating force. This is easily achieved, as an arbitrary time-dependent homogeneous force can be generated by phase-modulating one of the lattice beams.

Experimental demonstration

The first rocking ratchet for cold atoms was experimentally demonstrated in 2003 [7], using a spatially *symmetric* optical lattice and a bi-harmonic drive. The dynamics of rubidium atoms in the optical lattice was studied by direct imaging of the atomic cloud with a CCD camera. It was found that the center of mass of the atomic cloud can be set into motion with constant velocity despite the fact that the oscillating force has zero average. Furthermore, the velocity showed the expected sin(φ) dependence on the phase φ consistent with the symmetry analysis for a dissipationless system (see Box 1). This because this experiment was performed in the regime of relatively strong driving and small damping, which approximates well the dissipationless regime.

A similar set-up to the one described above was used in a subsequent experiment [8] to provide the first experimental evidence for dissipation-induced symmetry breaking in a rocking ratchet.

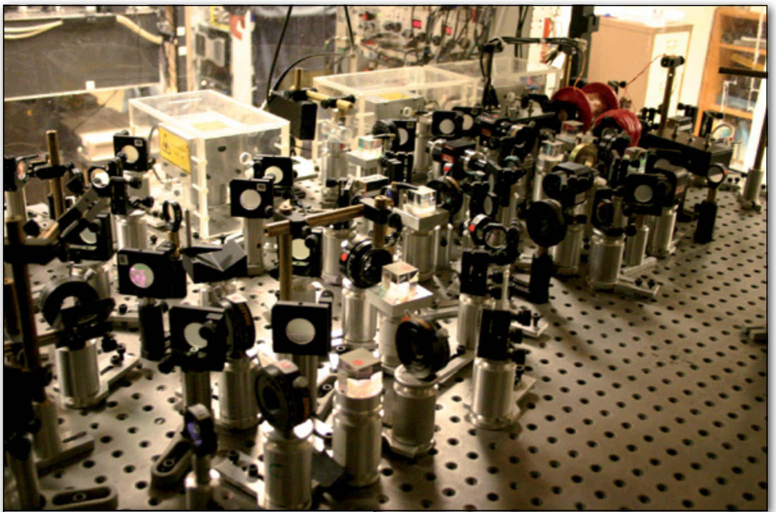
Different sets of measurements were performed for different values of the photon scattering rate, which characterizes quantitatively the level of dissipation in this system. The measured current of atoms (see Fig. 4) was found to be well approximated by $\sin(\varphi - \varphi_0)$. The measured phase shift φ_0 is zero, within the experimental error, for the smallest scattering rate examined in the experiment. In this case, no current is generated for $\varphi = n\pi$, with n integer, as for these values of the phase the system is invariant under time-reversal transformation. The magnitude of the phase shift φ_0 increases at increasing scattering rate, and differs significantly from zero. The nonzero phase shift corresponds to current generation for $\varphi = n\pi$, *i.e.* when the system Hamiltonian is invariant under the time-reversal transformation. This result demonstrates the predicted (Box) breaking of the system symmetry by dissipation.

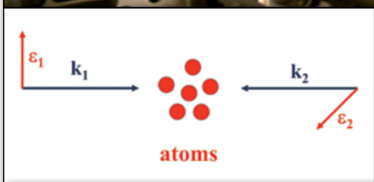
This demonstrated tunability of cold atom systems allows one to go beyond the basic ratchet corresponding to a bi-harmonic rocking force. Indeed, gating ratchets and quasiperiodically driven ratchets have also been demonstrated.

In the gating ratchet [9], particles experience an oscillating potential which is spatially symmetric. A zero-average and time-symmetric ac force is also applied. A current can be generated following a gating effect, with the lowering of the potential barriers synchronized with the motion produced by the additive force. This mechanism has to be contrasted with the previously discussed ac-driven ratchets with additive bi-harmonic driving, in which the underlying mechanism is harmonic mixing, where a nonlinear medium (the periodic potential) mixes the two harmonics producing a current. A gating ratchet for cold atoms was demonstrated experimentally by [10]. The ratchet was realized with cold atoms in a driven dissipative optical lattice. A single-harmonic periodic modulation of the potential depth was applied, together with a single harmonic rocking force. Whenever the relative phase between the modulation and the rocking force was such to break the relevant symmetries, directed motion was observed.

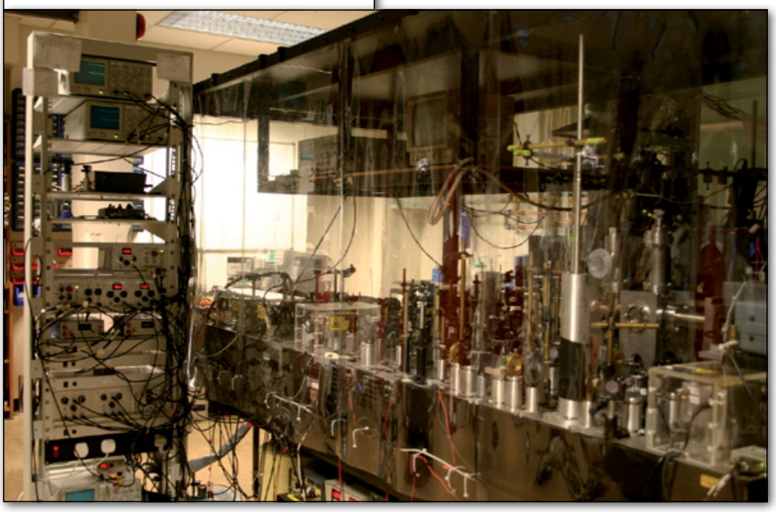
In the case of quasiperiodically driven ratchet [11], the interesting issue is how the symmetries, and the resulting current generation, are modified following the transition from periodicity to quasiperiodicity. Consider a generic driving force with two frequencies ω_1, ω_2. Quasiperiodic driving corresponds to an irrational value of the ratio ω_1 / ω_2. It can be shown that in the case of a quasiperiodic driving, the two phases $\omega_1 t, \omega_2 t$ can be treated as *independent* variables. In other words, there is an effective change in the dimensionality of the system in the transition from periodicity to quasiperiodicity. The symmetry of the system, and the corresponding conditions to observe a ratchet current, change accordingly. Clearly, in a real experiment ω_1 / ω_2 is always a rational number, which can be written as $\omega_1 / \omega_2 = p/q$, with p,q two coprime positive integers. However, as the duration of the experiment is finite, by choosing p and q sufficiently large it is possible to obtain a driving which is effectively quasiperiodic on the time scale of the experiment. Thus, in the experiment of Ref. [11], the transition from periodicity to quasiperiodicity was studied by increasing p,q, so to make the driving effectively more and more quasiperiodic. The measurements across the transition revealed a qualitative change in the conditions on the driving force parameters for which a ratchet current was observed. Such a change precisely corresponded to a change in the symmetry of the system. This observed change in the system symmetry

“ The first rocking ratchet for cold atoms was experimentally demonstrated in 2003 ”





▲ **FIG. 3:** (middle panel) Arrangement of laser fields which produce a 1D dissipative optical lattice, with blue arrows indicating the laser beams' directions and the red ones their polarizations; (up) close-up view of the laser set-up; (down) overall view of the experimental set-up.



was found to be consistent with the expected change in the effective dimensionality of the system, in agreement with the fact that the phases $\omega_1 t$, $\omega_2 t$ are effectively independent variables in the quasiperiodic limit.

Conclusions

This article reviewed recent experimental realizations of ac driven ratchets with cold atoms in optical lattices. Such a system allowed us to demonstrate several different ratchet schemes, and to investigate their fundamental properties. In particular, rocking and gating ratchets were demonstrated, and the transition from periodicity to quasiperiodicy was investigated in one of such systems. Although not discussed here, two-dimensional rocking ratchets for cold atoms have also been demonstrated [12], and new higher-dimensional rectification mechanisms revealed. This should also pave the way to the study of ratchet control of vorticity, a feature only present in higher dimensional systems. ■

Acknowledgements

The author thanks EPSRC, The Royal Society and the Leverhulme Trust for funding.

THE ROLE OF THE SYMMETRIES IN AC DRIVEN RATCHETS

Consider a Brownian particle in a spatially periodic potential. A time-dependent force F, of zero mean, is applied to the particle. The aim of the symmetry analysis is to determine the conditions for the equation of motion to be invariant under the transformations which map a trajectory into one with opposite momentum. Clearly, if the equation of motion is invariant under one of such transformations, trajectories with opposite momentum are equivalent and directed motion is not possible. To proceed further, one has to specify the general form of the potential and of the driving. We consider the case of a spatially symmetric periodic potential, as this is the case relevant to the experimental realizations reviewed in this work, with the symmetry of the system controlled by the ac driving. Specifically, we examine the case of a bi-harmonic driving force $F = A\cos(\omega t) + B\cos(2\omega t + \varphi)$. We first consider the dissipationless case, in which there is no friction mechanism which damps the particles' motion, which will then be extended to include weak dissipation. The relevant symmetry here is the time-reversal symmetry. For the case of a bi-harmonic drive, and neglecting dissipation, whether the time-reversal symmetry is broken depends on the value of the phase φ: for $\varphi = n\pi$, with n integer, the time-reversal symmetry is preserved, while for $\varphi \neq n\pi$ it is broken. Therefore for $\varphi = n\pi$ current generation is forbidden, while for $\varphi \neq n\pi$ it is allowed. Perturbative calculations show that the average current of particles is, in leading order, proportional to $\sin(\varphi)$, in agreement with the above symmetry considerations.
In the case of dissipative systems, the time-reversal symmetry is broken by dissipation. Therefore also for $\varphi \neq n\pi$ the generation of a current is not prevented, despite the symmetry of the driving. The generated current I still shows an approximately sinusoidal dependence on the phase φ, but acquires a phase lag φ_0: $I \sim \sin(\varphi - \varphi_0)$. Such a phase lag is the signature of dissipation-induced symmetry breaking.

About the author

Ferruccio Renzoni studied physics at the University of Pisa, Italy (M.Sc. 1993) and at the Technische Universität Graz (PhD 1998). He then spent two years at the University of Hamburg and three at Ecole Normale in Paris, where in 2003 he obtained his "Habilitation à diriger des recherches". Since 2003 he is at University College London, where he is a Professor of Physics.

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▼**FIG. 4:** Experimental results for the average atomic velocity in units of the recoil velocity v_r (equal to 3.52 mm/s in the present case), as a function of the phase φ between harmonics of the drive. Different data sets correspond to different scattering rates. The data are labelled by a quantity proportional to the scattering rate, reported in the bottom part.