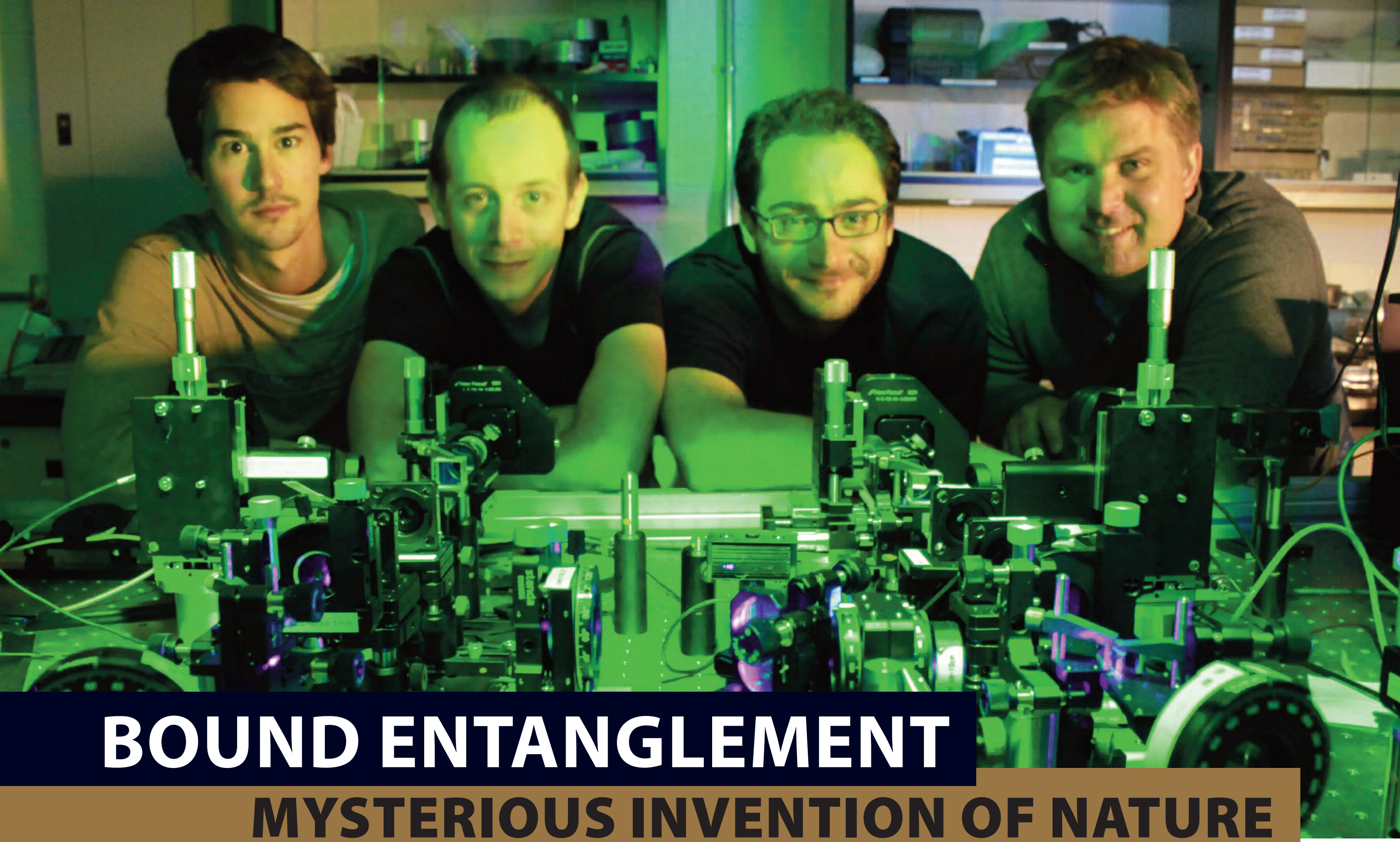

BOUND ENTANGLEMENT
MYSTERIOUS INVENTION OF NATURE

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When in 1998 the team of researchers of the Institute of Theoretical Physics and Astrophysics of the University of Gdańsk predicted the existence of bound entanglement, there was only one more thing to do: to believe that sooner or later this peculiar phenomenon would be confirmed empirically in laboratory conditions.

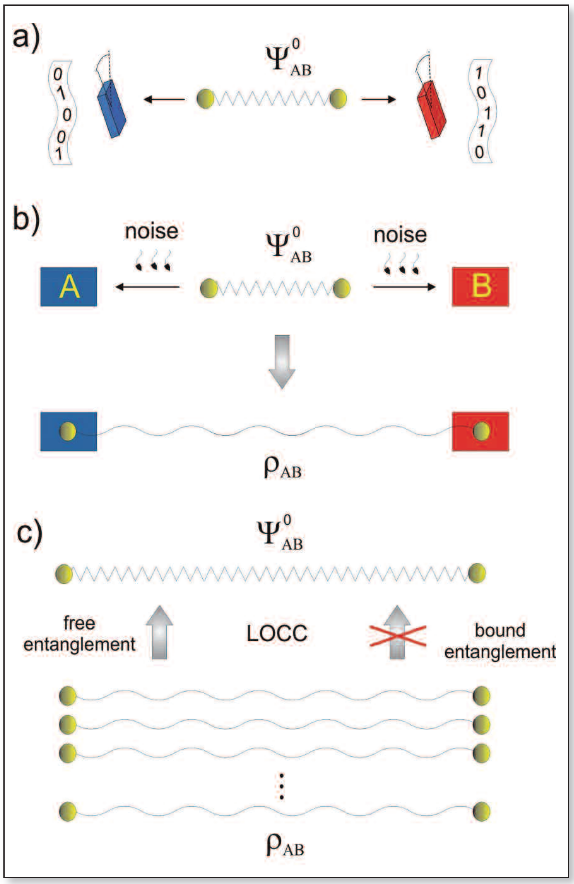
Eleven years later, on 23rd August 2009, *Nature Physics* [1] reported that the researchers of a laboratory in Stockholm had created a four-qubit bound entangled state. Another eight months later, on 30th April, another team from the laboratory in Dortmund reported the creation of pseudo-bound entanglement [2]. When that paper was in preparation, a team of researchers from Waterloo (see Fig.1) announced [3] that they had managed to experimentally create a genuine bound entangled state. At the same time the Innsbruck group presented an experiment implementing bound entanglement with ions [4]. The exciting course of these events already shows that experimental, unrecoverable imprisoning of quantum correlations, even in the simplest (four-qubit) arrangement, is no small challenge.

At the current stage, full understanding of bound entanglement – "a mysterious invention of nature" [5] – may be no more than an illusion. Nevertheless, it is interesting to follow the paths that led to that invention in a foggy bush of quantum formalism, in order properly to appreciate the efforts and inventiveness of experimentalists.

On the tracks of bound entanglement

Entanglement is "the characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought" [6]. Indeed, let us look for instance at the singlet state $|\psi^0_{AB}\rangle = 1/\sqrt{2}\,|0\rangle|1\rangle - 1/\sqrt{2}\,|1\rangle|0\rangle$ of a pair of polarization-entangled photons (the symbols 0 and 1 designate correspondingly the vertical (V) and horizontal (H) polarizations). It is a pure state, *i.e.*, it represents our maximum available knowledge about the arrangement of two polarization-entangled photons. As we know, the quantum formalism allows the existence of mixed states, which represent incomplete knowledge. The paradox consists of the fact that the whole system is in the pure state while its subsystems are in the maximum mixed states. In other words, if partners in two distant laboratories measure the polarisations of both photons using identically positioned polarisors, they will always receive opposite polarizations (Fig. 2a). Moreover, the results of these measurements cannot be known to anyone prior to measurement (in ▪▪▪

▲ **FIG. 1: Waterloo experiment team. Left to right Jonathan Lavoie, Rainer Kaltenbaek, Marco Piani, Kevin Resch**



▶ **FIG. 2:** **(a)** Pure entanglement produces perfect correlations. **(b)** The environment pollutes pure entanglement. **(c)** For distillable states, the reverse process is possible, while from bound entangled states one cannot obtain the pure entanglement back.

contrast to common, classical correlations, *e.g.*, momenta of slivers of an exploded projectile). On the other hand, the polarizations of photons independently measured by each partner are completely chaotic. Thus there is complete chaos in both subsystems, while the whole system is in perfect order. This unusual property of the state does not correspond to any situation in the classic world [7].

In an ideal situation, when quantum objects are entangled to the maximum extent, we can determine the state of one of them with certainty by measuring the state of the other. This, however, is only true for pure states which are not correlated with the rest of the world, that is, when we have "fuel" of the highest quality. In real situations, however, a system of entangled particles always interacts with its environment, which results in a situation of noised entanglement. This in turn results in a weakened quantum correlation. Formally, this means that the system of entangled particles passes into a mixed state ρ_{AB} (see Fig 2b), which can be interpreted as some information about system having leaked into the environment, resulting in degradation of entanglement. Thus, quite naturally, the following fundamental questions arise: a) How can we verify theoretically whether a given state (described on a sheet of paper) is entangled? b) Is direct detection of entanglement possible in a laboratory? c) Is it possible to reverse the process of degradation of quantum entanglement somehow?

Research into these issues was independently undertaken in the 1990s in several research centres [7]. While working on the first two issues in Gdańsk, we were astonished by the work of Asher Peres [8], who proposed an extremely strong criterion of entanglement, based on the so-called operation of partial transposition. Such an operation is carried out on one (A or B) subsystem of the state of a compound AB system. If the state subject to such an operation does not "survive" it, in the sense that it ceases to be positive, losing its probabilistic interpretation, it means that it was entangled. Mathematically speaking it means that its partially transposed density matrix has at least one negative eigenvalue.

Inspired by Peres' result we managed to prove [9] that a two-system state ρ_{AB} is entangled if and only if there is such an observable quantity W, that its average value in this state is negative, while it is always non-negative for all un-entangled states. This was the cornerstone of the method of detection of entanglement in laboratory conditions developed later, based on witnesses of entanglement [7]. While preparing a certain state in a laboratory we can always find a witness of entanglement corresponding to it, and then, by measuring it, verify whether or not the prepared state was entangled. The ceaseless popularity of detection of entanglement based on the method of witness of entanglement results from its simplicity and economy. To determine the presence of entanglement it suffices to measure only one or two observables (*e.g.*, the value of the spin or the polarization).

In spite of that, we were not completely satisfied. We did not know what happens when the system "survives" the operation of partial transposition. From the work of Peres it was possible to infer that if the state was un-entangled, then after such an operation it would still be un-entangled. But are there states in nature which are entangled and nevertheless have a positive partial transposition? Fortunately we managed to prove the existence of those strange states [9]. Our work has raised considerable interest among mathematicians, who considered related issues, expressed in a different language, as long ago as the 1970s. Of course physicists immediately expressed their wish to know what such states might look like.

Anna Sanpera (Barcelona) turned to Peres with this question, and he redirected it to our team. One of us (P. Horodecki) constructed the first entangled states with positive partial transposition [10]. The result raised interest at a conference in Torino (Italy) (1997), but in view of the lack of physical reference, the existence of such states was perceived as a mathematical peculiarity only. It would have remained such a peculiarity, if on the other side of the Atlantic, Bennett and his collaborators had not worked on the issue of (c): How to reverse the process of degradation of entanglement.

Their work [11] , published in 1996, had a key role to play in the theory of manipulation of entanglement. They introduced a natural class of operations of manipulation of entanglement by experimentalists located in remote laboratories. The experimentalists can only carry out any local operations on their entanglend particles and communicate via a classical communication such as a telephone (local operations and classic communication (LOCC)). On the basis of this concept they proposed a protocol of *distillation* of noisy entanglement: two partners in distant laboratories share n copies of the ρ_{AB} state, which contains noisy entanglement. Carrying out in their laboratories local operations on the subsystems of the shared state and using classic communications channels (*e.g.* a telephone) they can obtain the singlet state ψ^0_{AB}, which contains pure entanglement (Fig.2b). We then became interested in finding the answer to the question: Has the process of entanglement distillation a universal character? In other words: Can all the states in nature be distilled thanks to the application of the LOCC operations?

We managed quite quickly to demonstrate that all noisy entangled states of systems made of two qubits can be distilled. The results were very promising and there was a widespread belief that its generalisation would remain only a question of time. The idea that a purely mathematical operation of partial transposition without a clear physical interpretation could have anything in common with the "physical" protocol of distillation seemed absurd. However, when we considered more complicated systems, we obtained surprising results. Paradoxically, nature subject to a purely mathematical treatment revealed the physical peculiarity of entanglement: namely, that the environment may "pollute" the pure entanglement in such a way that it will not be possible to cleanse it with the help of LOCC (Fig.2c). Our conclusion was laconic: entangled states with positive partial transposition are LOCC undistillable [12]. This became known under the name of "bound entanglement".

This result astonished the physicists. It transpired that the structure of entanglement is not uniform! In nature there are at least two types of noisy entanglement: free, *i.e.* distillable entanglement and bound entanglement, which cannot be distilled with the help of the LOCC. In this way nature reveals the existence of a new type of quantum irreversibility which appears during the manipulation of entanglement. Namely to create bound entangled states by means of only local operations and classical communication, one has to have initially a certain amount of pure entanglement. However, using these operations, one cannot extract this pure entanglement back from the state.

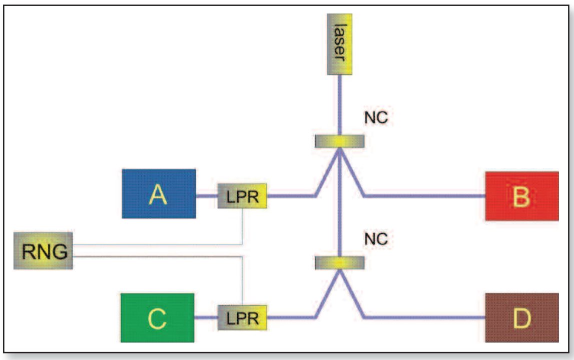
Bound entanglement in a laboratory

Since 1997 a number of states with bound entanglement have been constructed. One of the simplest and most elegant examples is Smolin's four-qubit state [13], which is an equal mixture of four Bell's states $|\psi^{0(1)}_{AB}\rangle = 1/\sqrt{2}\,|0\rangle|1\rangle \pm 1/\sqrt{2}\,|1\rangle|0\rangle$ and $|\psi^{2(3)}_{AB}\rangle = 1/\sqrt{2}\,|0\rangle|0\rangle \pm 1/\sqrt{2}\,|1\rangle|1\rangle$:

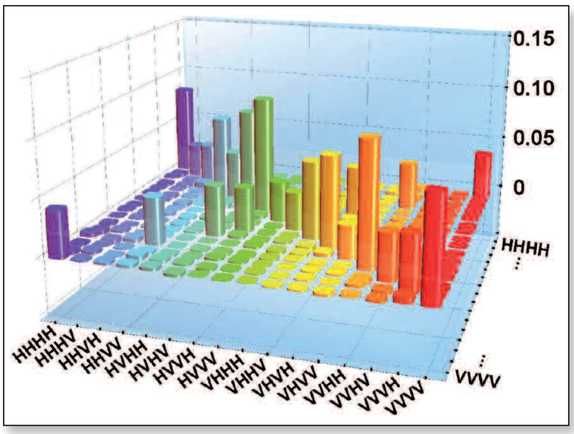
$$\rho_s = \frac{1}{4}\sum_{i=0}^{3}|\psi^i_{AB}\rangle\langle\psi^i_{AB}|\otimes|\psi^i_{CD}\rangle\langle\psi^i_{CD}|. \qquad (1)$$

Physically, the Smolin state can be interpreted in the following way: Partners A and B in distant laboratories share one of the four possible Bell states. Similarly, partners C and D share the same state. Each of those four Bell states appears with the probability of 1/4 and is unknown to the partners. Smolin demonstrated a specific property of this state: if four qubits are far from one another, it is not possible to distil the entanglement between any of them. If, however, only two qubits are in the same laboratory, it is possible to create a pure entanglement in the form of singlets between the two remaining qubits A and B with the help of the LOCC operations. Thus the state of Smolin contains bound entanglement. Its simplicity and high symmetry in relation to the exchange of qubits attracted the attention of experimentalists as an optimal candidate for quantum - optical implementation. In the experiments of the Stockholm and Waterloo teams the physical qubits are represented by polarized photons (see Fig. 3).

Experimentalists can hope that they will be able to create a four photon bound entangled Smolin state (2), but how can they subsequently verify it? As we already know, the state has to meet two criteria, *i.e.* it has to be entangled and not distillable. The first feature may be achieved by measuring an appropriate entanglement witness. If the measurement returns a negative average value,



◄ **FIG. 3:** The Smolin state in the Waterloo experiment was prepared in the following way: two sources, nonlinear crystals (NC This symbol is not used in the Figure) pumped by a single laser, generate two-photon pairs in the state $|\psi^1_{AB}\rangle$ encoded in the polarization of photons, where H and V correspond to the horizontal and vertical polarization, respectively. Then, according to the prescription of Smolin, the initial state $|\psi^1_{AB}\rangle|\psi^1_{CD}\rangle$ should randomly and with equal weight transform into the remaining states of the Bell base and be disseminated among the analysers A, B, C and D symbolising distant laboratories. The authors managed to accomplish this by applying the liquid crystal phase retarders (LPR) in each of the two sources. They were controlled by a computer using a pseudo-random number generator.



◄ **FIG. 4:** Tomographic picture of Smolin's state prepared by the Waterloo team. (The figure has been reprinted from [3]) it means that the prepared state was entangled. Secondly, the state, in order to be nondistillable, has to have a positive partial transposition. This may be checked by making a full tomography of the state (see Fig. 4). It appears that meeting the second criterion is quite difficult in the case of the state suggested by Smolin due to the fact that the property of positive partial transposition is extremely sensitive to imprecision in preparing the state and low data statistics. In the pioneering Stockholm experiment [1] under conditions of low ("natural") noise the minimum proper value was negative *i.e.* -0.02 ± 0.02. In this sense the demonstration of bound entanglement is not fully convincing. The Waterloo team (Fig. 1) then realised that by adding a large (nearly 50%) amount of white noise to the original state of Smolin the bound entanglement became much more robust. In the experiment, the observed average of the witness was -0.159 ± 0.008, while the smallest eigenvalue of the partially transposed state was positive: 0.0069 ± 0.0008.

In parallel with Waterloo experiment an independent realization of bound entanglement was carried out in Innsbruck [4] with ions. Their main advantage over the photonic experiments mentioned in the article is that the state is prepared unconditionally. Namely, in the Stockholm and Waterloo experiment, in gathering the statistics one must exclude some instances, *e.g.* when the photons didn't reach the detectors. Thus, in a sense, the experimenters get to know that they indeed prepared the valid state only after it is destroyed (*i.e.* after the photons are detected). This is called "post-selected" regime, and is common to almost all experiments with single photons. In contrast, in the Innsbruck experiment no such post-selection is applied.

Another ingenious experiment has independently been carried out in Dortmund [2]. A three-qubit pseudo-bound entanglement was created in a liquid using the method of Nuclear Magnetic Resonance. Using similar methods, *i.e.* the tomography and entanglement witnesses, the authors have shown that bound entanglement can exist in a laboratory.

The list of peculiarities and potential applications of bound entanglement is diverse and growing [7]. We already know that "bound entanglement is not a rare phenomenon" [15] since its presence was discovered in thermal spin systems [14,15]. At the same time, we are still far from complete understanding of the phenomenon of bound entanglement, which raises new experimental and theoretical challenges for the future.

► Gdańsk group. Left to right: Ryszard Horodecki, Michał Horodecki and Paweł Horodecki.

Note added

After completing this manuscript, further experiment realising bound entanglement has been announced by the Hannover group [J. DiGuglielmo *et al.*, http://arxiv.org/abs/1006.4651] with light in continuous variable regime. As in the Innsbruck experiment, no post-selection was applied. ■

About the author

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Selected awards:

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- Johannes Hevelius Scientific Award of the City of Gdansk
- Group Wojciech Rubinowicz Award granted by the Polish Physical Society

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