ON SUPERSOLIDITY

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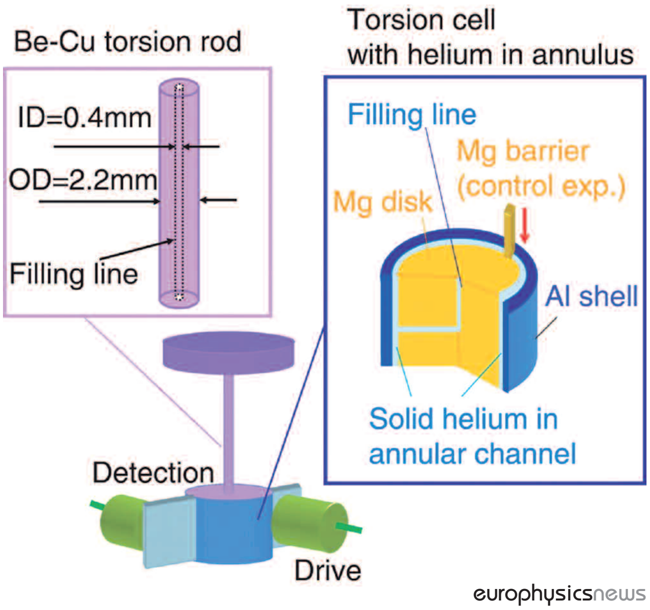
In the last few years a new term has become fashionable, supersolidity and again helium is at the centre of stage. With supersolidity one means a solid that displays some form of superfluidity, for instance that when you put in rotation a container filled with a solid sample, not all of this solid rotates but a part of it is not dragged along by the container and remains stationary with respect to the laboratory (this is called non-classical rotational inertia: NCRI). At first sight this seems at odds with common sense. A solid is a piece of matter that has its own shape, which resists shearing. Superfluidity means that a system can flow like a fluid but the "super" means that it is so prone to flow that it can do it without any dissipation. Notice the difference with a superconductor. Here the flow without dissipation is due to electrons. On the other hand a superconductor is a solid piece of matter but its solidity is due to a different kind of degrees of freedom, the ions. So in this case two different kinds of particles are responsible for the two aspects, the electrons for the flow and the ions for the solidity.

In a supersolid the situation is different: the same entities, neutral atoms, have at the same time to give the framework, which stands at the basis of solidity, and to give the particles that flow. Classically this is difficult to conceive but when Quantum Mechanics and indistinguishability of particles come into play this is not impossible. In fact supersolidity was suggested theoretically almost forty years ago [1,2] and the proposed mechanism was attributed to the possibility that the ground state of a quantum solid might contain a finite concentration of vacancies. This would mean that the probability of occupation of a unit cell of the crystal is less than unity. Often such a state is called an incommensurate crystal, in contrast to a commensurate crystal in which there is an integer (one) number of atoms in each cell of the crystal. If the atoms are bosons also, a vacancy, the absence of an atom, obeys the same statistics so at low temperature a Bose-Einstein condensation (BEC) is expected with the presence of some form of superfluidity.

The most likely candidate for such a state was identified in solid ^{4}He. ^{4}He atoms, due to the low atomic mass and very weak interatomic interaction, are very reluctant to solidify and, even at the lowest temperatures, they do so only if a substantial pressure is imposed. Nevertheless, when crystalline order is observed, solid ^{4}He is unique among all other solids in that the mean square oscillation of an atom around the equilibrium position is a very large fraction of the lattice parameter, of the order of 25%. All solids at finite temperature contain a finite concentration of point defects like vacancies but this concentration in a classical solid vanishes exponentially fast in 1/T, the inverse absolute temperature, as T goes to zero. In a quantum system a vacancy could tunnel to a neighbouring site, *i.e.* the vacancy becomes intrinsically mobile thus lowering its energy and it could even happen that the crystal with a vacancy has a lower energy than the crystal without vacancies. If this is the case, an incommensurate crystal would represent the ground state of the system and BEC should be present. This is a possible way to the supersolid state but not necessarily the only way. Since BEC implies off-diagonal order in the one-body density matrix, in such a state two different orders should be present at the same time: in real space (the crystalline order) and in momentum space (corresponding to the off-diagonal order). In any case for many years experiments did not find anything anomalous in the properties of solid ^{4}He so the interest in the supersolid faded away as a theoretical dream.

The situation abruptly changed in 2004 when Kim and Chan [3a] reported data on a torsional oscillator containing solid ^{4}He (Fig. 1). Below a temperature of a few tenths of a Kelvin the frequency of the oscillator dropped below the value at higher temperature, implying that the moment of inertia decreased below the value corresponding to a rigid rotation of the system (Fig. 2). The fact that there was a critical velocity effect (at large amplitudes of the oscillator no missing inertia was present), and that the effect was suppressed by the presence of a plug in an annular container or when the boson ^{4}He atoms were replaced by the fermions ^{3}He, all this led the authors to interpret these results as the manifestation of the long sought NCRI associated with the supersolid state in which about 1% of the ^{4}He atoms did not respond to the imposed oscillation of the container.

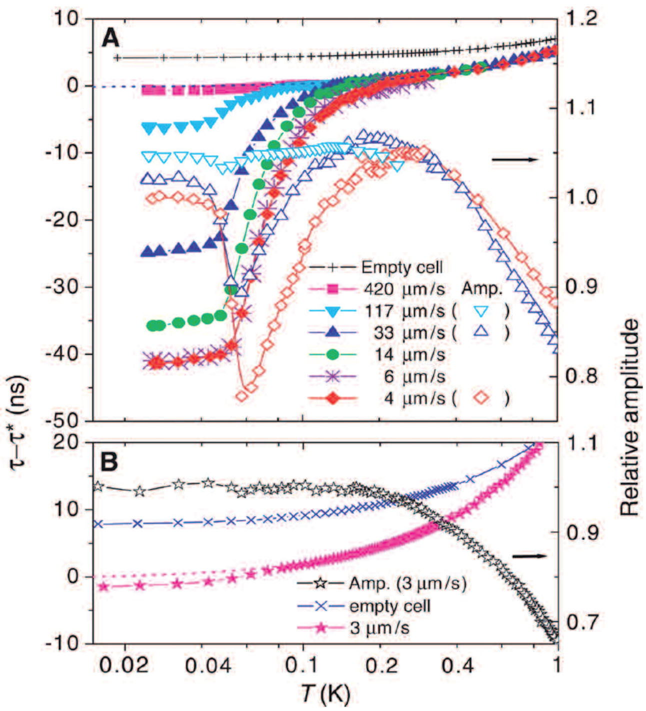
▼ **FIG. 1: Schematic torsional oscillator: The cylindrical drive puts into oscillation the cylindrical torsion cell and this oscillation induces an ac voltage on the detection electrode. A lock-in amplifier enables the oscillation to be kept in resonance. ^{4}He is introduced via the filling line and ^{4}He in the annular channel solidifies if a large enough pressure is applied. In the control experiment a barrier in the annular cell is present. (Figure from ref. 3a)**



These results spurred great excitement and many investigations, both experimental and theoretical, were launched. While in other laboratories NCRI was observed, these experiments made the situation very confusing. If the temperature range in which the NCRI effect was seen is common to the different experiments, this is not so for the superfluid fraction ρ_s. The value of ρ_s, which represents the fraction of atoms not responding to the torsional oscillator, differs by more than three orders of magnitude in different experiments; ρ_s is sensitive to annealing, to the cell geometry and composition, to even a minute concentration of ^3He impurities. Even in a single crystal, or almost so, it was found that ρ_s can differ by a large amount if the material of the container is changed. It is clear that defects have to play an important role in the phenomenon. Since the early measurements were performed in highly polycrystalline solids, grain boundaries were suspected to play a role. The fact that also single crystals gave NCRI [3b] implies that other defects have to be important and dislocations are reasonable candidates since it is known that even in a single crystal the concentration of dislocations can vary by a large amount depending on experimental details. This major role of defects on NCRI of solid ^4He led even to the conviction that the whole issue of supersolidity of ^4He was exclusively due to the presence of extrinsic defects that any real solid sample contains.

This position received a strong motivation also from theoretical results that in the meantime were obtained. If an ensemble of ^4He atoms represents a strongly interacting system, which is very difficult to study analytically, its boson nature makes the many-body problem much easier than the fermion case because there is no 'sign' problem here, at least for an equilibrium state, and this quantum system can be treated by robust simulation methods. Thus essentially exact results for bosons can be obtained from quantum simulations, even if one has to be careful that the results are with reference to a finite number of particles, usually less than a thousand, with periodic boundary conditions. Path-integral Monte Carlo methods have been widely used to study solid ^4He at finite temperatures and the result of such computations is that a perfect commensurate solid is not supersolid, at least in the temperature range of the simulations, a range that has some overlap with the one where NCRI is experimentally observed [4]. On the other hand, if some disorder is introduced, the simulations show that the system does become supersolid.

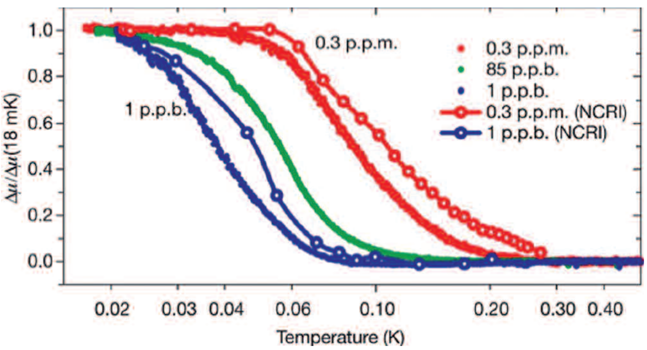
Liquid ^4He in the superfluid state has a number of peculiar properties in addition to NCRI. Obviously also in solid ^4He other signatures of superfluidity have been looked for, for instance flow without pressure drop, but none has been found so far. On the other hand, recently a new anomaly has been found in solid ^4He [5]. The elastic shear modulus has been measured and an anomalous stiffening of the solid has been observed at



▲ **FIG. 2: Resonant period (left axis, filled symbols) and amplitude (right axis, open symbols) of oscillations as a function of temperature (note the logarithmic scale) for the unblocked (A) and blocked (B) cell and for different maximum velocities as given in the legend. The period readings are shifted by v*, the resonant period at 300mK that is of order of 10^6. The resonant period of the empty cell at 300mK is about 3000 ns smaller than the filled cell volume due to the smaller moment of inertia of the empty cell. For the filled cell there is a drop of v at temperatures smaller that 300mK and this drop does not depend on velocity at low v. This drop is interpreted as due to a drop in the mass of ^4He coupled to the cell and this allows the superfluid fraction ρ_s to be extracted. The effect is drastically reduced for the blocked cell. (Figure from ref. 3a)**

low temperatures. The behaviour of the shear modulus has a remarkable similarity with the one of NCRI (Fig. 3). It has been proposed that this anomaly is due to dislocations, the same kind of disorder that has been invoked for NCRI, with dislocations being immobile at low temperature and becoming mobile at higher T. On the other hand this seems counterintuitive: the ability to move without dissipation seems at odds with a stiffening of the solid. The discovery of this stiffening is important new information on the system but we are far from an understanding of it and of the relation, if any, with NCRI.

If the establishment of NCRI represents a true phase transition one should expect to find some signature in the specific heat. It is important that very recently a peak in the specific heat has been reported at the temperature of the NCRI phenomenon [3c]. This is a very difficult experiment and it will be important to have an independent confirmation of this finding.



▲ FIG. 3: Shear modulus μ of solid ^{4}He has an anomalous increase of about 11% going from 300mK to 18mK. The variation Δμ=μ(T)-μ(300mK) normalized to the 18mK value is plotted as a function of T for samples with different concentration of ^{3}He impurities (filled symbols). Open symbols represent similarly scaled NCRI ρ_s data from torsional oscillator measurements. (Figure from ref. 5)

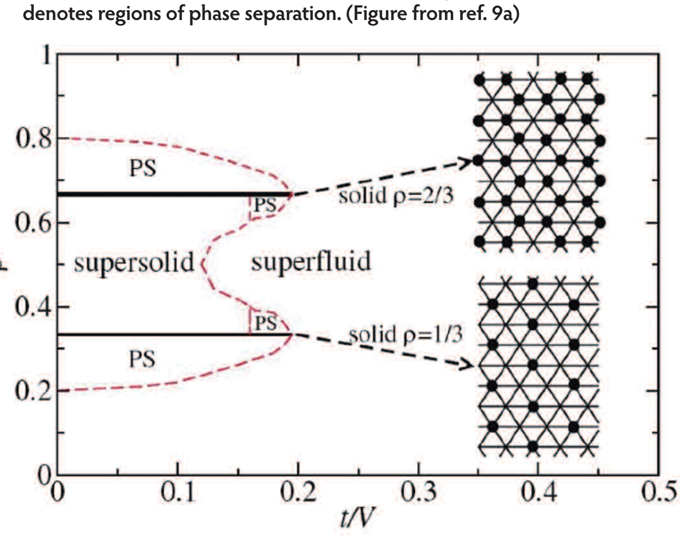
Some aspects of NCRI of solid ^{4}He, such as the value of the critical velocity and some observed metastability phenomena, are consistent with a vortex view of this state [6]: basic entities are assumed to be quantized vortices, possibly in a condensate which might correspond to the presence of BEC. However if the ground state of solid ^{4}He is commensurate, theory tells us that no BEC can be present. One possibility is that all "supersolid" phenomena in solid ^{4}He are due to the presence of some disorder induced by external conditions such as the condition of crystal growth. If this were the case this would be of interest per se but of course not as interesting as if supersolidity were an intrinsic property. In other words there is no doubt that defects have a very important role in NCRI but what happens if a more and more perfect crystal is grown? Will NCRI go away or will some remaining effect persist, i.e. the ground state of solid ^{4}He is qualitatively different from that of a normal solid?

As already mentioned there is strong theoretical evidence that a commensurate solid is normal, so the question is if some intrinsic form of disorder is present in the ground state. This leads back to from where we started forty years ago: are there vacancies in the ground state? My interpretation of experiments is that they exclude the presence of vacancies at a concentration of 1% or above but they cannot exclude the presence of vacancies at a lower concentration. Some argue that the presence of ground state vacancies is excluded by some theoretical computations giving a finite positive energy for the formation of a vacancy. To others such argument does not seem so convincing and they believe that the question of whether vacancies are present in the ground state of solid ^{4}He is still open both theoretically and experimentally [7]. But we should also be open to the possibility that intrinsic BEC and supersolidity might be associated not so much with vacancies but to some other kind of defects that, in any case, would make solid ^{4}He different from the textbook picture in which each atom occupies a crystal cell. We know for sure that the solid is periodic, this is demonstrated by the observation of Bragg peaks in the scattering of neutrons or of X-rays, but how the atoms populate the crystal cells might offer some surprise.

Looking in perspective, solid ^{4}He has turned out to be much more complicated than what was expected and we do not yet have a clear overall picture of the properties of this system. All attempts to explain the behaviour of solid ^{4}He in more conventional terms have failed so far and the presence of a non-conventional state appears to be the most likely explanation. But what this non-conventional state should be is unclear. Whether such a non-conventional state is only due to extrinsic defects or if it is an intrinsic property of the quantum solid is a fundamental question awaiting an answer. An argument in favour of the intrinsic nature of the state is the following. While, as already mentioned, the value of ρ_s changes by many orders of magnitude depending on the experimental details, the transition temperature is almost unaffected by such details. If supersolidity were entirely due to extrinsic defects it is difficult to understand how it can be that the transition temperature is almost unaffected when the amount of defects varies by many orders of magnitude. Consider now the other possibility. If the underlying ground state has BEC, associated with ground-state vacancies for instance, the system would have its own transition temperature but perhaps the intrinsic value of ρ_s could be very small. The presence of extrinsic defects might have an amplifying effect on ρ_s without necessarily affecting the transition temperature, *i.e.* extrinsic defects might act as leverage on ρ_s without much affecting the other properties of the system.

I believe that the issue of supersolidity will still be at the centre of attention in time to come. We have learnt something but much more has to be found out. And this topic will be at the centre of attention in another direction, that of trapped cold atoms. Cold atoms in a periodic potential is common trade nowadays and beautiful experiments [8] have shown how the system changes from superfluid to localized as

▼ FIG. 4: Zero-temperature phase diagram of hard core bosons on the triangular lattice in the density-interaction parameter t/V. t is the nearest neighbour tunnelling amplitude and V is the nearest neighbour repulsion energy. At low value of t/V there is a supersolid region for intermediate densities whereas the system is superfluid for larger values of t/V. PS denotes regions of phase separation. (Figure from ref. 9a)



the tunnelling probability of an atom from one site to the next decreases. Therefore the system is periodic and at the same time it can be superfluid. This however is not a true supersolid state because the periodicity is just the one imposed by the external field, for instance by a standing light beam. Theoretically there is strong evidence that bosons in a lattice can have a supersolid state [9]. Under certain conditions (Fig. 4) the spatial order of the atoms is not the simple one corresponding to that of the external potential but a more complex one resulting from the interplay between interatomic potential and external potential. Such complex spatial order can coexist with superfluidity so we can say now that supersolid order is present. At the moment this is a theoretical finding but we look forward to its experimental implementation with cold atoms. ■

References

[1] A.F. Andreev, I.M. Liftshitz, *Sov. Phy. JETP* **29** (1969), 1107

[2] G.V. Chester, *Phys. Rev. A* **2** (1970), 256

[3] a: E. Kim, M.H.W. Chan, *Science* **305** (2004), 1941; b: A.C. Clark *et al.*, *Phys. Rev. Lett.* **99**, 135302 (2007); c: X. Lin *et al.*, *Nature* **449** (2007), 1025

[4] M. Boninsegni *et al.*, *Phys. Rev. Lett.* **96** (2006), 105301; B.K. Clark, D.M. Ceperley, *Phys. Rev. Lett.* **96** (2006), 105302

[5] J. Day, J. Beamish, *Nature* **450** (2007), 853

[6] P. W. Anderson, *Nature Physics* **3** (2007), 160

[7] D.E. Galli, L. Reatto, *Phys. Rev. Lett.* **96** (2006), 165301

[8] M. Greiner *et al.*, *Nature* **415** (2002), 39

[9] S. Wessel, M. Troyer, *Phys. Rev. Lett.* **95** (2005), 127205; D. Heidarian, K. Damle, *Phys. Rev. Lett.* **95** (2005), 127206.