Toying with physics [DOI: 10.1051/EPN:2007010]

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In the early 19th century the popularisation of science became a vogue, expressed in books that often took the form of "conversations on natural philosophy". They may have echoed the discourse of the classical dialogue of debate but this was more the conversation of the drawing-room, albeit in the stilted parlance of the time.

> *The top, my boy is a subject which the great Mantuan bard did not consider beneath the patronage of his muse. (anon., quoted in* [1])

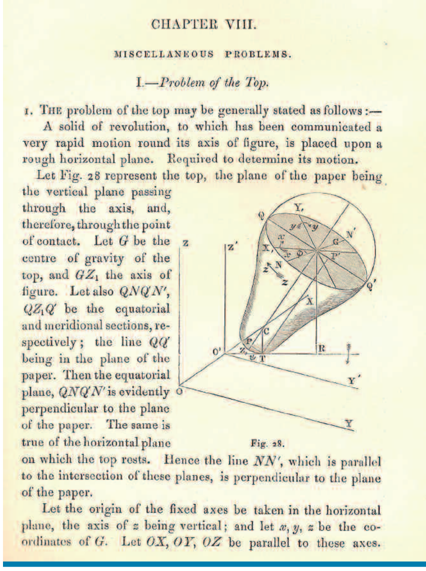
Such lessons were aimed at future scientists ("boys"), the workers ("operatives") and even the gentler sex ("little housewives"). Of three particularly influential texts [1,2,3], two were by women [2,3], one of whom preferred to remain anonymous for some time. The third, written by the early cancer researcher John Ayrton Paris, was splendidly entitled "Philosophy in Sport made Science in Action, being an Attempt to implant in the Young Mind the first Principles of Natural Philosophy by the aid of Popular Toys and Sports of Youth". Examples of the many playthings included as "instruments of philosophical instruction" are the seesaw, the kite, the top, and the Jew's Harp.

> "*And is it then possible", said the vicar, in a tone of supplication, "that you can seriously entertain such a wild and, I might add, kill-joy scheme?"* [1]

Paris' kill-joy scheme ran to many editions and imitations and helped found a pleasant tradition that was followed in later years by such popularisers as C. V. Boys in his lectures on soap bubbles, an early version carrying the flowery title "Soap-bubbles, their Colours and the Forces which mould them: Being the Substance of many Lectures delivered to Juvenile and Popular Audiences with the Addition of several new and Original Sections" [4]. Perhaps it faltered in the period of high seriousness about physics in the mid-20th century; nuclear physics had no place in the nursery. But today it emerges afresh in the minds of our physics teachers at every level (for a recent collection of experiments on soap bubbles see the books by Rämme [5]). Museum shops offer a variety of such diversions, from cheap plastic to executive-toy chrome, but many can be found in the kitchen, the toolbox and the sewing cabinet, or are easily constructed. Toys have become part of our cultural heritage, as one observes in archaeological digs, in the items for sale in antique shops, in museums, and the inclusion of the "slinky" in the US Postal Services "Celebrate the Century" series of postage stamps.

◄ **Fig. 1:** Slinky on a US postage stamp commemorating the ``Slinky craze" of 1945.

One of the present authors essayed a lecture [6] on such teasing trivialities at the "International Conference on Theoretical and Applied Mechanics" in Warsaw (2004). An eager audience overflowed the capacity of the lecture theatre. At the preceding congress in Chicago (2000), as part of a Science Teachers Day, Professor Raymond C. Turner of Clemson University told an appreciative crowd of local physics and science teachers about using toys to teach. Many in the audience were very conversant with this mode of instruction, but keen to learn more.



CHAPTER VIII.

MISCELLANEOUS PROBLEMS.

I.—*Problem of the Top.*

1. THE problem of the top may be generally stated as follows:—

A solid of revolution, to which has been communicated a very rapid motion round its axis of figure, is placed upon a rough horizontal plane. Required to determine its motion.

Let Fig. 28 represent the top, the plane of the paper being the vertical plane passing through the axis, and, therefore, through the point of contact. Let G be the centre of gravity of the top, and GZ_1 the axis of figure. Let also $QNQ'N'$, QZ_1Q' be the equatorial and meridional sections, respectively; the line QQ' being in the plane of the paper. Then the equatorial plane, $QNQ'N'$ is evidently perpendicular to the plane of the paper. The same is true of the horizontal plane on which the top rests. Hence the line NN', which is parallel to the intersection of these planes, is perpendicular to the plane of the paper.

Fig. 28.

Let the origin of the fixed axes be taken in the horizontal plane, the axis of z being vertical; and let x, y, z be the coordinates of G. Let OX, OY, OZ be parallel to these axes.

▲ **Fig. 2:** A page from Jellett's *Treatise on the Theory of Friction* (Macmillan, London, 1872) where *Jellett's constant* is introduced.

Childlike absorption

Many famous personalities of physics have had a taste for toys and tricks. Von Kármán [7] described Prandtl's "childlike absorption":

> *In subsequent months I came to know Prandtl quite well. He was a man of strange contrasts. Sophisticated and gifted in science, and a great teacher on a person-to-person basis, he was nonetheless naïve about life and childlike in behavior. He could not pass a toy in a shop, for instance, without curiously fingering it, and he was spell-bound by routine magicians tricks. Once at a party the guests decided out of fun to test Prandtl's well-known love of toys. In one corner of the room somebody placed a child's gyro, in such a position that it would be in full view. Everybody awaited Prandtl's arrival to see whether the professor would live up to expectations and go directly to the toy. He didn't disappoint them. As soon as he entered the room and his eye fell on the toy, he had no interest in anything else or in anyone at the party. It was almost half an hour before Prandtl realized that the party had formed around him and that everyone was secretly enjoying his childlike absorption.*

Other well-known scientists who have written about the analysis of toys and games include G.-G. Coriolis, today principally known for the acceleration that bears his name, but a distinguished mechanician who in 1835 wrote "Théorie mathématique des effets du jeu de billard"; F. Klein and A. Sommerfeld, •••

▲ **Fig. 3:** The so-called Russian *rattleback* in which the two small control masses (shaped as turtles) can be adjusted to create an asymmetry that dramatically affects it motion.

••• seminal figures of the late 19th and early 20th century in mathematics and physics, respectively, who collaborated on "Über die Theorie des Kreisels", publishing all four volumes in Leipzig between 1897 and 1910; H. Bondi, J. L. Synge, and K. Stewartson all wrote about spinning tops and so on. This tradition of studying toys seriously has, fortunately, survived to the present day, and we find in the current literature and in premier journals contributions on the "Levitron" [8], on "Newton's cradle" [9] (and references therein), and on "Euler's disk" [10].

Maddening mechanics

With so much to choose from, we shall concentrate on mechanics, a subject misleadingly presented as rather trivial in elementary physics classes.

The tractability of many problems of rational mechanics may be its attraction: even the spinning rigid body bows to theory for the special cases usually considered. But add a bit of asymmetry, or a dash of friction or other contact forces, and you soon have a recipe for strange, counter-intuitive gravity-defying behavior. Unpalatable mathematical formalism is needed to explain it.

Many effects were well known already in the nineteenth century. For example, a *spinning* top will rise to the vertical "sleeping" position: why? In his popular work "Spinning Tops" John Perry (1890) waxed sarcastic on the subject of Cambridge men who thought they surely understood but had forgotten the details. It had something to do with the action of friction on the rounded tip of the top - it is fairly evident that reducing it to a point removes the hope of an explanation. For a proper elucidation Perry was directed by another Irish phyicist, George Francis Fitzgerald, to the work of his father-in-law. John Jellett's "Treatise on the Theory of Friction" contains much wisdom on the role of friction that endures today, in particular "Jellett's constant".

The constant arrives unheralded in a section of miscellaneous applications (see Fig. 2). It is the result of judicious approximation: if the rotation about the symmetry axis of the top is regarded as dominant, an additional conserved quantity emerges. Jellett used it in a neat argument to show that, as friction reduces the energy of the top, it must rapidly bring it to the upright position.

But with both friction and asymmetry we are still in real trouble. The classic example is the celt (pronounced "selt"), rattleback or wobblestone, guaranteed to stun a class that sees it reverse its direction of rotation on an overhead projector. In his inspiring book "The Flying Circus of Physics" Jearl Walker writes [11]:

> *Some of the stone instruments made by primitive men... display curious personalities...These stones, called celts, are generally ellipsoidal in shape. When you spin them about a vertical axis some behave as you would guess, but others act normally only when spun in one direction about the vertical. If you spin them in the other sense, the rebellious stones will slow to a stop, rock for a few seconds, and then spin in their preferred direction...If you tap one of these stones on an end, ... it will rock for a while. But soon the rocking ceases, and the stone begins to rotate about the vertical axis. What causes such personalities?*

Figure 3 depicts a neat variation. A "tunable" rattleback, commercially known as the Russian rattleback (probably because of the style of decoration). The two small turtles may be rotated on the "deck" of the otherwise completely symmetric "hull". When they face one another, the object rotates with equal ease in the clockwise or the counterclockwise direction. However, turn the turtles so that they are perpendicular to the axis of the hull, facing in opposite directions, and the mysterious behavior emerges at once. They like to ride in the forward direction, and wobble and reverse direction if spun so they are riding backwards.

Perry describes another variation: an ellipsoidal stone spun on a surface, about one of its shorter axes: it staggers into the upright position, spinning about its long axis. A further elaboration is the additional end-to-end asymmetry of an egg, which, if spun with its blunt end down will flip over, and spin on its sharp end. This is the same behavior seen in the "Tippe-Top" toy, which has fascinated amateur and professional physicist alike. A famous snapshot (Fig. 4) shows Wolfgang Pauli and Niels Bohr experimenting with the apparent angular-momentum-reversing behavior of the tippe-top.

According to Perry, Lord Kelvin would experiment with oddly shaped stones when he went to the seaside, exploring such peculiar dynamics. He was also fond of eggs, for undergraduate demonstrations, baffling the class with the different properties of identical eggs (one of which was hard-boiled).

In a modern study of eggs worthy in its erudition of an emeritus Cambridge professor, Keith Moffatt has addressed not only some of these extraordinary feats, but also the astonishing capacity of an egg to jump vertically in the course of its rotational manoeuvres [12].

Moffatt came to the problem via a simpler one, that of the Euler disk. This is just a metal disk, the heavier the better. When spun on a flat surface it gradually slows, but its audible frequency of rotation increases dramatically. It rises inexorably towards a crescendo and stops abruptly, like the best symphonies.

Who said dissipation causes motion to die away slowly? Here it provokes a mathematical crisis called a finite-time singularity, important in modern fluid dynamics.

In relatively modern times, the rotational toy that has made the biggest move in the stock market is surely the "yo-yo". Like many of our toys it has a pedigree measured in millennia, and there have been periodic crazes in the modern era. (Invest now.) Another device of ancient origin is the diabolo. James Clerk Maxwell was given one as a child and took it with him to Cambridge (Fig. 5).

All of the above is certainly abstruse - some would say that it is more of a lesson in humility for the teacher than elementary physics for the pupil. So let us retreat to something simpler - "Newton's cradle", as shown in Figure 6. A ten-second experiment with a ten-second explanation in terms of conservation of energy and angular momentum? But there is a greater lesson to be learned: do the experiment, and make careful observations...

We observe that, if one ball is pulled aside and released, it is not precisely true that only one ball is ejected from the other side. Merely some misalignment in manufacture? It turns out that the cause is not to be found there, but rather in the finite compressibility of the balls, which renders collisions non-instantaneous. A compressive elastic pulse of finite width is propagated down the line. When it ejects the last ball, the previous two are still slightly compressed together, so they fly apart too in a clearly visible displacement (the remaining displacements are not usually discernible). Recognizing this, we see that the subsequent motion cannot long conform to what is expected (and taught by many). Indeed it does not, and several papers address this motion; see [9] and references therein. To observe it unmodified by air drag, build a big Cradle, and rock it. Discarded bowling balls would be good.

Funny fluids

Fluid mechanics has its share of toys, executive and otherwise. The "Ooze Tube" and its variants allow one to watch the coiling of a thread of viscous liquid falling through a small circular hole, see Figure 7. Barnes and Woodcock [13] call this the "liquid rope-coil effect". In his paper at the "12th International Congress of Applied Mechanics", held at Stanford University in 1968, the father of modern fluid dynamics, G. I. Taylor showed and analyzed examples of instabilities of jets, threads and sheets of viscous fluids.

The thin thread with a circular cross-section coils neatly into a growing mound. A flat ribbon, on the other hand, will fall in layers. When placed horizontally, and thus constrained to move two-dimensionally, the thread will buckle. Taylor gives the following explanation [14]:

> *The reason for the instability is clear. If the stream is very thin, the longitudinal compression acts in the same way as end compression in a thin elastic rod. The rod becomes unstable at a certain load, and less force is needed to move the ends towards one another when the rod is bent than when it is straight. This is called Euler instability.*

Experiments [13] show that the effective height of fall and the frequency of coiling of a viscous thread are proportional. Is this clear from elementary considerations? Simple dimensional analysis, using just the coiling frequency, the height of fall, and the acceleration of gravity, inevitably yield a frequency inversely proportional to the square root of the height - not at all the observed relationship. However, introduce the kinematic viscosity of the liquid, as would immediately be suggested by simple fluid mechanics, and dimensional analysis yields two dimensional groups, one of which is a "Reynolds number" for the flow. For small values of this dimensionless number or slow flow, we obtain a coiling frequency proportional to the fall height as observed.

The so-called "Brazil nut effect" is the observation that in a mixture of granular material of different sizes, *e.g.*, a can of mixed nuts, the large particles are often observed to end up on top. One intuitive explanation is that when the container is shaken, the small particles flow to fill in voids with greater probability and facility than large particles, hence achieving •••

▶ **Fig. 4:** Two giants of modern physics, Wolfgang Pauli and Niels Bohr, fascinated by the counter-intuitive behavior of the tippe-top. (Photograph by Erik Gustafson, courtesy AIP Emilio Segre Visual Archives, Margrethe Bohr Collection) (reproduced with kind permission of AIP).

▲ **Fig. 5:** Maxwell's diabolo. (Photograph courtesy of the University of St. Andrews.)

▼ **Fig. 6:** Newton's cradle.

··· segregation. There is a mechanical toy, the "Sand wand", that exploits this effect, see Figure 7. Enclosed in a transparent cylindrical tube is a roughly monodisperse granular material and a single steel ball that almost spans the tube. Move the ball, one is instructed, to one end of the tube. Gentle manipulation, rotation, tapping, and so on will not do the trick. A few sturdy shakes with the rod essentially vertical and the desired result is achieved, the glittering ball emerging from the smaller grains and settling on top of them.

A perennial favorite is the "Cartesian diver" or sometimes Cartesian devil - although the attribution to Descartes is, apparently, misplaced - often artfully crafted as a small demon-like creature, submerged in a vertical glass cylinder filled with water, see Figure 7. The cylinder has a flexible membrane at the top. Depress the membrane and the diver descends as if magically heeding instructions from a distance. Release the pressure and he ascends. With a bit of practice he can be made to swirl and dance in the water column. What is the source of this magic? The diver is constructed with a small air-filled chamber inside, open at the bottom and there in contact with the water. When the water column is compressed, the air pocket within the diver is much more easily compressed. The buoyancy of the diver (and the air pocket with him) is reduced and he descends. As with the rest of us, already struggling to stay afloat, a bit more pressure applied from above and we are totally submerged!

Positive play

There is a creative playfulness to good science, and using toys in instruction seems to bring this full circle. They are approachable, intuitive, familiar, we are tempted to say "easy". We feel that we understand how they work. We certainly know how we expect them to work. When they turn out to challenge our intuition and our deepest analytical abilities, do we become insecure or intrigued? Hopefully the latter and, hopefully, that curiosity breeds interest, engagement, creativity, and, ultimately an affection for scientific inquiry. ■

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Denis Weaire is Erasmus Smith Professor of Natural and Experimental Philosophy at Trinity College Dublin and has a keen interest in the history of Physics. Together with **Stefan Hutzler**, who is a lecturer at TCD, he has written a book on the Physics of Foams, containing further examples how child-like fascination (in this case with soap bubbles) can be turned into science.

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► **Fig. 7:** Fluid mechanical toys. **up**, the "Ooze Tube" illustrating the instability of a viscous liquid thread. **Center**, the "Sand wand" - how does one move the steel ball to the to top of the granular material in the tube? **down**, the "Cartesian diver" illustrating simple principles of buoyancy.