Alexandru Proca (1897-1955)
and his equation of the massive vector boson field

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"One of the most prodigious phenomenon of the recent history of physics was the extraordinary influence of Einstein's ideas not only on the contemporary scientific theory, but also on its applications". These words have been written for the French hebdomadal *Le Figaro Litteraire* at the first jubilee in 1955 of one of the greatest landmarks of physics development – the *Annus Mirrabilis* 1905. The author, Alexandru Proca, at the time Directeur de Recherche at the CNRS France, was one of the great minds who entered physics at the right time. He belonged to the golden generation that shaped the structure not only of physics, but also of the science in its entirety. Proca devoted the most creative years of his life to the development of a sound relativistic quantum theory. During the effervescent years of the fourth decade of the 20th century, among scientists such as de Broglie, Schrödinger, Pauli, Heisenberg, Bohr and Dirac, Proca found an elegant and original path that remains even today actual and valid. It is an act of gratitude to revive his memory after another half century since he paid homage to the memory of Einstein just a few months before his own death.

Alexandru Proca was born in Bucharest in 1897 into a family of intellectuals. A brilliant school boy, mastering a few modern languages as well as Latin and some old Greek as was the curricula of any modern high school at that time, he showed an early appetite for mathematics. Soon after graduation, in the middle of World War I, he was mobilized (1917) and, after a brief instruction in a Military School, was sent to the front. 1918 was a triumphant year for his country, Romania, which saw most of its historic and ethnic borders settled. However the infrastructure for higher education, especially in the sciences and engineering, was quite weak and many young people used to go abroad, mainly to France and later to Germany, to acquire a proper training and qualification. Proca graduated from the newly inaugurated Polytechnic School (PS) in 1922 as an Electromechanical Engineer. He was employed by the Electrical Society Campina, a company at the centre of a rich oil field. At the same time he served as assistant professor at the chair of Electricity of the PS, which was headed by Professor Vasilescu Karpen a celebrity among many generations of engineers. The young Proca felt however that he was betraying his natural aptitude for the fundamental sciences, mathematics and physics. Already familiar with Einstein's papers he anticipated a major breakthrough and thought that he had something to say in this field. It is amazing that as early as 1920, still a student in engineering, he wrote a first paper on relativity, one of not many in the world at that time. So he left a promising and lucrative career to go to Paris (1923), full of expectations. Here, to his disappointment, he discovered that his diploma was useless. What he had to do was to matriculate from a French University and to pass the examinations for the 4-years curricula. This he did brilliantly and one year later he was *Licencié ès Sciences* at Sorbonne, Paris. In 1925 Marie Curie offered him his first job in her *Institut du Radium* where he was assigned to make measurements of β rays emitted by thorium descendants. He completed successfully the subject with a publication in Comptes Rendus (Nov. 1926), but that marked the end of his experimental investigations. Though he enjoyed much sympathy and appreciation from Mme Curie, she allowed him to pursue his natural calling towards theoretical physics that persisted during all these years in spite of his official daily duties. He was attracted by fundamental problems, such as the intimate nature of light quanta, the atomicity of entropy and even of time; he was one of the first thinkers about a discontinuous spatio-temporal frame.

◀ Alexandru Proca (1897-1955).

It was however de Broglie who directed him towards the mainstream of theoretical pursuits at that time, namely the Dirac equation and the quantum relativistic fields that had just started to take shape in the pioneering works of Born and Jordan, Dirac himself and Heisenberg. Proca engaged seriously in the programme and after a series of six papers devoted to the Dirac equation published in C. R Acad. Sci. Paris, (1930-33), and two more on the properties of the photon, he submitted an exceptional doctorate thesis to a commission: Jean Perrin, Louis de Broglie, Léon Brillouin and Aimé Cotton. In 1930 Proca received French citizenship and married Marie Manolesco with whom he had a son, George Proca [1-3].

From 1929, when *Les Annales de l'Institut Henri Poincaré* was founded, Proca was the editor of this famous journal. In 1934 he spent one year with E. Schrödinger in Berlin and a few months with N. Bohr in Copenhagen, where he met Heisenberg and Gamow. From 1936 to 1941 he developed his masterpiece work, the theory of massive vector (spin 1) boson fields governing the weak interaction and the motion of spin-1 mesons. Prestigious scientists such as Yukawa, Wentzel, Taketani, Sakata, Kemmer, Heitler, Fröhlich and Bhabha, reacted favourably to his equations in 1938. W. Pauli [4-6] mentioned Proca's theory in his Nobel lecture. As a particular sign of his world-wide recognition one can mention his invitation to attend in 1939 the *Solvay Congress*. To his misfortune, this Congress could not take place due to the outbreak of World War II.

During the war he was for a short time Chief Engineer of the French Radio broadcasting Company. In 1943 he moved to Portugal where he lectured at the University of Porto. In 1943-45 he was in the United Kingdom at the invitation of the Royal Society and the British Admiralty to join the war effort. After the war he started in 1946 the *Proca seminar* series in Paris with many prestigious invited speakers from France and abroad including A. Einstein, H. Yukawa and W. Pauli. This seminar contributed very much to the education of young French particle physicists. He accepted to organize with

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P. Auger in 1950 the Theoretical Physics Colloquium of CNRS and in 1951 to be the French delegate at the General Meeting of the International Union of Physics. By that time the Proca equation was indeed famous as the only sound theoretical basis of the Yukawa meson. Starting in 1953 Proca began a fight with a laryngeal cancer that lasted until December 13, 1955 when he passed away. He left a major heritage in theoretical physics that by its actuality goes beyond historical interest.

Proca (massive vector boson) field

Proca extended the Maxwell equations to quantum field theory. For a massive vector boson (spin1) field the Proca equation

$$\Box A^{\nu} - \partial^{\nu}(\partial_{\mu} A^{\mu}) + m^2 A^{\nu} = j^{\nu}$$

is obtained as a Euler-Lagrange equation emerging from the Lagrangian density

$$\mathcal{L} = -(1/4)\, F_{\mu\nu} F^{\mu\nu} + (1/2) m^2 A_{\mu} A^{\mu} - j_{\mu} A^{\mu}$$

after expressing the antisymmetric field-strength tensor,

$$F^{\mu\nu} = \partial^{\mu} A^{\nu} - \partial^{\nu} A^{\mu},$$

in terms of the four potential A^{μ}. One assumes the Einstein's convention: summing over repeated indices.

$$\Box = (\partial^2/\partial t^2) - \nabla^2$$

is the d'Alembertian operator while ∇^2 is the Laplacian. The 4-potential and current density are

$$A^{\mu} = (\varphi; \boldsymbol{A}), \quad j^{\mu} = (\rho; \boldsymbol{j})$$

Scalar j and vector $\boldsymbol{A}$ potentials are introduced via

$$E = -\nabla\varphi - \partial \boldsymbol{A}/\partial t, \quad \boldsymbol{B} = \nabla \times \boldsymbol{A}$$

In his Nobel lecture [4], W. Pauli noted: "The simplest cases of one-valued fields are the scalar field and a field consisting of a four-vector and an antisymmetric tensor like the potentials and field strengths in Maxwell's theory. While the scalar field is simply fulfilling the usual wave equation of the second order in which the term proportional to m^2 has to be included, the other field has to fulfill equations due to Proca which are generalization of Maxwell's equations."

After Yukawa's hypothesis of a particle mediating the nuclear interaction this particle was initially called the mesotron, or alternatively the Proca particle. Yukawa won the Nobel Prize in 1949 for his prediction (in 1934) of the existence of mesons on the basis of theoretical work on nuclear forces. By analogy with photons mediating the electromagnetic interaction he assumed the nuclear forces, acting between nucleons, are mediated by such bosons. As he suggested, the study of cosmic rays gave the first experimental evidence of the new particles. Yukawa employed a scalar field equation. He based his initial argument on the Klein Gordon scalar equation (zero spin) which not only was in conflict with relativity, but led to wrong quantitative results. It was Proca's equation that rendered Yukawa's genial intuition the theoretical ground suitable for a spin one particle.

Pauli [5] wrote: "This case holds the centre of current interest since Yukawa supposed the meson to have spin 1 in order to explain the spin dependence of the force between proton and neutron. The theory for this case has been given by Proca". Around 1970 there were many theories attempting to explain the nuclear interaction. Presently according to quantum chromodynamics, the strong interaction, mediated by massless gluons, affects only quarks and antiquarks; it binds quarks to form hadrons (including the proton and neutron).

Nonzero photon mass, the graviton and the superluminal radiation field.

The possibility and the effects of a non-zero photon rest mass is incorporated into the Proca equations. Implications of a massive photon are extraordinary: the variation of light velocity c; a longitudinal electromagnetic component of radiation and gravitational deflection; the possibility of charged black holes; the existence of magnetic monopoles; the modification of the standard model, etc. An upper limit for the photon rest mass [7] is $m_{\gamma} \leq 10^{-49}$ g. The hypothesis of a massive photon has consequences that might transcend the limits of physics itself. One such consequence is that the speed of the photon would depend on its frequency according to the theory of relativity. The speed of light will remain a simple universal constant. Other consequences refer to the violation of the Coulomb law, light propagation and the spatial extension of the electromagnetic field. These facts also provide a clue on how independent searches should be conducted, from laboratory bench tests of electrostatic screening to intergalactic effects. In one of the most spectacular experiments, undertaken by the Pioneer 10 spacecraft, the magnetic field decay around Jupiter [8] was measured.

One should point out that the m^2 term in the Proca equation is not a fancy idea of the author; its presence is in perfect agreement with the relativistic invariance of the electromagnetic field and the relativistic conservation of energy. It is the m=0 hypothesis, so much familiar to us, that comes as a supplementary condition and has to be checked.

The concept of a graviton of non-zero rest mass may be defined in two ways: phenomenologically, by a mass term in the linear Lagrangian density, as in Proca electrodynamics, and self-consistently, by solving Einstein's equations in the conformal flat case. The rest mass of the graviton was given in terms of the three fundamental constants: the gravitational, Planck, and the light velocity. The Einstein-Proca equations, describing a spin-1 massive vector field in general relativity, have been studied [9] in the static spherically-symmetric case. It has been shown that a special case of the gauge theory of gravity is effectively equivalent to the coupled Einstein-Proca theory. The Einstein-Proca field equations are frequently discussed in connections with dark matter gravitational interactions. At the level of string theories there are hints that non-Riemannian models, such as Einstein-Proca-Weyl theories may be used to account for the dark matter. Superluminal (faster than light) particles, *tachyons*, with an imaginary mass of the order of $m_e/238$, can be described by a real Proca field with a negative mass square [10]. They could be generated in storage rings, in the Jovian magnetosphere and in supernova remnants.

Proca's heritage is well established. Maxwell and Proca theories may be found in textbooks [11] as important examples of relativistically invariant formulation of the field equations for a free field. There are many publications referring to Proca directly in their title: the Einstein-Proca model (*e.g.* [9,13]), the Einstein-Proca-Weyl theories [12], or the Maxwell-Chern-Simons-Proca [14] electrodynamics. His physics opens new roads for theoretical and experimental investigations in the 21st century, proving once more that surprises are still in store. There were great moments of physics in the past as there will be in the future. ■

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Alexandru Calboreanu, president of the Romanian physical society, has received his PhD from the University of Oxford (1970). His main works are in studies of nuclear reactions and the stability of nuclei. He has taught at the University of Constanta, which has conferred on him the title of Doctor Honoris Causa.

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Both authors have fulfilled terms as scientific directors of their institute in Bucharest.

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Water from Heaven

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Large rain drops fall faster than small ones, that much is obvious for any physicist. But let's be a bit more precise. The terminal velocity follows from the balance between the weight of the drop and its air resistance. What exactly is the resistance of a drop falling through the atmosphere? We have to distinguish two regimes here. If the droplets are very small, like cloud droplets (or fog particles, if you wish), the Reynolds number is so small that Stokes' formula applies: the air resistance is proportional to viscosity, radius and velocity: $F = 6\pi\eta Rv$. For a typical cloud droplet having a radius of 0.01 mm, we find a terminal velocity of about 1 cm/s. That is very small indeed. But it goes up rapidly with size: since its weight is proportional to R^3 and the resistance only to R, the terminal velocity increases with the square of the size for such droplets. That applies to droplets up to about 0.1 mm in diameter, according to the handbooks (good old Ludwig Prandtl's book *Führung durch die Strömungslehre*, for example).

For ordinary raindrops, above about 1 mm diameter, turbulent flow dominates. Here the weight is balanced by drag $F_D = C_D\pi R^2.1/2\rho v^2$, where πR^2 is the frontal area, C_D is the drag coefficient, which is about 0.5 for a sphere at the relevant Reynolds numbers, and ρ the density of air. For a rain drop of 1 mm in diameter, we find a terminal velocity of 16 km/h. Note that in this regime the velocity is proportional to the square root of the diameter. Consequently, a 3 mm raindrop reaches 28 km/h. And so on, we would guess. Given the above, we may expect that for the biggest drops - 5 mm, say - the terminal velocity is well above 35 km/h.

Wrong! Something interesting happens, as already noticed by the German physicist, Philipp Lenard, a century ago. Using a vertical wind tunnel to balance the speed of the drop, he noticed that drops larger than about 3 mm diameter become deformed to the shape of a small pancake, and have a flat bottom. Consequently, their frontal area is larger than for spherical droplets having the same mass. As a result of the increased drag, the terminal velocity hardly increases any further: for raindrops of 4 and 5 mm it reaches an asymptotic value of about 29 km/h, which is practically the same speed as that already reached by the 3 mm drops.

And beyond 5 mm? As soon as the diameter reaches about 5.5 mm, the forces become so large that surface tension cannot hold the drop together, so it breaks up into pieces. For raindrops, there is no life beyond 5 mm! ■