# Loop quantum gravity

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The revolution brought by Einstein's theory of gravity lies more in the discovery of the principle of general covariance than in the form of the dynamical equations of general relativity. *General covariance* brings the relational character of nature into our description of physics as an essential ingredient for the understanding of the gravitational force. In general relativity the gravitational field is encoded in the dynamical geometry of space-time, implying a strong form of universality that precludes the existence of any non-dynamical reference system—or non-dynamical background—on top of which things occur. This leaves no room for the old view where fields evolve on a rigid preestablished space-time geometry (e.g. Minkowski space-time): to understand gravity one must describe the dynamics of fields with respect to one another, and independently of any background structure.

General relativity realizes the requirements of general covariance as a classical theory, i.e., for $\hbar = 0$. Einstein's theory is, in this sense, incomplete as a fundamental description of nature. A clear indication of such incompleteness is the generic prediction of space-time *singularities* in the context of gravitational collapse. Near space-time *singularities* the space-time curvature and energy density become so large that any classical description turns inconsistent. This is reminiscent of the foundational examples of quantum mechanics—such as the UV catastrophe of black body radiation or the instability of the classical model of the hydrogen atom—where similar singularities appear if quantum effects are not appropriately taken into account. General relativity must be replaced by a more fundamental description that appropriately includes the quantum degrees of freedom of gravity.

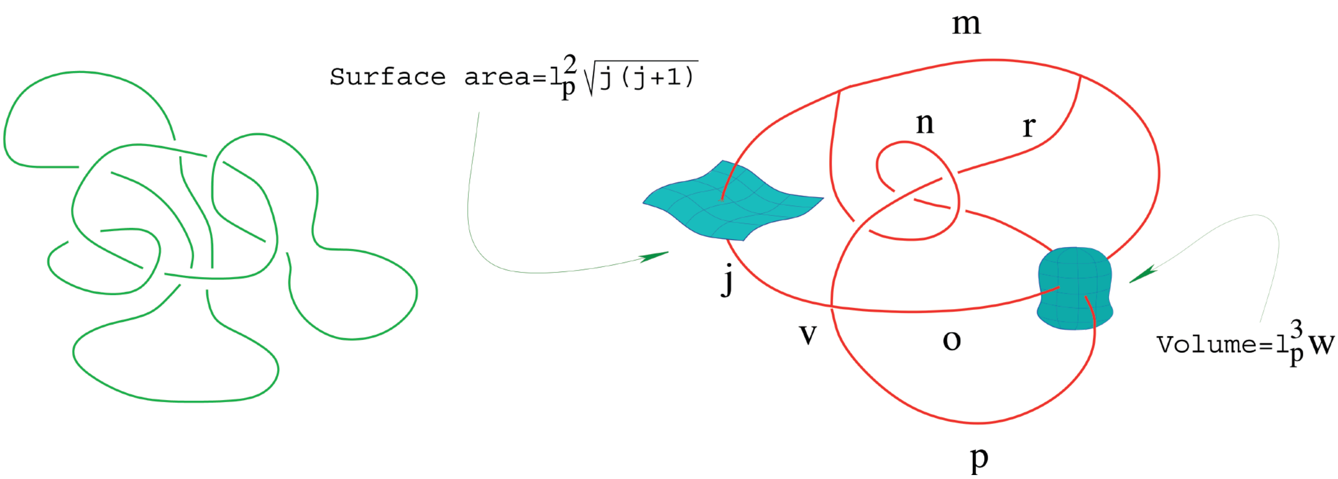
At first sight the candidate would be a suitable generalization of the formalism of quantum field theory (QFT). However, the standard QFT's used to describe other fundamental forces are not appropriate to tackle the problem of quantum gravity. Firstly, because standard QFT's are not generally covariant as they can only be defined if a non-dynamical space-time geometry is provided: the notion of particle, Fourier modes, vacuum, Poincaré invariance are essential tools that can only be constructed on a given space-time geometry. This is a strong limitation when it comes to quantum gravity since the very notion of space-time geometry is most likely not defined in the deep quantum regime. Secondly, quantum field theory is plagued by singularities too (UV divergences) coming from the contribution of arbitrary high energy quantum processes. This limitation of standard QFT's is expected to disappear once the quantum fluctuations of the gravitational field, involving the dynamical treatment of spacetime geometry, are appropriately taken into account. But because of its intrinsically background dependent definition, standard QFT cannot be used to shed light on this issue. A general covariant approach to the quantization of gravity is needed.

This is obviously not an easy challenge as in the construction of a general covariant QFT one must abandon from the starting point most of the concepts that are essential in the description of 'no-gravitational' physics. One has to learn to formulate a quantum theory in the absence of preferred reference systems or pre-existent notion of space and time. Loop quantum gravity (LQG) is a framework to address this task. In this article I will illustrate its main conceptual ideas, and established results. We will see that if the degrees of freedom of gravity are quantized in accordance to the principles of general covariance both the singularity problems of classical general relativity as well as the UV problem of standard QFT's appear to vanish providing a whole new perspective for the description of fundamental interactions.

This is a brief overview of the theory aimed at non experts where nothing is explicitly proved. The interested reader can consult the book [1] and the references therein for more details.

## Why background independence?

The remarkable experimental success of the standard model of particle physics is a great achievement of *standard* QFT. The standard model unifies the principles of *quantum mechanics* and *special*




▲ **Fig. 1:** The basic loop excitations of geometry are combined into states of an orthonormal basis of the Hilbert space called *spin network states*. These states are labelled by a graph in space and assignment of spin quantum numbers to edges and intersections ($j, n, m, r, o, p \in \mathbb{Z}/2$). The edges are quantized lines of *area*: a spin network link labelled with the spin $j$ that punctures the given surface is an eigenstate of its area with eigenvalue $\sqrt{j(j+1)}$ times the fundamental Planck area. Intersections can be labelled by discrete quantum numbers of volume ($v$, and $w$ here). In order for this page to have the observed area one would need about $10^{68}$ spin network punctures with $j = 1/2$!



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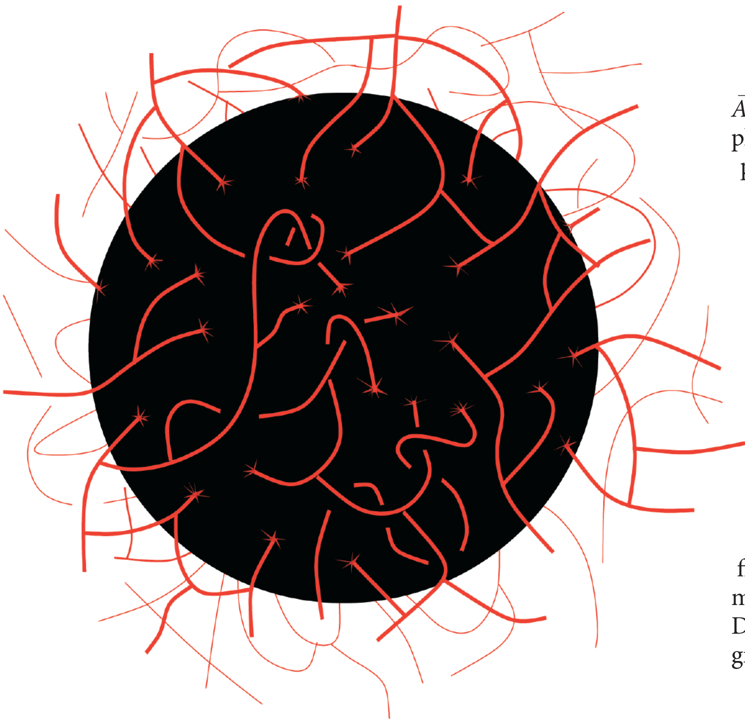


▲ **Fig. 2:** In loop quantum gravity the Bekenstein-Hawking entropy formula for a black hole of area $A$, $S_{bh} = A/(4\ell_p^2)$, can berecovered from the quantum theory as $S_{bh} = \log(N)$, where $N$ is the number of microstates (spin network states puncturing the 2-dimensional horizon with arbitrary spins) compatible with the macroscopic horizon area of the black hole.

*relativity* in the description of the strong, and electro-weak interactions. It is therefore valid as an approximation when the dynamics of the gravitational field is negligible. This limitation is implicit in the definition of standard QFT through the assumption of the existence of inertial coordinates in terms of which the field equations are defined.

In this regime it is easy to construct the idealized physical systems used to define inertial coordinates by starting from an array of test particles at rest with respect to one another, separated by some fixed distance, and carrying clocks which can be synchronized using light signals. By using neutral matter in the construction, the reference system will not be affected by the physical process being studied. Thus its dynamics is trivial, and its properties can be completely hidden in the definition of the inertial coordinates together with a notion of Minkowskian background geometry. In terms of these physical reference systems one writes (or discovers) the laws of physics (either classical or quantum) as long as gravity is neglected.

When the dynamics of the gravitational field cannot be neglected the situation changes dramatically. Due to the fact that everything is affected by gravity one can no longer construct a reference system whose dynamics is known beforehand: whatever physical system one chooses as reference it will be affected by the gravitational field involved in the processes of interest. It is no longer possible to identify any meaningful notion of non-dynamical background. One has no choice but to represent the dynamics of the system in a relational manner where the evolution of some degrees of freedom are expressed as functions of others. Processes do not happen in a god given space-time metric, they define the space-time geometry as they occur.

Except for very special situations, coordinates cannot be associated to physical entities so they are introduced as mere parameters labelling space-time events with no intrinsic physical meaning. As in electromagnetism where the choices of vector potential $\vec{A}$ and $\vec{A}+\vec{\nabla}\chi$ represent the same physical configuration, in gravitational physics a choice of coordinates is a choice of gauge. Any physical prediction in electromagnetism must be gauge-independent; similarly, in gravity they must be coordinate-independent or *diffeomorphism invariant*.

In classical gravity the importance of diffeomorphism invariance is somewhat attenuated by the fact that there are many interesting physical situations where some kind of preferred reference systems can be constructed (e.g., co-moving observers in cosmology, or observers at infinity for isolated systems). However, the necessity of manifest *diffeomorphism invariance* becomes unavoidable in the quantum theory where simple arguments show that at the Planck scale ($\ell_p \approx 10^{-33}$ cm) the quantum fluctuations of the gravitational field become so important that there is no way (not even in principle) to make observations without affecting the gravitational field. In this regime only a background independent and diffeomorphism invariant formulation can be consistent.

Despite all this one can try to define quantum gravity as a background dependent theory by splitting the space-time metric $g_{ab}$ as

$$g_{ab} = \eta_{ab} + h_{ab} \tag{1}$$

where $\eta_{ab}$ is a flat Minkowski metric fixed once and for all and $h_{ab}$ represents small fluctuations. Now if the field $h_{ab}$ is quantized using standard techniques the resulting theory predicts UV divergent amplitudes that cannot be controlled using the standard renormalization techniques. The background dependent attempt to define quantum gravity fails. According to our previous discussion, the key of this problem is in the inconsistency of the splitting [1]. This statement is strongly supported by the results of the background independent quantization proposed by *loop quantum gravity*.

## Loop quantum gravity

LQG is a background independent approach to the construction of a quantum field theory of matter fields and gravity. The theory was born from the convergence of two main set of ideas: the old ideas about background independence formulated by Dirac, Wheeler, DeWitt and Misner in the context of Hamiltonian general relativity, and the observation by Wilson, Migdal, among others, that Wilson loops are natural variables in the non-perturbative formulation of gauge theories. The relevance of these two ideas is manifest if one formulates classical gravity in terms of suitable variables that render the equations of general relativity similar to those of standard electromagnetism or Yang-Mills theory.

The starting point is the Hamiltonian formulation of gravity where one slices space-time arbitrarily in terms of *space* and *time* and studies the evolution of the space geometry along the slicing. In the standard treatment the metric of space and its conjugate momentum—simply related to its time derivative—are the phase space variables of general relativity. By a suitable canonical transformation one can obtain new variables consisting of: a triplet of electric fields $\vec{E}_i$ whose conjugate momenta are given by a triplet of vector potentials $\vec{A}_i$ with $i = 1, 2, 3$. The (unconstrained) phase space of general relativity is equivalent to that of an $SU(2)$ Yang-Mills theory (a non Abelian generalization of electromagnetism).

What is the physical meaning of the *new variables*? The triplet of vector potentials $\vec{A}_i$ have an interpretation that is similar to that of $\vec{A}$ in electromagnetism: they define the notion of parallel transport of spinors encoded in the 'Aharonov-Bohm phase' acquired by matter when parallel transported along a path $\gamma$ in space—affecting all forms of matter due to the universality of gravity. Unlike in electromagnetism, here the 'phase' is replaced by an element of $SU(2)$ associated with the action of a real rotation in space on the displaced spinor.

This is mathematically encoded in the Wilson loop (related to the circulation of the magnetic fields $\vec{B}_i$) along the loop $\gamma$ according to

$$W_\gamma[A] = P\exp\int_\gamma \tau^i\,\vec{A}_i\cdot d\vec{s} \;\in SU(2), \qquad (2)$$

where $P$ denotes the path-ordered-exponential, $\tau^i$ are the generators of $SU(2)$, and $s$ is an arbitrary parameter along $\gamma$.

The electric fields $\vec{E}_i$ have a novel physical interpretation: they encode the (dynamical) geometry of 3-dimensional space. More precisely the triplet of electric fields $\vec{E}_i$ define at every point of space an (densitized) orthonormal local frame, which in turn can be used to reconstruct the space metric. Therefore, any geometric property of space can be written as a functional of $\vec{E}_i$.

There are two geometric quantities that one can construct in terms of simple functionals of $\vec{E}_i$ that will play an important role in the quantum theory. The first (and the simplest) one is the area $A(S)$ of a two dimensional surface $S$—corresponding to the 'absolute value of the flux' of the electric field across $S$—embedded in space, while the second is the volume $V(R)$ of a three dimensional region $R$ in space. In equations,

$$\begin{aligned} &\textit{Area of } S \;\rightarrow\; A(S) = \int_S |E_i^\perp E_i^\perp|, \\ &\textit{Volume of } R \;\rightarrow\; V(R) = \int_R \sqrt{\vec{E}_i\cdot(\vec{E}_j \times \vec{E}_k)\,\epsilon^{ijk}} \end{aligned} \qquad (3)$$

where $\perp$ denotes the component of $E_i$ normal to $S$, $\epsilon^{ijk}$ is the Levi-Civita skew-symmetric tensor (repeated indices are summed). As anticipated, both the area of a surface $S$ and the volume of a region $R$ are written as functionals of the dynamical variable $\vec{E}_i$.

Einstein's equations are encoded in relations among the phase-space variables. They are given by to the so-called *kinematic constraints*, related to certain manifest gauge symmetries,

$$\begin{aligned} &\textit{Gauss law} \;\rightarrow\; \mathrm{Div}\,(\vec{E}_i) = 0, \\ &\textit{Vector constraint} \;\rightarrow\; \vec{E}_i \times \vec{B}_i(A) = 0 \end{aligned} \qquad (4)$$

and the *Hamiltonian constraint*, encoding the non trivial dynamics of general relativity,

$$\frac{(\vec{E}_i \times \vec{E}_j)\cdot \vec{B}_k(A)\,\epsilon^{ijk}}{\sqrt{\vec{E}_i\cdot(\vec{E}_j \times \vec{E}_k)\,\epsilon^{ijk}}} = 0 \qquad (5)$$

where $\vec{B}_i(A) = \vec{\nabla}_A \times \vec{A}_i$ is the triplet of magnetic fields constructed from the $SU(2)$ connection $\vec{A}_i$.

## Quantization

The quantization is performed following the canonical approach, i.e., promoting the phase space variables to self adjoint operators in a Hilbert space $\mathscr{H}$ satisfying the canonical commutation relations according to the rule $\{\ ,\ \} \rightarrow -i/\hbar\,[\ ,\ ]$. The classical constraints are imposed as operator-equations on the states of the theory. These are the *quantum Einstein's equations*. The kinematical conditions (4) are directly applied in the construction of $\mathscr{H}$. *Quantum dynamics* is governed by the quantum version of the Hamiltonian constraint, which formally reads

$$\widehat{\frac{(\vec{E}_i \times \vec{E}_j)\cdot \vec{B}_k(A)\,\epsilon^{ijk}}{\sqrt{\vec{E}_i\cdot(\vec{E}_j \times \vec{E}_k)\,\epsilon^{ijk}}}}\;\Psi[A] = 0 \quad \text{for} \quad \Psi[A] \in \mathscr{H} \qquad (6)$$

and can be viewed as the the analog of the Schroedinger equation of standard quantum mechanics.

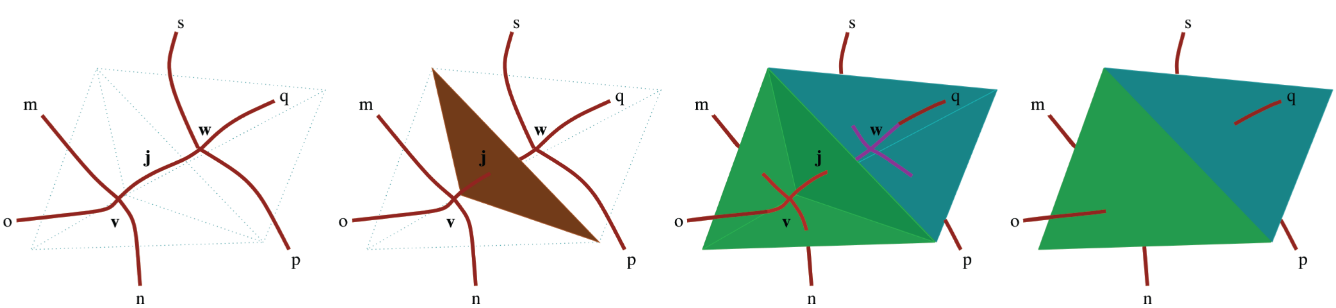
As there is no background structure the notion of *particle*, as basic excitations of a *vacuum* representing a state of minimal energy, does not exist. However, there is a natural *vacuum* associated with the state of no geometry $\widehat{\vec{E}_i}\,|0\rangle = 0$. This state represents a very degenerate quantum geometry where the distance between any pair of points is zero. The quantum version of (2), $\widehat{W_\gamma}[A]$ acts on the vacuum by creating a one-dimensional flux tube of electric field along $\gamma$. As $\vec{E}_i$ encodes the geometry of space, these fundamental *Faraday lines* represent the building blocks of a notion of *quantum geometry* as we shall see below.

These one-dimensional excitations are however not completely arbitrary as they must be subjected to the kinematical restrictions (4). For instance, $\mathrm{Div}(\vec{E}_i) = 0$ requires the flux of electric field through any arbitrary closed surface to vanish. This means that only those excitations given by closed lines of quantized electric field are allowed by quantum Einstein's equations, i.e., *loop states*. The construction of the Hilbert space of quantum gravity is thus started by considering the set of arbitrary multiple-loop states, which can be used to represent (as emphasized by Wilson in the context of standard gauge theories) the set of gauge invariant functionals of $\vec{A}_i$.

## Spin network states

Multiple-loop states can be combined to form an orthonormal basis of the Hilbert space of gravity. The elements of this basis are labelled by: a closed graph in space, a collection of spins—unitary irreducible representation of $SU(2)$—assigned to its edges, and a collection of discrete quantum numbers assigned to intersections. As a consequence of $\mathrm{Div}(\vec{E}_i) = 0$ the rules of addition of angular momentum must be satisfied at intersections. They are called *spin-network states*, see Fig. 1.

*Spin network states* are eigenstates of geometry as it follows from the rigorous quantization of the notion of area and volume (given by equations (3)). For instance, given a 2-dimensional surface $S$ one




▲ **Fig. 3:** Spin network intersections are quantum excitations of space volume. They are fundamental *atoms* of space related to one another through spin network links carrying quanta of the area associated to the extension shared by neighbouring *atoms*. The information about how the atoms are interconnected to form a quantum geometry is contained in the combinatorics of the underlying abstract graph. Here we show two 4-valent vertices connected by a link carrying spin $j$. We can interpret this portion of a spin network as being represented by two *tetrahedra* of volume $\ell_P^3 v$ and $\ell_P^3 w$ respectively sharing a face of their boundary (the brown triangle in the second diagram) with area $\ell_P^2\sqrt{j(j+1)}$.

▲ **Fig. 4:** The regulated Hamiltonian acts by attaching a Wilson loop to vertices and the loop size plays the role of an UV regulator. The regulator must be removed shrinking the loop. When the latter is small enough (last two diagrams on the r.h.s.) it can no longer entangle the gravitational field exitations around the vertex and the further reduction of the loop does not have any physical effect according to diffeomorphism invariance. The combinatorial structure of the quantum states in loop quantum gravity provides a *physical* cut-off regularizing all the interactions in LQG!

can define the quantum operator $\widehat{A(S)}$ associated to its area. It turns out that a spin network state that punctures $S$ with an edge carrying spin $j$ is an eigenstate of $\widehat{A(S)}$ with eigenvalue $\ell_P^2\sqrt{j(j+1)}$, see Fig. 1. In LQG the area of a surface can only take discrete values in units of Planck scale! Similarly, the spectrum of the volume operator $\widehat{V}(R)$ can be shown to be discrete and to be associated to the presence of spin network intersections inside the region $R$. Hence, the theory predicts a quantization of geometry.

The discovery of the discrete nature of geometry at the fundamental level has profound physical implications. In fact before completely solving the quantum dynamics of the theory one can already answer important physical questions. The most representative example (and early success of LQG) is the computation of black hole entropy from first principles in agreement with the semiclassical predictions of Hawking and Bekenstein (see Fig. 2).

Another profound implication of discreteness concerns the UV divergences that plague standard QFT's. It is well known that in standard QFT the UV problem finds its origin in the difficulties associated with the quantization of product of fields at the same point (representing interactions). A first hint of the regulating role of gravity is provided by the fact that, despite their non-linearity in $\vec{E}_i$, area and volume are quantized without the appearance of any UV divergences. This mechanism will become more transparent when we present the quantization of the Hamiltonian constraint.

## The combinatorial nature of LQG

So far we have avoided UV divergences but perhaps at too high of a price, since the Hilbert space spanned by spin network states is so large—in fact two spin networks differing by a tiny modification of their graphs are orthogonal states!—that would seem to make the theory intractable. However, one must still impose the vector constraint given in (4). This is where the crucial role background independence starts becoming apparent as the vector constraint—although is not self-evident—implies that only the information in spin network states up to smooth deformations is physically relevant. Physical states are given by equivalence classes of spin networks under smooth deformations: these states are called *abstract spin networks*.

*Abstract spin network* states represent a quantum state of the geometry of space in a fully combinatorial manner. They can be viewed as a collection of 'atoms' of volume (given by the quanta carried by intersections) interconnected by edges carrying quanta of area of the interface between adjacent atoms. This is the essence of background independence: the spin network states do not live on any preestablished space, they define space themselves.

The details of the way we represent them on a three dimensional 'drawing board' do not carry physical information. The degrees of freedom of gravity are in the combinatorial information encoded in the collection of quantum numbers of the basic atoms and their connectivity (see Fig. 3).

## Quantization of the Hamiltonian constraint

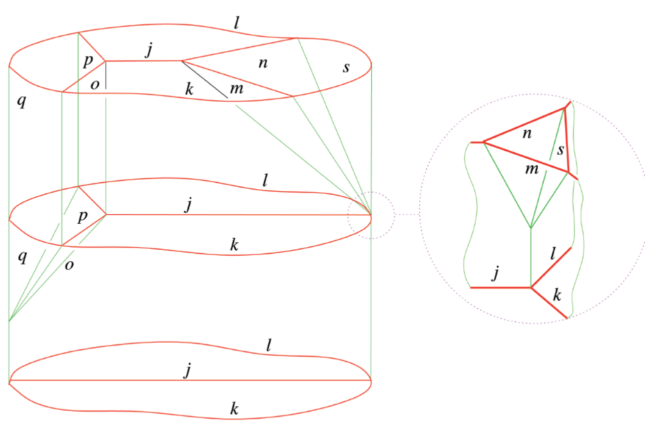
Up to this point we have constructed the kinematic setting of LQG by defining the Hilbert space satisfying the conditions (4). In order to impose the dynamical equation (6) one must quantize the Hamiltonian constraint (5). However, the non linearity of the latter brings in again the question of UV divergences in the quantum theory. From (3) and (6) one can write the Hamiltonian constraint as

$$\widehat{H} = -\frac{i}{\hbar}[\widehat{V}, \vec{A}_i]\vec{B}_i(A) \qquad (7)$$

which allows all the non linearities in $\vec{E}_i$ to be hidden in the commutator of the (free of UV singularities) volume operator and $\vec{A}_i$. As the magnetic field $\vec{B}_i(A)$ is given by the circulation of $\vec{A}_i$, one can express the non linear $A$-dependence by a non-local Wilson loop $W_{\gamma\epsilon}[A]$ around an infinitesimal loop $\gamma_\epsilon$ of size $\epsilon$. In the quantum theory $W_{\gamma\epsilon}[A]$ creates a loop excitation and $\epsilon$ is an *UV-regulator*.

As the volume operator, the regulated Hamiltonian acts only on spin network intersections, and it does so by creating a new flux excitation $W_{\gamma\epsilon}[A]$. Due to background independence the regulated Hamiltonian depends on $\epsilon$ only through the position of the newly created loop $\gamma_\epsilon$. Its action for different values of the regulator $\epsilon$ is shown in Fig. 4.

The physical Hamiltonian constraint is obtained by taking the limit $\epsilon \to 0$. In standard QFT's this process brings in all the well




▲ **Fig. 5:** A systematic control of the space of solutions is necessary to fully understand the dynamics implied by LQG. Feynman's path integral formulation can be adapted to the formalism in order to investigate this issue. Transition amplitudes that encode the dynamics of quantum gravity can be computed as sums of amplitudes of combinatorial objects representing histories of spin network states. These histories can be interpreted as quantum space-time processes and are called *spin foams*. In the figure we show a simple spin foam obtained interpolating between an 'initial' and 'final' spin network. An intermediate spin network state is emphasized as well as a vertex where new links are created as a result of the action of the quantum Hamiltonian constraint.

known UV problems that require renormalization. However, in LQG, background independence in fact assures that this limit exits without any UV divergences. For finite value of the $\epsilon$ the extra loop created by the quantum constraint can entangle the weave of links and nodes in the given *spin network* around the intersection (first three diagrams on the *l.h.s.* of Fig. 4). As $\epsilon$ becomes smaller the added loop shrinks and there is a critical value $\epsilon_c$ after which it can no longer wind around any of the neighboring links. At this point changing the value of $\epsilon$ amounts to a trivial deformation of the extra loop that, according to the combinatorial nature of the quantum states of gravity, has no physical effect. Therefore, for sufficiently small $\epsilon$ the action of the regulated constraint becomes regulator independent and the limit is defined without need of renormalization (see Fig. 4). This result also holds when coupling gravity with the matter of the standard model; the combinatorial nature of the states of quantum gravity provides a physical regulator for all interactions.

## Perspectives and Conclusions

We have discussed how the dynamical equation of quantum gravity can be promoted to a quantum operator, and how the dynamics of the theory is in the solutions of the quantum Hamiltonian constraint. Although many solutions to the equation are known, there is no complete control of the space of solutions at present. A systematic approach to investigate the solution space is the path integral representation which in the case of LQG is known as the *spin foam* approach. In it, physical transition amplitudes are computed as sums of amplitudes associated with histories of spin network states (Fig. 5). These histories can be interpreted as the quantum counterpart of space-time: they represent the quantum evolution of the quantum states of space geometry.

Despite the fact that a full understanding of the dynamics of LQG has not yet been achieved, there are interesting physical situations where one can bypass these limitations. One of these is the computation of black hole entropy briefly mentioned above and the other is the application of the framework to systems with additional symmetry. Important examples are the study of quantum cosmology and the near-singularity regime in black hole physics, where due to symmetry assumptions most of the technical problems of the full theory can be overcome. Even though these symmetry-reduced models must be regarded as toy models, as an infinite number of degrees of freedom are ignored in the treatment, there are interesting results that indicate that the singularity problem of classical general relativity would be resolved due to the fundamental discreteness predicted by LQG.

Another important open issue in LQG is the semiclassical limit: how to recover from the fundamental polymer-like excitations of LQG the smooth physics of general relativity and the standard model at low energies.

LQG realizes a unification between the principles of general covariance and those of quantum mechanics. The approach suggests that the outstanding problem of divergences in QFT's and singularities in classical general relativity are resolved when the quantum dynamical degrees of freedom of gravity are included in a background independent manner. The results obtained so far are encouraging; future research will tell whether all this is consistent with the so far elusive nature of quantum gravity. ■

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# Physics in daily life: seeing under water

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Most physicists realize that the human eye is not made for seeing under water. For one thing, if we open our eyes under water to see what's going on, our vision is blurred. The reason is obvious: since the index of refraction of the inner eye is practically that of water, we miss the refractive power of the strongly curved cornea surface. With its $1/f$ of about 40 diopters it forms an even stronger lens than the actual eye lens itself. Could we repair that with positive lenses? In view of the strong curvature of the cornea surface (radius 8 mm), the idea of replacing it by a glass lens in a water environment is beyond hope. We really need to restore the air-water interface in front of the cornea, and that is precisely what our diving mask does.

But there is more to it: under water, our field of vision is reduced dramatically. Whereas we normally have a field of more than 180° due to the refraction at the air-cornea interface, we loose that benefit once we're under water. The diving mask does not repair that, as schematically indicated in the figure.

So, scuba divers, beware! You have to turn your head much further than you may think necessary, if you want to be sure that you are not followed by a shark. ■