Rotating Bose-Einstein condensates

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At low temperature Quantum Physics can manifest itself at the macroscopic scale in many spectacular ways. Some of its most remarkable features occur when a fluid is stirred. Indeed, as a consequence of the existence of a macroscopic wave-function describing the system, a quantum fluid cannot sustain rigid body rotation by contrast with a classical fluid. Rather, the stirring generates quantized vortices. The recent observation of novel quantum fluids formed by ultra-cold atomic gases allows us to revisit the properties of vortex systems, and to address pending questions of condensed matter physics.

Quantum physics tells us that particles can be classified into two families called respectively bosons and fermions. The fermions like the electrons have a half-integer spin and obey the Pauli Principle, according to which no two identical fermions can occupy the same quantum state. By contrast, bosons have an integer spin and they can occupy the same state. Following the work of S.N. Bose on blackbody radiation, A. Einstein showed in 1924 that this gregarious bosonic trend leads at low temperature to a phase transition for a gas of independent particles, the Bose-Einstein condensation. The condensation threshold is reached when the quantum statistics effects start to be prominent, i.e. when the interatomic distance is of the order of the coherence length of the matter waves. The Bose-Einstein condensate which forms below a critical temperature contains a macroscopic number of particles, all occupying the same quantum state. These particles are thus described by the same macroscopic wave function and the system constitutes a so-called *quantum fluid*. Quantitatively the Bose Einstein condensation in an ideal gas occurs when the temperature T and the atomic density ρ satisfy the relation

$$\rho\Lambda^3_{dB} \simeq 2.6 \qquad (1)$$

where $\Lambda_{dB} = \hbar\sqrt{2\pi} / \sqrt{mk_BT}$ is the thermal wavelength, $\hbar$ and k_B are the Planck and Boltzmann constants, and m is the mass of a particle.

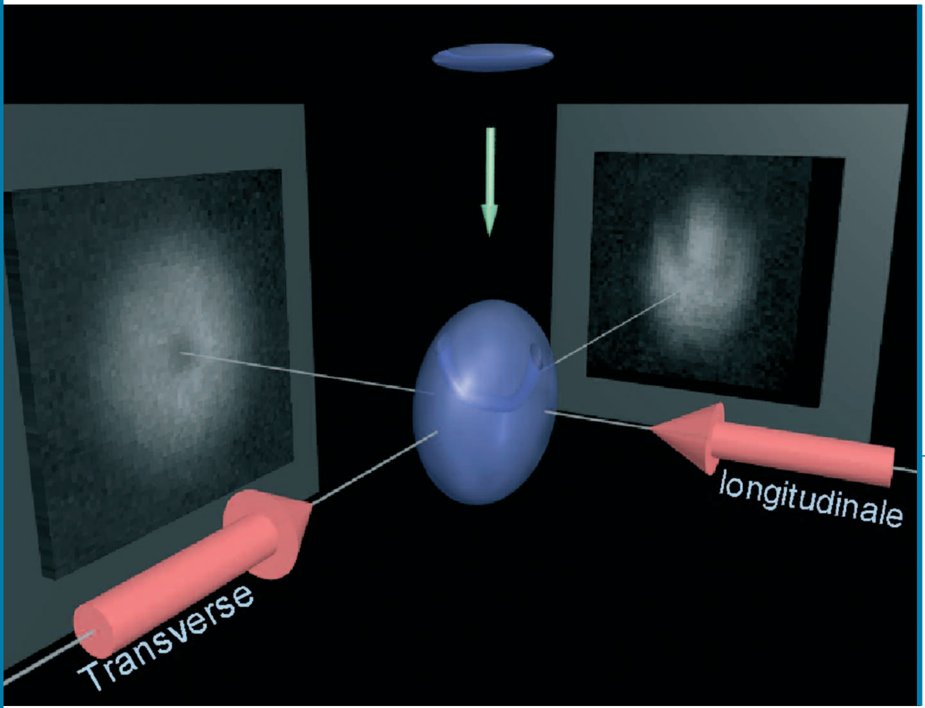
Einstein's prediction for a phase transition in a non interacting gas was first considered dubious by some physicists, and it remained unproved experimentally for more than ten years. Finally in 1938, Kapitza, Allen and Misener showed that the viscosity of liquid helium vanishes suddenly below 2.17 K. London immediately related this superfluid behaviour with Einstein's prediction. This discovery constitutes a milestone in the history of statistical physics. It marked the beginning of a fruitful research, in which many of the greatest physicists of the middle of the 20th century, such as L. Landau or R.P. Feynman, were involved. Despite some major successes, the theoretical understanding of superfluid helium remained however severely hindered by the strength of the atomic interactions in a liquid phase. The physics of liquid helium is indeed very far from the ideal gas situation considered by Einstein, which makes any *ab initio* prediction very difficult.

Gaseous Bose-Einstein Condensates.

A recent breakthrough in the history of quantum fluids was the observation in 1995 of the first gaseous Bose-Einstein condensates [1,2]. This major finding was achieved by the groups of E. Cornell and C. Wieman at Boulder, and a few months later by W. Ketterle at MIT, using rubidium and sodium atoms, respectively. These three physicists were awarded the 2001 Nobel Prize in physics for this discovery. With these systems it became possible to study Einstein's prediction in a regime of low density, thus very close to the situation addressed in the 1924 article [3-5].

These experiments have now been reproduced using several other atomic species. A few different recipes to achieve quantum degeneracy exist, but they are all based on the same basic tool called *evaporative cooling*. The atoms are trapped in a potential well, created either by a magnetic field or a focused laser beam, that one deliberately truncates at energy U_t. Consider an elastic collision between two trapped atoms: if the final energy of one of the two partners is larger than U_t, it can escape from the trap. Thanks to the repetition of such processes, the energy of the remaining particles decreases and the gas thermalizes at a temperature of the order of a fraction of U_t/k_B. The sample is cooled even further by decreasing slowly the value of U_t. When the elastic collision rate between trapped atoms is large enough, the ratio T/U_t stays constant as U_t decreases, and the condensation threshold can be reached.

In the experiments described below, we start with a rubidium vapour at room temperature. Using standard laser trapping techniques, we confine and cool 10^9 atoms in



◄ **Fig. 1:** Absorption imaging scheme. The trapping potential is switched off abruptly, letting the cloud fall and expand. We measure the absorption of two beams propagating in directions parallel and perpendicular to the symmetry axis of the trap. The laser beams are resonant with the atomic resonance transition and the density profile of the cloud is obtained from the absorption of light by the atoms.

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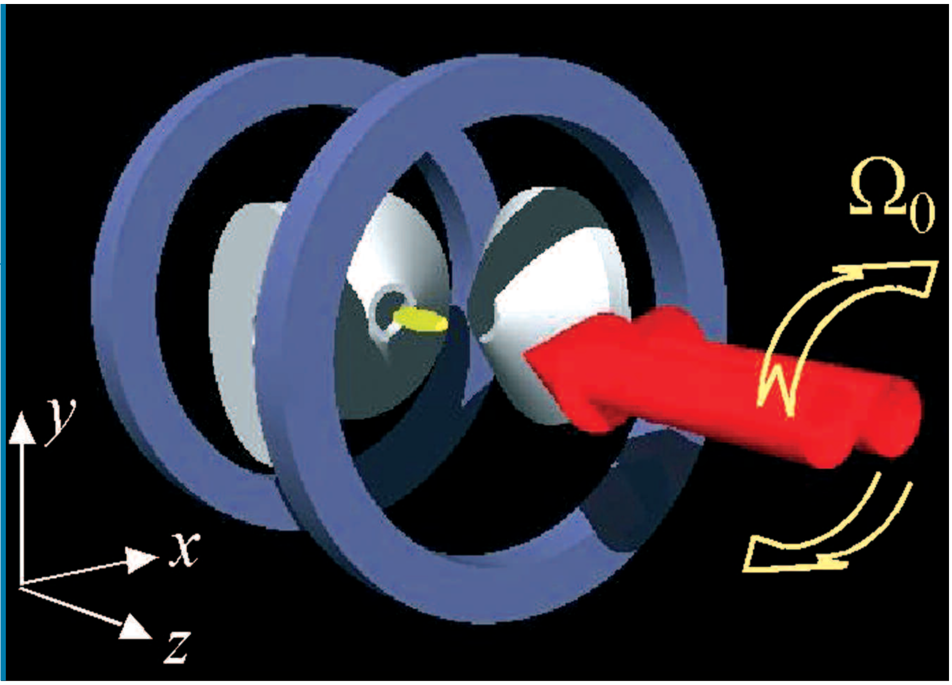
▶ **Fig. 2:** Principle of the atom spoon. We shine two laser beams (red) on the trapped atoms (yellow cigar sitting at the center of the white and blue coils that create the magnetic trapping potential). The combination of the two beams creates an optical potential whose axes rotate along the *z* axis at an angular velocity Ω_0.



a magneto-optical trap, and we transfer them in an elongated, cylindrically symmetric magnetic trap. The longitudinal and transverse frequencies of the trap are typically $\nu_{\parallel} = 10$ Hz and $\nu_{\perp} = 100$ Hz, and the initial temperature of the atom cloud is 200 microkelvins, corresponding to a phase space density $\rho\Lambda^3_{dB}$ of 10^{-6}. After a 20 second evaporation phase, we reach the threshold (1) of Bose-Einstein condensation at a temperature T_c~500 nK. By pushing the evaporation a bit further, we produce a quasi-pure condensate with N_c~3 10^5 atoms. The cloud is cigar shaped, with a 100 µm length and a 10 µm diameter. The atoms are observed by measuring the absorption of a resonant probe laser beam: we image the shadow imprinted by the atoms on this probe beam onto a CCD camera. In order to obtain a good spatial resolution, we switch off the trap abruptly and let the cloud expand freely for 25 ms before shining it with the probe laser. During this free fall, the transverse dimensions of the cloud are scaled by a factor 15, while the longitudinal dimension changes only weakly [1]. Using two orthogonal probe beams we have access to the column density of the atom gas both in the longitudinal and transverse directions (Fig. 1).

Quantized vortices

One of the most spectacular manifestations of the existence of a macroscopic wave function describing a Bose-Einstein condensate is the nucleation of quantized vortices when the system is set in rotation [6,7]. To understand why these vortices emerge, we write the condensate wave-function as $\psi(\mathbf{r}) = \sqrt{\rho(\mathbf{r})}\, e^{i\theta(\mathbf{r})}$. The quantity $\rho(\mathbf{r}) = |\psi(\mathbf{r})|^2$ is the particle density in the condensate and the phase θ is defined everywhere that the density is non zero. The "conservation of probability" yields the following relationship between the phase and the macroscopic velocity field $\mathbf{v}$

$$\mathbf{v} = \frac{\hbar}{m}\nabla\theta. \tag{2}$$

This equation leads to a paradox when one tries to predict the behaviour of a superfluid that is set into rotation at an angular velocity Ω_0, for instance when the fluid is kept in a rotating bucket. In the case of a classical fluid, the viscous drag between the walls of the vessel and the fluid generates a velocity field analogous to the one of a rotating solid, that is $\mathbf{v} = \Omega_0 \times \mathbf{r}$. The vorticity $\Omega = \mathbf{curl}(\mathbf{v})/2$ is uniform and equal to Ω_0. A well known effect of this rotation is the characteristic parabolic shape of the free surface of the liquid. However, this scenario is incompatible with equation (2) that yields a curl free velocity field. Therefore, one could expect naively that a rotating bucket experiment performed on a superfluid should leave its free surface undisturbed. This prediction is however contradictory with experimental observations that show without any doubt that the free surface of a superfluid held in a fast rotating vessel is close to a parabola!

This non-intuitive result was explained by Onsager and Feynman, who showed that equation (2) allows the vorticity to enter a Bose-Einstein condensate along phase singularity lines. Noting that the phase of a wave function is defined within 2π only, equation (2) implies that the circulation of the velocity field along a closed contour must be quantified in units of h/m, that is

$$\Gamma = \oint \mathbf{v}.\mathbf{dl} = p\,\frac{h}{m}\ . \tag{3}$$

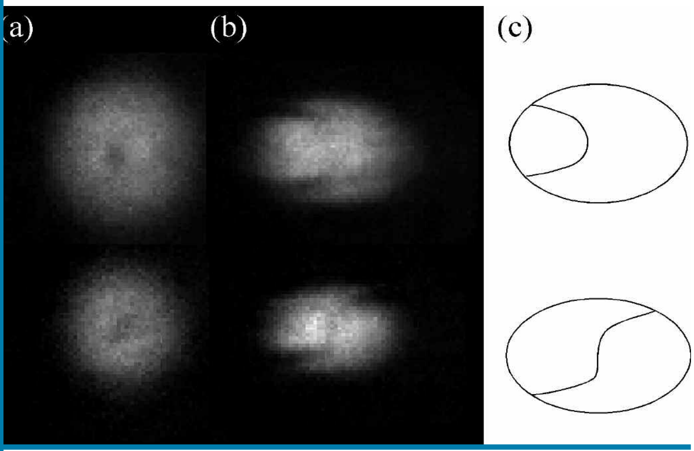
p is an integer number called the *topological charge* of the flow, and it corresponds to the winding number of the phase along the contour. In order to get a non-zero circulation, it is necessary that the contour winds around a line of zero density, along which the phase, and hence the velocity field, is no longer defined. Otherwise, Stokes theorem implies the cancellation of Γ. These zero density lines carry the vorticity of the flow and are called *quantized vortices*.

The nucleation of such quantized vortices is an experimental proof of the existence of the macroscopic wave function characterizing a Bose-Einstein condensate. Their observation was quite difficult in the case of superfluid liquid helium due to the smallness of the size of the vortex core. As shown by L. Pitaevskii in 1961, this size is of the order of the so called *healing length*

$$\xi = \frac{\hbar}{mc} \tag{4}$$

where c is the sound velocity in the fluid. In the case of helium 4, this length is of the order of a few angstroms. Vortices were observed in this system by Packard and his group in 1979. They were visualized by means of electrons trapped in the vortex cores and accelerated on a phosphorescent screen. By contrast the quantization of the circulation was demonstrated as soon as 1958 by Vinen, who studied the vibration modes of a quartz wire immersed in rotating superfluid helium. Let us also mention the experiment performed in 1985 by Avenel and Varoquaux, which proved the 2π phase slip between two superfluid helium buckets coupled by a capillary tube, when a vortex crosses the tube.

[1] This inversion of the ellipticity is a consequence of the repulsive interactions between rubidium atoms. The transverse directions being the most compressed explode faster when the trap is switched off.



▲ **Fig. 3:** Density profile of the vortex line. a) Longitudinal imaging. The vortex core is identified by the density dip at the center of the cloud. b) Transverse imaging. The vortex line (darker line in the density profile) is bent, which explains the contrast reduction in longitudinal imaging. c) Sketch of images b).

A spoon for an atomic gas

In the case of a gaseous Bose-Einstein condensate, the sound velocity is of the order of a few cm/s (to be compared with hundreds of m/s in helium), yielding a healing length in the micrometer range. The vortices are then directly detectable by optical means and were actively sought as soon as the first alkali Bose-Einstein condensates were obtained. Two strategies have been developed. The first one is based on a direct imprinting of the 2π phase shift on the macroscopic wave function and was successfully implemented by the group of E. Cornell in Boulder [8]. The second method that we developed in Laboratoire Kastler Brossel, in collaboration with V. Bretin, K.W. Madison, P. Rosenbusch and S. Stock [9] is an adaptation of the rotating bucket experiment to a gas of trapped bosons. We confine the atoms in the magnetic trap described above, and we superimpose in the transverse plane an anisotropic potential created by two laser beams propagating along the axis of the trap (Fig. 2). The axes of the anisotropic potential rotate at an angular velocity Ω_0, and this potential acts like a spoon in a cup of coffee. This stirring method has been used later at MIT (W. Ketterle's group [10]), Oxford (C. Foot's group [11]) and Boulder (E. Cornell's group [12]).

Above a certain critical angular velocity of the spoon, a first vortex is nucleated and is detected as a depression in the density profile (Fig. 3). The contrast of the density dip is however not 100%. This can be understood when looking at the transverse density profiles which clearly reveal that the vortex line is bent. This bending is a spontaneous symmetry breaking of the system; it originates from the atom interactions, which introduce a non-linear term in the Schrödinger equation satisfied by the macroscopic wave function $\psi(\mathbf{r})$.

Vortex lattices and semi-classical approximation

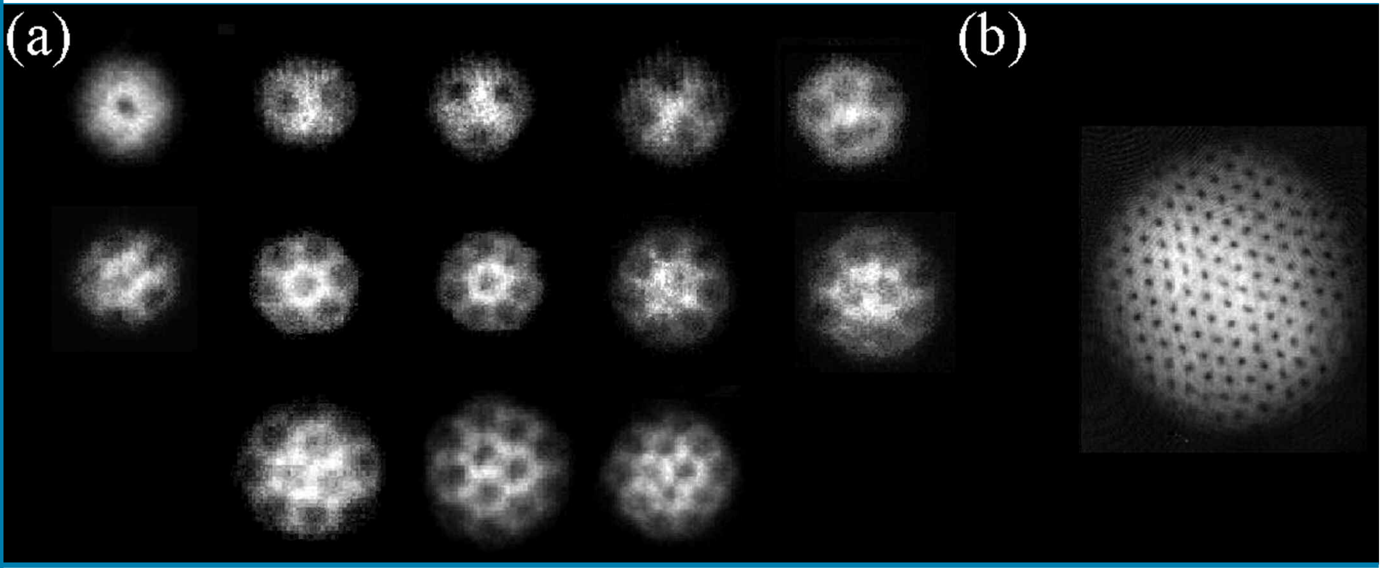
When one increases the rotation frequency above the threshold for the appearance of the first vortex, new vortices enter the condensate and form a regular lattice known as the Abrikosov lattice in the physics of superconductors (see figure 4). The equilibrium shape of this lattice results from the competition between trapping and Magnus forces. The role of the trapping force is to attract the vortices to the center of the magnetic trap (see for example figure 3 for the case of a single vortex). The Magnus force, which is well known in classical hydrodynamics, induces a repulsion between two corotating vortices.

When the number of vortices is large compared to unity, the parameters of the vortex lattice can be deduced from the correspondence principle, as first explained by Feynman. The vortices arrange themselves to form a lattice with a uniform surface density n_v, so that the coarse-grain average of the velocity field mimics the rigid body rotation for an angular frequency Ω. More precisely, although the local velocity field remains highly singular at the core of a vortex, the average velocity field has a uniform vorticity equal to 2Ω. Since each vortex corresponds to a single quantum of circulation h/m, one can deduce the relation between Ω and n_v:

$$2\Omega = \frac{h}{m} n_v \qquad (5)$$

This relation is nicely confirmed by experimental observations.

In the regime where many vortices are present inside the trapped condensate, the analogue of the parabolic profile of the free surface of a rotating liquid is the increase of the transverse diameter of the cloud. The centrifugal force reduces the transverse



▲ **Fig. 4:** Abrikosov lattice in a fast rotating Bose-Einstein condensate. The formation of this regular triangular pattern is a consequence of the repulsive Magnus force between vortices. a) Vortex lattices (with 1 to 13 vortices) obtained at ENS. b) Giant vortex lattice observed at MIT [10].

confinement of the cloud which occupies a larger volume than when it is at rest.

The regime of fast rotations

When the rotation frequency Ω is increased to a value close to the transverse trapping frequency $\omega_\perp$, the transverse size of the condensate tends to infinity since the quadratic confinement potential is nearly balanced by the centrifugal potential, which is also quadratic. The atom density ρ drops down and the healing length (which varies as $c^{-1} \propto \rho^{-1/2}$) can become arbitrarily large. Since the vortex density increases with the rotation speed, there exists a rotation frequency for which the size of a vortex core becomes comparable to the vortex spacing. Above this rotation frequency, the vortex lattice is tightly packed and the size of a vortex core is not anymore related to the healing length. It saturates to a value comparable with the distance between two adjacent vortices, which is of the order of $a_0 = \sqrt{\hbar/(m\omega_\perp)}$.

In this fast rotation regime, the physics of a rotating Bose-Einstein condensate is very reminiscent of that of a charged particle in a magnetic field. Indeed the transverse force exerted on a single atom located in $\vec{r} = (x, y)$ in the rotating frame is the sum of the trapping, centrifugal and Coriolis terms:

$$\vec{F} = -m\omega_\perp^2\,\vec{r} + m\Omega^2\,\vec{r} + 2m\vec{\Omega}\times\vec{v}.$$

For $\Omega = \omega_\perp$, we are left only with the Coriolis term $2m\vec{\Omega}\times\vec{v}$, which is formally equivalent to the Lorentz force exerted by a uniform magnetic field on a charged particle (the cyclotron frequency being equal to $2\omega_\perp$). In quantum terms, the energy eigenstates of a particle evolving in a uniform magnetic field are known as Landau levels. The regime of fast rotation, in which the vortex core size saturates to a_0, corresponds to a situation where interactions and temperature are so low that only the lowest Landau level (LLL) is populated.

The experimental investigation of this fast rotation regime is not an easy task. The stirring method described above fails when $\Omega \simeq \omega_\perp$ because of a parametric instability of the center of mass of the gas when it is stirred at a frequency close to the trapping frequency. To circumvent this problem two ways have been explored. At ENS, we have added an extra confinement potential, described by a small quartic term, which eliminates the center of mass instability. We could thus explore the region of fast rotations up to $\Omega = 1.05\ \omega_\perp$ and investigate the structure of the vortex lattice in this quadratic+quartic potential [13]. The Boulder group has kept a purely harmonic potential and implemented an "evaporative spin up" technique: the atoms with less angular momentum than average are evaporated so that the remaining atoms thermalize at a faster rotation speed. The Boulder group could then reach rotation speeds up to $\Omega \sim 0.99\ \omega_\perp$ and confirm the predictions made for the size of the vortex core (figure 5 and ref. [14]).

The regime of fast rotation in a harmonically trapped gas is very different from what is expected from an incompressible superfluid in a rigid container, or from what is known for a type II superconductor placed in a large magnetic field. In the latter case, the regime of overlapping vortices corresponds to a loss of superconductivity, whereas the harmonically trapped Bose gas simply expands over a large transverse area while keeping its coherence properties (at least as long as the number of vortices remains smaller than the number of atoms).

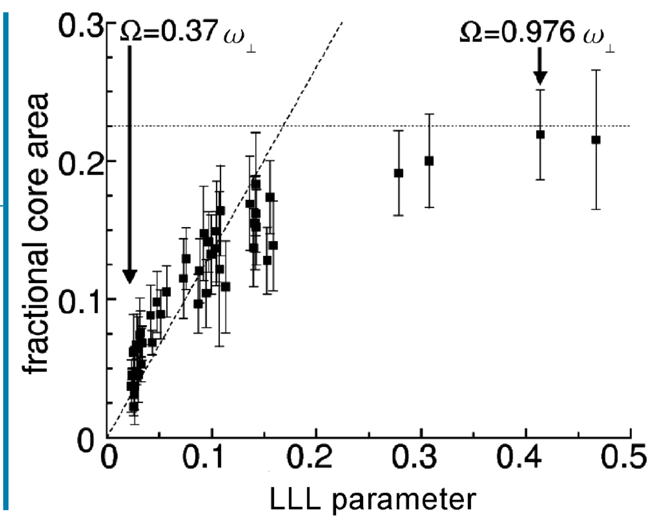
Perspectives

We have presented in this paper only a few illustrations of the fascinating physics of rotating Bose Einstein condensates. The experiments that we did not present include in particular the interferometric detection of the phase slip of the wave function around the vortex core, or the full elucidation of the vortex nucleation mechanism.

Very recently, the group of W. Ketterle (MIT) has observed a vortex lattice in a rotating *fermionic* cloud [15]. This finding might seem surprising at first sight. Indeed, as stated in the beginning of this paper, fermions cannot occupy the same quantum state, hence they should not be able to condense and form a vortex lattice. This paradox is solved by considering the attractive van der Waals interactions existing between atoms. It leads to the formation of pairs of fermions, which can themselves condense in a macroscopic quantum state. This state is very much akin to the many-body quantum state introduced by Bardeen, Cooper and Schrieffer in the 1950's to explain the superconducting behaviour of electrons in metals at low temperature. The observations of vortices in a cloud of fermions constitute a dramatic demonstration of the coherent nature of this assembly of ultra cold gases of atom pairs.

For the future, one of the most promising perspectives of this field of research deals with the regime of extremely fast rotations, where the number of vortices becomes comparable with the number of atoms. It corresponds to rotation frequencies Ω even larger than those required for reaching the lowest Landau level regime. Several theoretical studies have been performed recently on these systems, but no experimental result is yet available. In this ultra-fast rotation regime, one leaves the domain of simple Bose-Einstein condensation, where all atoms share the same macroscopic wave function. The system is expected to reach a strongly correlated state, similar to those appearing in the description of the fractional quantum Hall effect.

To summarize, the rotations of ultracold bosonic and fermionic gases have already been the subject of several studies, with topics of interest that go much beyond the simple illustration of macroscopic quantum mechanics. In particular it is quite remarkable that the investigation of vortex lattices can now be used as a tool to



▶ **Fig. 5:** Fractional core area as a function of the LLL parameter. The fractional core area is the ratio of the square of the core size (measured using a Gaussian fit of the density dip at the vortex locations) and the area of the unit cell of the vortex lattice. The LLL parameter is the ratio of the energy splitting between two Landau levels ($2\hbar\omega_\perp$) and the interaction energy characterized by the chemical potential μ. The entrance in the LLL occurs for an LLL parameter of order unity. The broken line is the prediction obtained assuming that the vortex core is proportional to the healing length ξ. The dotted line corresponds to the expected limit for the LLL (figure obtained by the group of E. Cornell at Boulder [14]).

investigate pending outstanding questions of condensed matter physics, either for strongly correlated fermions or bosonic fractional Quantum Hall systems. ■

About the authors

Frédéric Chevy after studying rotating Bose-Einstein condensates during his PhD, spent two years at College de France working on surface hydrodynamics of classical fluids. He is now part of the cold atom group at École Normale Supérieure where he works on ultra cold fermions and teaches quantum mechanics and laser physics. **Jean Dalibard** has been working in the field of quantum optics and cold atoms since 1982. He is now involved in research on quantum gases and Bose-Einstein condensates. He teaches quantum physics at École Polytechnique and École Normale Supérieure.

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Blue skies, blue seas

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For the sky, it's simple. Most physicists know that the blue colour of the sky is due to the 1/l4 dependence of Rayleigh scattering. But what about the blue of the sea? Could it be simply reflection of the blue skies by the water surface? That certainly cannot be the main story: even if the sky is cloudy, clear water from mountain lakes and seas can look distinctly blue. Moreover: those of us who like to dive and explore life under water will have noticed that, a few meters under the surface, bluish colours tend to dominate. Indeed, if we use an underwater camera and take pictures of those colourful fish, we notice that the nice red colours have almost completely disappeared. And – unlike our eyes - cameras don't lie. We need a flash to bring out the beautiful colours of underwater life. In other words: absorption is the key: sunlight looses much of its reddish components if it has to travel through several meters of water. Or ice, for that matter: remember the bluish light from ice caves or tunnels in glaciers. And even the light scattered back from deep holes in fresh snow is primarily blue.

What causes the selective absorption of visible light by water? Spectroscopists know that the fundamental vibrational bands of H-atoms bound to a heavier atom, such as in H_2O, are typically around 3 µm. This is way too long to play a role in the visible region. But wait: because of the large dipole moment of H_2O also overtone and combination bands give an appreciable absorption. And they happen to cover part of the visible spectrum, up from about 600 nm, as seen in the figure. The strong rise near 700 nm is due to a combination of symmetric and asymmetric stretch ($3\nu_1 + \nu_3$), slightly red shifted due to hydrogen bonding (see, *e.g.*, C.L. Braun and S.N. Smirnov, *J. Chem. Edu.*, 1993, **70**(8), 612). We notice that the absorption coefficient in the red is appreciable: it rises to about 1 m^{-1} around 700 nm, an attenuation of a factor of *e* at 1 m. It is no wonder that our underwater pictures turn out so bluish.

It is interesting to note: the spectrum of D_2O is red shifted by about a factor 1.4, since the larger mass of the deuterons makes for much more slow vibrations. It is therefore shifted out of the visible region.

But that is not the whole story about the 'deep blue sea'. For the water to look blue from above, we need backscattering. For shallow water, this may be from a sand bottom or from white rock. In this case the absorption length is twice the depth. For an infinitely deep ocean, however, we have to rely on scattering by the water itself and by possible contaminants. This even enhances the blue color by Rayleigh scattering, as long as the contaminants are small.
If the water gets really dirty, things obviously become more complex. Scattering from green algae and other suspended matter may shift the spectrum towards green, or even brown.

But clear water is blue. Unless it's *heavy* water, of course… ■