differences, that are critical for clinical purposes. However, the mean JGWS values are significatively larger in the ictal than in the pre- and post-ictal epochs for all $q \geq 1$.

The present article described informational tools derived from the orthogonal discrete wavelet transform and their application to the analysis of brain electrical signals. The quantifier (relative wavelet energy) RWE provides information concerning the relative energy associated with different frequency bands that are to be found in the EEG and enables one to ascertain their corresponding degree of importance. Our second quantifier, normalized wavelet entropy (GWS), carries information about the degree of order/disorder associated with a multi-frequency signal response. Finally, our third quantifier, the statistical wavelet complexity (JGWC), provides us with a measure that reflects the intricate structures hidden in the brain-dynamics.

In particular, it becomes clear that the ERR behavior reported by Gastaut and Broughton [1] for generalized TCES is accurately described by the RWE quantifier. Moreover, the reported study does not require the use of curare or of digital filtering. In addition, a significant decrease in the entropy was observed in the recruitment epoch, indicating a more rhythmic and ordered behavior of the EEG signal, compatible with a dynamical process of synchronization in the brain activity. In addition the recruiting phase also exhibits larger values of statistical complexity.

It is well established that an EEG is directly proportional to the local field potential recorded by electrodes on the brain's surface. Furthermore, one single EEG electrode placed on the scalp records the aggregate electrical activity from up to 6 cm^2 of the brain surface, and hence from many millions of neurons. With such large numbers, is seems quite natural to model the neocortex as a continuous sheet of neurons (neuronal matter) whose activity varies with time. Taking into account the available results for (*i*) the chaoticity index (the largest Lyapunov exponent with stationary constraints removed) as a function of time and (*ii*) the largest Lyapunov exponent for selected portions of the EEG signal, one can confidently assert that a chaotic behavior can be associated with the whole EEG signal. This chaoticity becomes smaller during the recruiting phase [2]. As pointed out by many authors (see for instance [9]), the coexistence of chaos with ordering and increasing complexity for extended system is a manifestation of self-organization. We can thus suggest, on the basis of experimental EEG data and using appropriate statistical tools, that in the case of tonic-clonic epileptic seizures, the epileptic focus triggers a self-organized brain state characterized by both order and maximal complexity.

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Long-range memory and nonextensivity in financial markets

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Perhaps one of the most vivid and richest examples of the dynamics of a complex system at work is the behavior of financial markets. The price formation process of a publicly traded asset is clearly the product of a multitude of evasive interactions. Individuals around the globe post orders to buy or sell a particular stock at a particular price. Transactions are cleared at a certain price at a given time, either by passing through the hands of a specialist on the trading floor, or automatically on the many electronic markets which have flourished along with technological advances over the past few years (Fig. 1). Apart from fundamental properties of the company whose stock is being traded, factors such as supply and demand clearly must affect the price of stocks, as well as general trends in the particular industry in question. Stock specific events, such as mergers and acquisitions, have a big impact, as do world events, such as wars, terrorist attacks and natural disasters.

Time series of financial data exhibit highly nontrivial statistical properties. What is quite fascinating is that many of these anomalous properties appear to be universal, in the sense that they are present in a variety of different asset classes, ranging for example from commodities such as wheat or oil, to currencies and individual stocks. Furthermore they are present across the geographical borders, and can be observed among others in US, European and Japanese markets.

Finding a somewhat realistic model of price variations that can capture the spectrum of interesting statistical features inherent in real data is a challenging task, important for many real-world reasons, such as risk control, the development of trading strategies, option pricing and the pricing of credit risk to name a few. Bachelier's random walk model in 1900 was the first attempt of a mathematical model of price variations. While a century ago this Gaussian stochastic process was state-of-the-art, and indeed lies at the bottom of the celebrated Black-Scholes option pricing formalism, we now know Figure 2: The empirical distribution of daily returns from the stocks comprising the SP 100 (red) is fit very well by a q-Gaussian with $q = 1.4$ (blue). that it is way too simple to describe the properties of real data. In fact, during the past decade, there has been an increasing and widespread access to data extracted from financial markets. This includes for example every single trade and quote of all stocks traded on the New York Stock Exchange, records from various electronic markets, the entire order book data from the London stock exchange, to name just a few sources. These vast amounts of historical stock price data have helped establish a variety of so-called stylized facts [1, 2], which can be seen as statistical signatures, of financial data.

The best known stylized fact is perhaps the distributions of returns (defined as logarithmic relative price changes). On time scales ranging from minutes to weeks these have fat tails, exhibiting

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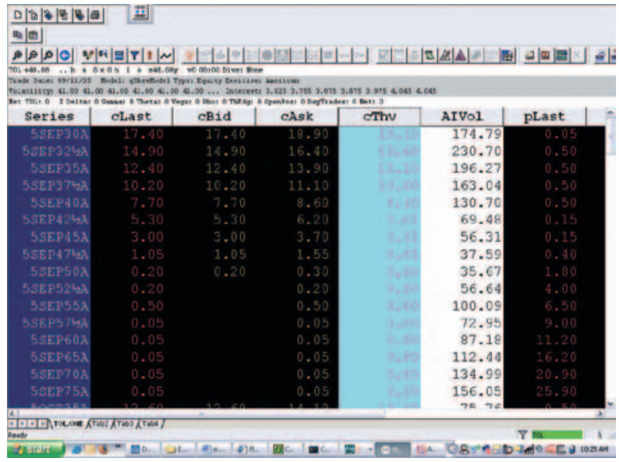
power-law decay and are modeled quite well by the Tsallis or Student form [1] (Fig. 2).As the time-scale over which one calculates the price changes increases to months or years, the distribution does become closer to a Gaussian. In addition, there is a long range memory in volatility fluctuations, evident because the autocorrelation of the volatility decays only slowly as a power law. This leads to bursts of higher or lower volatility in the time series of returns, a phenomenon also referred to as volatility clustering, and furthermore the distribution of the instantaneous volatility is close to log-normal. Also, there are certain asymmetric correlations in that large past price changes imply large future volatilities, an effect called leverage. In addition to these stylized facts, are also more subtle ones which have been elucidated in recent years. Examples are multifractal scaling, a financial analogue of the Omori law for earthquakes (in other words, large volatility shocks tend to be followed by after-shocks decaying in magnitude according to a power law), as well as the statistical asymmetry under time reversal, implying the rather obvious fact (which however is not present in most models of price fluctuations!) that financial time series differentiate the past from the future.

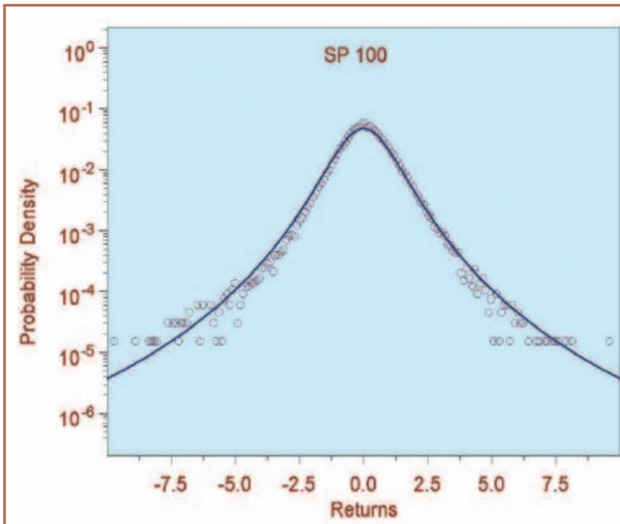
Several different models have been proposed [3] in an attempt to capture fat tails and volatility clustering which don't exist in the Gaussian Bachelier model. Popular approaches include Levy processes, which induce jumps and thus fat tails on short time-scales, but convolve too quickly to the Gaussian distribution as the time-scale increases and do not present volatility clustering. Stochastic volatility models, such as the Heston model where the volatility is assumed to follow its own mean-reverting stochastic process, reproduce fat tails, but not the long memory observed in the data. The same holds true for the simplest of Engle's Nobel prize winning GARCH models in which the volatility is essentially an autoregressive function of past returns. Multifractal stochastic volatility models (similar to cascade models of turbulent flow) are another promising candidate (cf [2]), reproducing many of the stylized facts, lacking mainly in that they are strictly time reversal symmetric in contrast to empirical evidence.

In addition, most of the above mentioned models are difficult if not impossible to deal with analytically. Analytic tractability is desirable for reasons such as efficiently calculating the fair price of options or other financial derivatives which in their own right are traded globally in high volumes. They fill important financial functions with respect to hedging and risk control, as well as offer purely speculative opportunities. In short, options are financial instruments which depend in some contingent fashion on the underlying stock or other asset class. The simplest example is perhaps the European call option. This is the right (not obligation) to buy a stock at a certain price (called the strike) at a certain time (called the expiration) in the future. Contracts similar to options were exploited already by the Romans and story has it that Thales, the Greek mathematician, used call options on olives to make a huge profit when he had reason to believe that the harvest would be particularly good. In Holland in the 1600s, tulip options were traded quite a bit by speculators prior to the famous tulip bubble. But it wasn't until 1974 that the fair price of options could be calculated somewhat reliably with the publication of the Nobel-prize winning Black-Scholes formula. This is still the most widely used option pricing model, not because of its accuracy (since it is based on a Gaussian model for stock returns which, as we discussed above, is unrealistic) but rather due to its mathematical tractability (which exists due to the same Gaussian assumptions). In fact, an impressive school of mathematical finance has been developed over the past three decades, and is based largely on notions stemming from the famous Black-Scholes paradigm.

Because real stock returns exhibit fat tails, yet the Black-Scholes pricing formula is based on a Gaussian distribution for returns, the probability that the stock price will expire at strikes far from its current price will be underestimated. Traders seem to correct for this intuitively; for the Black-Scholes model to match empirical option prices, higher volatilities must be used the farther away the strike price is from the current stock price value. A plot of these Black-Scholes implied volatilities as a function of the strike price is thus not constant but instead most typically a convex shape, often referred to as the volatility smile. This way of representing option prices in terms of the Black-Scholes volatility is so widely used that prices are often quoted just in terms of this quantity, most often referred to simply as *the vol.*



▲ **Fig.1:** A typical electronic trading screen. Here, live quotes on options are fed in from the market and the trader can execute electronically.



▲ **Fig.2:** The empirical distribution of daily returns from the stocks comprising the SP 100 (red) is fit very well by a *q*-Gaussian with $q = 1.4$ (blue).

[1] The Tsallis distribution (also referred to as a q-Gaussian) is equivalent to the Student distribution whenever q is a rational of the form (3 + n)=(1 + n), where n is a positive integer denoting the number of degrees of freedom.

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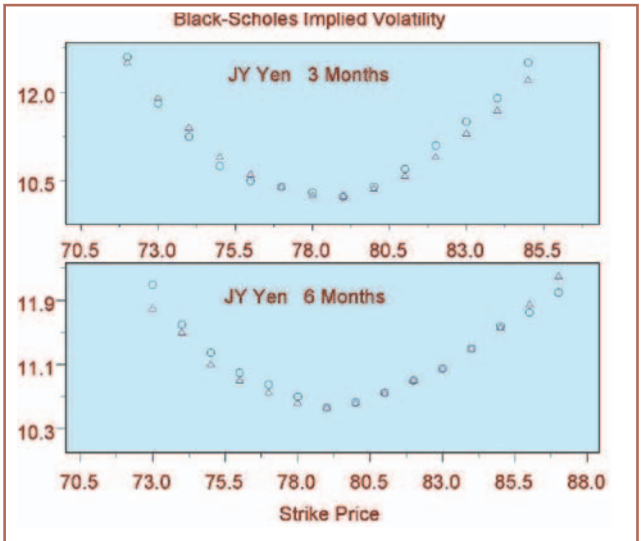

▲ **Fig.3:** Theoretical implied Black-Scholes volatilities from the $q = 1.4$ model (triangles) match empirical ones (circles) very well, across all strikes and for different times to expiration.

From all that has been said up to now, it is really quite clear that the *true* model of stock price fluctuations has many challenging statistics to reproduce, in addition to correctly pricing derivative instruments. Furthermore, it would be desirable that the mechanisms of such a model are somewhat intuitive. With these goals in mind, the field of nonextensive statistical mechanics has made some progress in recent years although of course there is still a long way to go both within and beyond this framework.

As already mentioned in passing, returns (once demeaned and normalized by their standard deviation) have a distribution that is very well fit by q-Gaussians with $q \approx 1.4$ [4], only slowly becoming Gaussian ($q \rightarrow 1$) as the time scale approaches months or years. Another interesting statistic which can be modeled within the nonextensive framework, is the distribution of volumes, defined as the number of shares traded. A q-exponential multiplied by a simple power of the volume presents power laws at both high and low volumes and fits very well to the data [4]. These results are encouraging, albeit they are macroscopic descriptions of the data and a dynamical description of the underlying processes is of course desirable. For the volumes, such a model was recently proposed. For stock prices, a class of models that has had some success in option pricing was introduced a few years ago [5], based upon a statistical feedback process. Recently, that model was extended to incorporate memory over multiple time-scales [6] (recovering a class of long-ranged GARCH models [7]) and seems to reproduce most of the stylized facts of financial time series. Other interesting models related to the nonextensive thermostatistics include an ARCH process with random noise distributed according to a q-Gaussian as well as some state-dependent additive-multiplicative processes [8]. These models do capture the distribution of returns, but not necessarily the empirical temporal dynamics and correlations.

In the statistical feedback model, price fluctuations are assumed to evolve such that the Tsallis entropy is maximized. This leads to an instantaneous volatility which is proportional to a power of the probability of the most recent price: It is large when price moves are exceptionally large (or rare); conversely, the volatility is smaller if the price moves are more moderate (or common). This mechanism is an attempt to model the collective behavior of market players. The statistical feedback tries to reflect market sentiment. Mathematically, it leads to a non-linear diffusion equation for the price. Exact time-dependent solutions to this equation can be found resulting in a Tsallis distribution for price changes at all times, and volatility clustering is also present. The entropic index q which characterizes the resulting distribution depends on the power of the statistical feedback term. If $q = 1$, the power vanishes so there is no statistical feedback and the standard Gaussian model is recovered. If $q > 1$, the power is negative and fat tails are present. This model has been quite successful for the purpose of option pricing, again largely due to the fact that one can actually calculate a lot of things analytically, and in particular one obtains closed-form solutions for European options.

Since a value of $q = 1.4$ nicely fits real returns over short to intermediate time horizons (corresponding to 4 degrees of freedom with the Student formulation), this model is clearly more realistic than the standard Gaussian model. Using that particular value of q as calibrated from the historical returns distribution, fair prices of options can be calculated easily and compared with empirical traded option prices, exhibiting a very good agreement. In particular, while the Black-Scholes equation must use a different value the volatility for each value if the option strike price in order to reproduce theoretical values which match empirical ones, the $q = 1.4$ model uses just one value of the volatility parameter across all strikes. One can calculate the Black-Scholes implied volatilities corresponding to the theoretical values based on the $q = 1.4$ model, and a comparison of with the volatility smile observed in the market will reflect how closely the $q = 1.4$ model fits real prices (Fig. 3).

Although quite successful, this model is not entirely realistic. The main reason is that there is one single characteristic time in that model, and in particular the effective volatility at each time is related to the conditional probability of observing an outcome of the process at time *t given what was observed at time* $t = 0$. For option pricing this is perfectly reasonable; one is interested in the probability of the price reaching a certain value at some time in the future, based entirely on one's knowledge now. But this is a shortcoming as a model of real stock returns; in real markets, traders drive the price of the stock based on their own trading horizon. There are traders who react to each tick the stock makes, ranging to those reacting to what they believe is relevant on the horizon of a year or more, and of course, there is the entire spectrum in-between. Therefore, an optimal model of real price movements should attempt to capture this existence of multiple time-scales and long-range memory.

Indeed, by including a kind of statistical feedback over multiple timescales a model is obtained which seems to account for most stylized facts of financial time series (Fig. 4). The distribution of returns are fit well by Tsallis-Student distributions. As the time horizon of returns increase, the distribution approaches Gaussian in the same way as empirical data. Long range volatility clustering is present, with a decay that matches real data. The distribution of instantaneous volatility is close to log normal. Subtle effects like the multifractal spectrum and Omori analogue are reproduced. In particular, the time-reversal asymmetry is inherent. Although some of these statistics can be calculated analytically, most are obtained through numerical simulation. In principle, the model can be used for option pricing via Monte-Carlo simulations, but analytic option pricing formulae would be very welcome. Obtaining these is still an open problem.

A final interesting remark on the implications of this model is that the parameters which calibrate to empirical data put the model close to an instability. This suggests that the dynamics of financial markets are operating on the brink of non-stationarity. In fact, this mathematically perhaps undesirable property is philosophically quite desirable; in reality, financial markets indeed appear only quasi-stationary and as we have seen historically, they can completely break down and crash. Another implication of the model is that it predicts a large correlation between the present volatility and past price changes, which was verified empirically. This questions the efficient market hypothesis which states that all information relevant to the stock is immediately absorbed and reflected in the price and that the past price history can have no influence on investor behavior. The long-memory volatility model states otherwise however.

To bring an analogue to physics, this fact is in a sense akin to the notion that Boltzmann-Gibbs statistical mechanics works well for systems with short-range interactions and short-term memory. If long-range interactions or long-memory is present, one must go beyond the standard framework. In a similar fashion, the long-memory in financial markets forces us beyond the standard paradigm and we must perhaps rethink some very well-established ideas in a new light.

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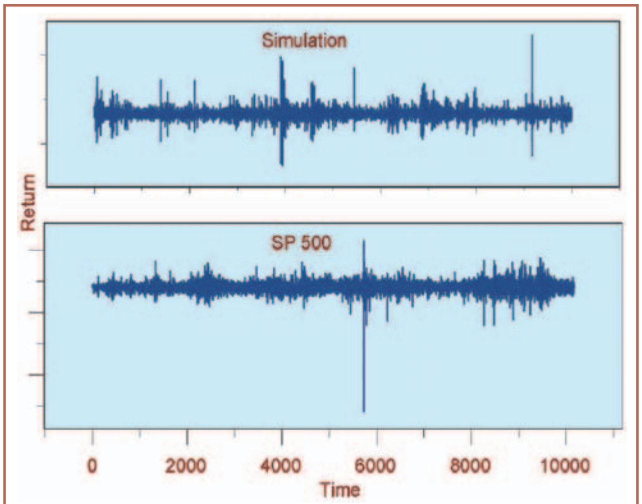

▲ **Fig.4:** A time series of returns simulated with the long-memory multiple time-scale feedback model (top) and the daily returns since 1965 of the SP 500. The simulated series reproduces most of the stylized facts of the real data.

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