same values for both the ferromagnetic and paramagnetic phase. Instead, in the QSS regime magnetization correctly vanishes with $N^{-1/6}$ while polarization remains constant at a value 0.24 ±0.05. This does strongly indicate that we can consider the QSS regime as a sort of glassy phase for the HMF model. Again, it is important to stress the role of the initial conditions in order to have dynamical frustration and glassy behaviour. Actually, glassy features are very sensitive to the initial kinetic explosion that produces the sudden quenching and dynamical frustration. In particular, reducing M_0 from 1 to 0.95 the polarization effect and the hierarchical clusters size distributions become much less evident, until they completely disappear decreasing further M_0 [10]. In this sense, the M_0=1 initial conditions seems to select a special region of phase space where the system of rotators described by the HMF Hamiltonian behaves as a glassy system. We note in closing that this result gives also a further support to the broken ergodicity interpretation of the QSS regime of Tsallis themostatistics.

Conclusions

Summarizing the HMF model and its generalization, the a-XY model, provide a perfect benchmark for studying complex dynamics in Hamiltonian long-range systems. It is true that several questions remain still open and need to be further studied with more detail in the future. However the actual state of the art favours the application of Tsallis thermostatistics to explain most of the anomalies observed in the QSS regime. The latter seems to have also very interestings links to glassy dynamics.

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Complexity of seismicity and nonextensive statistics

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Seismic time series is basically composed of the sequence of occurrence time, spatial location and value of magnitude of each earthquake. In other words, seismic moments (their logarithms being values of magnitude) as amplitudes are defined at discrete spacetime points. Seismicity is therefore a field-theoretic phenomenon. However, unlike a familiar one such as an electromagnetic field, it is inherently probabilistic in both magnitude and locus in spacetime. It is characterized by diverse phenomenology, accordingly. Known classical examples are the Gutenberg-Richter law and the Omori law. The former states that the frequency of earthquakes with magnitude larger than M, $N(>M)$, is related to M itself as $\log N(>M) = A - bM$, where A and b are constants and, in particular, $b \approx 1$ empirically. Magnitude is related to the emitted energy, E, as $M \sim \log E$. The Omori law describes the temporal rate of aftershocks following a main shock. The number of aftershocks between t and $t + dt$, $dN(t)$, after a main shock at $t = 0$, empirically decays in time as $(t + t_0)^{-p}$, where t_0 and p are constants with p varying between 0.6 and 1.5 according to real seismic data. It is noticed that both of them are power laws, i.e., they have no characteristic scales, pronouncing complexity and criticality of the phenomenon.

Nonextensive statistics (Tsallis statistics or q-statistics) [1] has been attracting continuous attention as a possible candidate theory of describing statistical properties of a wide class of complex systems. It is a generalization of Boltzmann-Gibbs equilibrium statistical mechanics, and is concerned with nonequilibrium stationary states of complex systems. Since complex systems reside at the edge of chaos in the language of dynamical systems, ergodicity, which is a fundamental premise in Boltzmann-Gibbs statistics, may be broken. That is, at the level of statistical mechanics, long-time average and ensemble average of a physical quantity do not coincide. Clearly, the concept of ergodicity does not apply to seismicity because of the absence of the ensemble notion in its nature.

Though the complete dynamical description of seismicity is still out of reach, some models have been proposed in the literature to explain some features. Among others, the spring-block model, the self-organized criticality model and mean-field models such as the coherent noise model are frequently discussed (the last one will be visited here later).

Criticality and nonergodicity of real seismicity may lead to the question if some of its physical aspects can phenomenologically be described by nonextensive statistics and related theoretical treatments. The answer to this question turns out to be affirmative.

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Abe and Suzuki [2] have studied long-time statistics of the spatial distance between two successive earthquakes by using two sets of the data taken in Japan and California. They have found that, over the whole ranges, both of the data are fitted extremely well by the q-exponential distributions with $0 < q < 1$. Here, the q-exponential distributions are the maximum Tsallis entropy distributions [1], where q is the index of the Tsallis entropy [1]. Then, they have further analyzed long-time statistics of the time interval between two successive earthquakes, termed "calm time" (or, interoccurrence time), and have ascertained [2] that, as the spatial distance, the calm time also obeys the q-exponential distributions both in Japan and California with $q > 1$ over the whole ranges. The q-exponential distribution has the form: $\sim \exp_q(-\beta Q)$, where β is a positive constant and Q is a positive random variable, i.e., the distance or the time interval between two successive earthquakes, in the present case [see Box in the Editorial by J. P. Boon and C. Tsallis for the definitions of the q-exponential function, $\exp_q(x)$ and its inverse, the q-logarithmic function, $\ln_q(x)$].

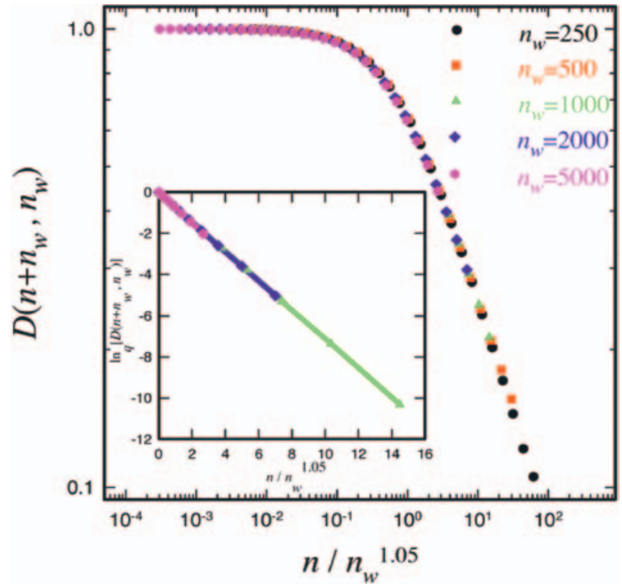
The q-exponential distribution with $q > 1$ is equivalent to the so-called Zipf-Mandelbrot power-law distribution, which is known to be derivable from several independent approaches including the maximum Tsallis entropy principle. On the other hand, the q-exponential distribution with $0 < q < 1$ has a cutoff at a finite value of the random variable under consideration. Simultaneous derivation of both $q > 1$ and $0 < q < 1$ cases is considered to be realized only by the maximum Tsallis entropy principle. Another point of interest is that the sum of the q-indices of the distributions of the spatial distance and the calm time in real seismicity is found to be close to 2, both in Japan and California. In nonextensive statistics, the theories with $q(> 0)$ and $2 - q(> 0)$ are said to be *dual* to each other. Therefore, seismicity exhibits "spatio-temporal duality".

Thus, nonextensive statistics well explains spatio-temporal complexity of real seismicity in its long-time behavior. In the short-time scale, however, the seismic data is nonstationary and highly structured, in general. Such features are mainly due to aftershocks following main shocks. A time interval of the seismic time series, in which the events obey the Omori law for aftershocks, is referred to as "Omori regime". Recently, Abe and Suzuki have discovered [3] that nonstationarity of the time series of aftershocks obeys a peculiar law for event-event correlation. The event-event correlation function proposed in Ref.[3] is given by

$$C\left(n+n_w, n_w\right) = \frac{\left\langle t_{n+n_w} t_{n_w}\right\rangle - \left\langle t_{n+n_w}\right\rangle \left\langle t_{n_w}\right\rangle}{\left(\sigma^2_{n+n_w}\sigma^2_{n_w}\right)^{1/2}} \qquad (1)$$

In this equation, t_n is the time when the nth shock after a given main shock occurs. The averages and the variance are defined by $\langle t_m\rangle = (1/M)\sum_{k=0}^{M-1} t_{m+k}$, $\langle t_m t_{m'}\rangle = (1/M)\sum_{k=0}^{M-1} t_{m+k} t_{m'+k}$ and $\sigma_m^2 = \langle t_m^2\rangle - \langle t_m\rangle^2$, respectively. M is the number of the successive events taken inside the Omori regime under consideration. Comparing Eq. (1) with the ordinary autocorrelation function, one sees that the basic random variable is t_n, which is the occurrence time of the nth event. In other words, n plays a role of a certain time parameter of the discrete time series. Such a time parameter is called "natural time", which has been introduced by Varotsos and his collaborators [4]. Accordingly, n_w in Eq. (1) is termed "natural waiting time".

A striking property reported in Ref.[3] is that event-event correlation of aftershocks exhibits the *aging phenomenon* with respect to the natural waiting time (this phenomenon will be explained below). In addition, it also obeys a definite scaling property. The correlation function in Eq. (1) is found to satisfy the functional relation: $C(n + n_w, n_w) = \tilde{C}(n/f(n_w))$. The relation of this kind is called the scaling relation, and the associated function $\tilde{C}$ of a single argument is termed the scaling function. For earthquake aftershocks, $f(n_w)$ is empirically given by $f(n_w) \sim (n_w)^\gamma$ with a positive exponent γ. It is emphasized that these properties are revealed by the use of the natural time, not the conventional time.



▲ **Fig. 1:** Data collapse of the aging curves of the event-event correlation functions for different values of the natural waiting time. Inset: the semi-q-log plot of the collapsed curve (see Box in the editorial paper for the definition of the q-logarithmic function). The straight line implies that the scaling function is of the q-exponential form with $q \simeq 2.98$. The ensemble average is performed over 120000 numerical runs. All quantities are dimensionless.

The above result sheds new light on the physical nature of aftershocks. The crust has a complex landscape regarding the stress distribution at faults. A main shock can be regarded as a kind of quenching process. Aftershocks following a main shock give rise to nonequilibrium nonstationary process. It is a slow relaxation process due to the power-law nature of the Omori law. Combining these features with the above-mentioned aging phenomenon and scaling relation, one recognizes that the mechanism governing aftershocks may be of *glassy dynamics*. This observation is of particular interest since, according to the recent investigations of Baldovin and Robledo (e-print cond-mat/0504033) and of Pluchino, Rapisarda and Latora (e-print cond-mat/0507005), there is some evidence which suggests the existence of a deep connection between nonextensive statistical mechanics and glassy dynamics.

The Gutenberg-Richter law has been a touchstone for any model of earthquakes. Real seismicity is however characterized by much richer phenomenology. The novel laws, phenomena and relations mentioned above give stringent criteria for modeling. From the physical viewpoint, what is more important is to examine how these properties are universal for complex systems with catastrophes. Model simulation may be useful for this purpose.

Recently, Tirnakli and Abe [5] have numerically reanalyzed a simple mean-field model called the coherent noise model in order to examine if the aftershocks described by it can exhibit the aging and scaling properties. This model is already known to describe both the Gutenberg-Richter law and the Omori law. In the analysis, a main shock is identified and the associated Omori regime is fixed. As expected, the model is nonergodic and, accordingly, the time average and the ensemble average of a physical quantity are different from each other. We have ascertained that, for the event-event correlation function (defined by the natural-time average) in Eq. (1), the model well reproduces the aging and scaling properties (together with the form of the scaling function, $\tilde{C}$) discovered in Ref.[3] for real seismicity. On the other hand, an intriguing feature was found for the event-event correlation function defined by the ensemble average with respect to numerical runs. To distinguish such a correlation function from the one with the natural-time averages in Eq. (1), it is denoted here by $D(n + n_w, n_w)$. This quantity also turned out to exhibit the aging phenomenon with respect to the natural waiting time, that is, *the smaller the value of the natural waiting time is, the faster correlation decays.* So, the system has an internal clock. Fig. 1 shows that the aging curves become collapsed to a single curve by the rescaling of the natural time: $D(n + n_w, n_w) = \tilde{D}(n/(n_w)^{1.05})$, establishing the scaling property. The inset presents its semi-q-log plot. The straight line there implies that the scaling function, $\tilde{D}$, is given by the q-exponential function.

Now, according to Tsallis [6], there may be "q-triplet" $\{q_{stat}, q_{sen}, q_{rel}\}$ for a system described by nonextensive statistics, where q_{stat} is the entropic index appearing in the maximum Tsallis entropy distribution as well as the Tsallis entropy itself, q_{sen} is the index characterizing sensitivity of a nonlinear dynamical system to the initial condition and q_{rel} controls the rate of relaxation and decay of correlation. In the case of a simple system described by Boltzmann-Gibbs-type statistics, the q-triplet may be given by $\{q_{stat}, q_{sen}, q_{rel}\} = \{1, 1, 1\}$. In the case of a complex system, physical quantities are often expressed empirically in terms of the q-exponential function: e.g., probability distributions (q_{stat}), the distance between two trajectories of a dynamical system (q_{sen}) and relaxation or correlation (q_{rel}), with the values all different from unity. An example is provided by the recent work done by the people from NASA [7], who have discovered a non-Boltzmann-Gibbs case in a single physical setup. Analyzing the fluctuating magnetic field strength observed by Voyager 1 in the solar wind, they have found that $\{q_{stat}, q_{sen}, q_{rel}\} = \{1.75 \pm 0.06, -0.6 \pm 0.2, 3.8 \pm 0.3\}$.

Therefore, the q-exponential scaling function obtained for aftershocks in the coherent noise model, $\tilde{D}$, with $q_{rel} \simeq 2.98$, which is notably different from unity, could be seen as a fingerprint of further relevance of nonextensive statistics.

Science of complexity certainly enables one to reveal novel aspects of real seismicity. Nonextensive statistics is expected to offer a guiding principle for a deeper understanding of complex dynamical systems with catastrophes, in general and complexity of seismicity, in particular.

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S_q entropy and self-gravitating systems

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More than fifteen yeas ago, a generalized thermostatistical formalism (usually referred to as nonextensive statistical mechanics) based on a power-law entropic measure S_q was advanced by Constantino Tsallis [1]. Increasing attention has been paid to the Tsallis proposal in recent years, because it has been hailed by many researchers as a useful tool for the description of certain aspects of physical scenarios exhibiting atypical thermodynamical features due, for instance, to the presence of long range interactions. When the Tsallis formalism appeared in 1988, it was not at all clear what possible physical applications it might have. The first hint pointing towards a relationship between the Tsallis' ideas and the thermodynamics of systems with long range interactions came in 1993 [2], when it was realized that the Tsallis entropic functional is closely related to a family of distribution functions well known by astronomers studying the dynamics of stellar systems: the polytropic stellar distributions. In point of fact, the discovery of the connection between Tsallis entropy and the stellar polytropic distribution constituted the first application of the Tsallis' formalism to a concrete physical problem. Stellar systems, such as stellar clusters or galaxies, are important examples of astrophysical self-gravitating systems, where the gravitational interaction between the constituents of the system play a fundamental role in determining the system's properties. The exploration of the connections between the Tsallis thermostatistical formalism and the physics of self-gravitating systems has been the focus of a considerable research activity in recent years [3-7].

Nonextensive statistical mechanics is built up on the basis of the nonextensive, power-law entropy S_q [1]. The entropic index q (also called the Tsallis' entropic parameter) characterizes the statistics we are dealing with. In the limit $q \rightarrow 1$ the usual Boltzmann Gibbs (BG) expression is recovered: $S_1 = S_{BG}$ (for the definitions of the basic quantities associated with the q-entropy see the Box in the editorial introduction by Tsallis and Boon to the present Issue). Optimizing the entropic measure S_q under the constraints imposed by normalization and the mean value of the energy, one obtains the probability distribution associated (in the context of Tsallis' formalism) with the thermal equilibrium or metaequilibrium of the system under consideration. The main property of this q-generalized maximum entropy distribution is that it exhibits a power-law like dependence on the microstate energy ϵ_i, instead of the exponential dependence associated with the standard Boltzmann-Gibbs thermostatistics.

Galaxies can be regarded as self gravitating N-body systems that are trapped for a long time in a non-collisional regime (until collisional effects finally become important) where the stars move under the influence of the mean potential Φ generated by the whole set of stars, Φ being a function of the spatial position $\mathbf{x}$.