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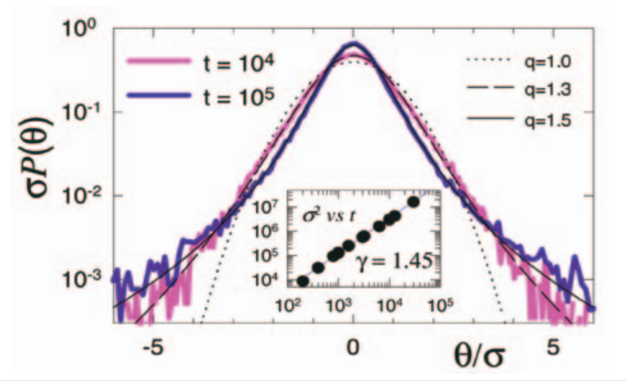

▲ Fig. 3: Histogram of normalized angles at different times of the HMF dynamics. Parameters and initial conditions are the same used in previous figures. Notice that at long times, the histogram is of the q-Gaussian form. Inset: squared deviation as a function of time. It follows the law $\sigma^2 \sim t^\gamma$, with $\gamma > 1$, signaling superdiffusion.

Noise induced phenomena and nonextensivity

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During the last few decades of the 20th century the scientific community has recognized that in many situations (and against everyday intuition) noise or fluctuations can trigger new phenomena or new forms of order, like in noise induced phase transitions, noise induced transport [1], stochastic resonance [2], noise sustained patterns, to name just a few examples. However, in almost all the studies of such noise induced phenomena it was assumed that the noise source had a Gaussian distribution, either white (memoryless) or colored (that is, with "memory"). This was mainly due to the difficulties in handling non Gaussian noises and to the possibility of obtaining some analytical results when working with Gaussian noises. In addition to the intrinsic interest in the study of non Gaussian noises, there has been some experimental evidence, particularly in sensory and biological systems, indicating that at least in some cases the noise source could be non Gaussian.

This article is a brief review on recent studies about some of those noise induced phenomena when submitted to a colored (or time correlated) and non Gaussian noise source. The source of noise used in those works was one generated by a q-distribution arising within a nonextensive statistical physics framework [3]. In all the systems and phenomena analyzed, it was found that the system's response was strongly affected by a departure of the noise source from the Gaussian behavior, showing a shift of transition lines, an enhancement and/or marked broadening of the systems response. That is, in most of the cases, the value of the parameter q optimizing the system's response resulted $q \neq 1$ (with $q = 1$ corresponding to a Gaussian distribution). Clearly, this result would be highly relevant for many technological applications, as well as for some situations of biological interest.

Non gaussian noise

In order to introduce the form of the non Gaussian noise to be used, we start considering the following form of a Langevin or stochastic differential equation (that is, a differential equation with random coefficients), with additive noise

$$\dot{x} = f(x, t) + \eta(t), \qquad (1)$$

where $\eta(t)$ is the stochastic or noise source. Usually, it is assumed that such noise source corresponds to a Gaussian distributed variable, having a correlation $C(t - t') = \langle \eta(t)\eta(t') \rangle$. If the noise is "white" (a particular form of Markovian or memoryless process), we have $C(t - t') \sim \delta(t - t')$, while for a typical Ornstein-Uhlenbeck process, we have $C(t - t') \sim \exp[-(t - t')/\tau]$, with τ the "correlation time".

However, motivated by previous work based on a nonextensive thermostatistics distribution [3], it was assumed that the noise $\eta(t)$ was a non Gaussian and non Markovian process (that

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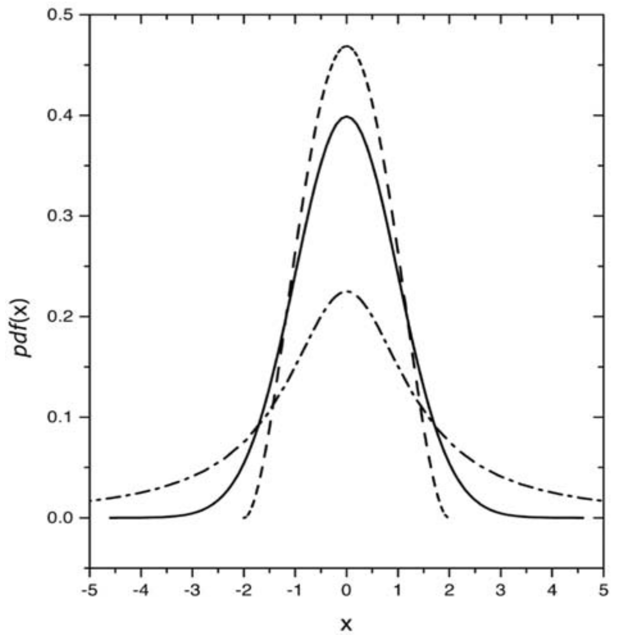


▲ **Fig. 1:** The stationary probability distribution function for the non Gaussian distribution given by Eq. (3), for the value $\tau/D = 1$. The solid line indicates the Gaussian case ($q = 1$); the dashed line corresponds to a bounded distribution ($q = 0.5$); while the dashed-dotted line corresponds to a wide distribution ($q = 2$).

is with "memory"). It was shown that such non Gaussian and non Markovian noise could be generated through the following Langevin equation

$$\dot{\eta} = -\frac{1}{\tau}\frac{d}{d\eta}V_q(\eta) + \frac{1}{\tau}\xi(t); \tag{2}$$

where $\xi(t)$ is a standard Gaussian white noise of zero mean and correlation $\langle\xi(t)\xi(t')\rangle = D\delta(t-t')$, while the potential

$$V_q(\eta) = \frac{D}{\tau(q-1)}\ln[1 + \frac{\tau}{D}(q-1)\frac{\eta^2}{2}].$$

As this article is not the appropriate place to refer to all the properties of the process η, we make reference to [4] for details. However, it is instructive to show the stationary probability distribution function, which is given by

$$P_q^{st}(\eta) = \frac{1}{Z_q}\exp_q\left[-\frac{\tau}{D}\frac{\eta^2}{2}\right] \tag{3}$$

where $\exp_q(x)$ was defined in Box 1 (see the introduction by C.Tsallis and J.P.Boon in this same issue), Z_q being the normalization constant. This distribution can be normalized only for $q < 3$, its first moment is $\langle\eta\rangle = 0$, while the second moment,

$$\langle\eta^2\rangle = \int\eta^2 P_q^{st}(\eta)d\eta = \frac{2D}{\tau(5-3q)} \equiv Dq,$$

is finite only for $q < 5/3$. Also, τ_q, the correlation time of the process η, diverges near $q = 5/3$ and can be approximated over the whole range of values of q by $\tau_q \approx 2\tau/(5 - 3q)$. When $q \to 1$ the limit of η being a Gaussian, Ornstein-Uhlenbeck colored noise, is recovered, with noise intensity D and correlation time τ. Furthermore, for $q < 1$, the probability distribution function has a cut-off and it is only defined for $|\eta| < \sqrt{\frac{2D}{\tau(1-q)}}$. In order to visualize the form of the probability distribution as function of η, in Fig. 1 it is shown for different values of q.

The process η was analyzed in [4], and an *effective Markovian approximation* was obtained via a path integral procedure. Such an approximation allows different (quasi) analytical results to be obtained. Those results and their dependence on the different parameters in the case of a double well potential, were compared with extensive numerical simulations with excellent agreement.

We will now briefly review some of the results obtained when studying a few of the noise induced phenomena indicate above.

Stochastic resonance

The phenomenon of *stochastic resonance* shows the counterintuitive role played by noise in nonlinear systems as it enhances the response of a system subject to a weak external signal [2]. It was first introduced by Benzi and coworkers to explain the periodicity of Earth's ice ages (see [2] and references therein). The study of stochastic resonance has attracted considerable interest due to its potential technological applications for optimizing the response in nonlinear dynamical systems, as well as to its connection with some biological mechanisms. A large number of the studies on stochastic resonance have been done analyzing a paradigmatic bistable one-dimensional double-well potential. In almost all descriptions the transition rates between the two wells were estimated as the inverse of the Kramers' time (or the typical mean passage time between the wells), which was evaluated using standard techniques. In almost all cases, noises were assumed to be Gaussian.

Consider the problem described by Eqs. (1) and (2), where $f(x,t) = -\frac{\partial U(x,t)}{\partial x} = -\frac{\partial U_0(x)}{\partial x} + S(t)$, the external signal is $S(t) \sim A\cos(\omega t)$ and $U_0(x)$ is a double well potential. This problem corresponds (for $A = 0$) to the case of diffusion inside the potential $U_0(x)$, induced by the colored non Gaussian noise η. We will not describe here the details of the *effective Markovian Fokker-Planck equation* (see [4, 5]); but it is worth indicating that such an approximation allowed us to obtain the probability distribution function of the process η, and to derive expressions for the Kramers time. Another useful approximation, the so-called two-state approach [2], was also exploited in order to obtain analytical expressions for the power spectral density and the *signal-to-noise ratio*.

Figure 2 shows some of the main results. In the upper part is depicted the theoretical results: on the left hand part for R – the signal-to-noise ratio – versus D, for a fixed value of the time correlation τ and various q. It is apparent that the general trend is that the maximum of the signal-to-noise curve increases when the system departs from the Gaussian behavior ($q < 1$). The right hand part again shows R vs D, but for a fixed value of q and several values of τ. The general trend agrees with previous results for colored Gaussian noises [2]: an increase of the correlation time induces the maximum of the signal-to-noise ratio decrease as well to shift towards larger values of the noise's intensity. The latter fact is a consequence of the suppression of the switching rate for increasing τ. Both qualitative trends were confirmed by Monte Carlo simulations of Eqs. (1) and (2). The lower part of Fig. 2 show the simulation results. The left hand side corresponds to the same situation and parameters indicated in the upper left part. In addition to the increase of the maximum of the signal-to-noise ratio curve for values of $q < 1$, it is also seen to be an aspect that is not well reproduced or predicted by the effective Markovian approximation: the maximum of the signal-to-noise ratio curve flattens for lower values of q, indicating that the system, when departing from the Gaussian behavior, does not require a fine tuning of the noise intensity in order to maximize its response to a weak external signal. On the right hand side, simulation results for the same situation and parameters indicated in the upper right part are also shown.

The numerical and theoretical results can be summarized as follows: (a) for a fixed value of τ, the maximum value of the signal-to-noise ratio increases with decreasing q; (b) for a given value of q, the optimal noise intensity (that one maximizing the signal-to-noise ratio) decreases with q and its value is approximately independent of τ; (c) for a fixed value of the noise intensity, the optimal value of q is independent of τ and in general it turns out that $q_{op} \neq 1$.

Using a simple experimental setup, in [6] the stochastic resonance phenomenon was analyzed but using a non Gaussian noise source built up to exploit the form of noise introduced above, for the particular case of non Gaussian white noise. Those results confirmed most of the predictions indicated above.

Brownian motors

Brownian motors or "ratchets" – where the breaking of spatial and/or temporal symmetry, induces directional transport in systems out of equilibrium – is another noise induced phenomenon that attracts the attention of an increasing number of researchers due to both its potential technological applications and its biological interest [1]. The transport properties of a typical Brownian motor are usually studied analyzing the following general stochastic differential or Langevin equation

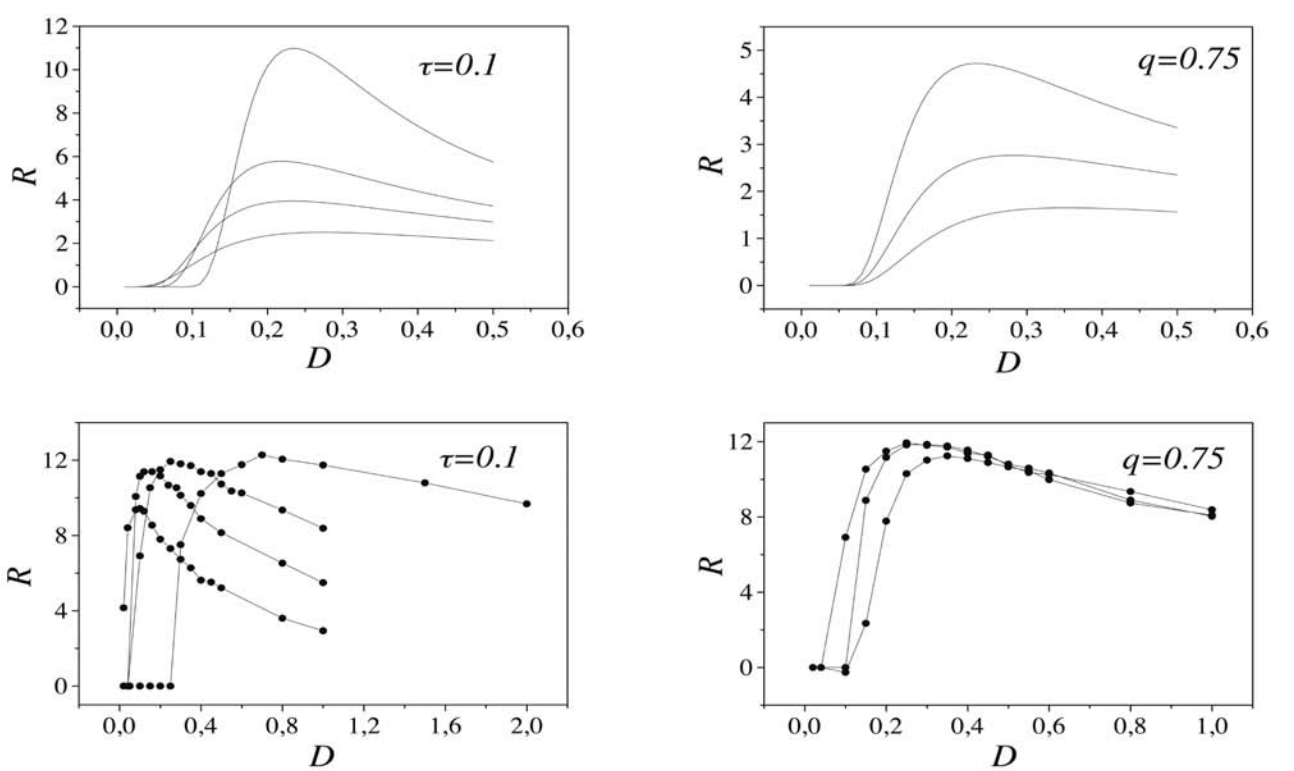
$$m\frac{d^2x}{dt^2} = -\gamma\frac{dx}{dt} - V'(x) - F + \xi(t) + \eta(t), \qquad (4)$$

where m is the mass of the particle, γ the friction constant, $V(x)$ the (sawtooth-like) ratchet potential, F is a constant "load" force, and $\xi(t)$ the thermal noise satisfying $\langle\xi(t)\xi(t')\rangle = 2\gamma T\delta(t-t')$. Finally, $\eta(t)$ is the time correlated forcing (with zero mean) that keeps the system out of thermal equilibrium allowing the motion to be rectified. For this type of ratchet model several different kinds of time correlated forcing have been considered in the literature [1].

The effect of the class of the non Gaussian noise introduced before on the transport properties of a typical Brownian motor, was analyzed in [7], with the dynamics of $\eta(t)$ described by the Langevin equation (2). As discussed before, for $1 < q < 3$, the probability distribution decays as a power law, that is slower than a Gaussian. Hence, keeping D (the noise intensity) constant, the width or dispersion of the distribution increases with q, meaning that, the higher q, the stronger the "kicks" that the particle will receive when compared with the Gaussian Ornstein-Uhlenbeck process.

By setting $m = 0$ and $\gamma = 1$, the overdamped regime was initially analyzed. The main objective of the studies was to analyze the dependence of the mean current $\left(J = \left\langle \frac{dx}{dt} \right\rangle\right)$ and the efficiency (ε) on the different parameters, in particular, their dependence on q. For the efficiency, defined as the ratio of the work (per unit time) done by the particle "against" the load force F to the mean power injected into the system through the external forcing η, a closed expression using an adiabatic approximation was obtained [7].

Figure 3, on the left hand side, shows typical analytical results - obtained through the adiabatic Approximation - for J and ε as functions of q, together with results of numerical simulations.



▲ **Fig. 2:** Signal-to-noise ratio (R) vs the noise intensity (D) for the double-well potential $U_0(x) = \frac{x^4}{4} - \frac{x^2}{2}$. Theoretical results are shown in the upper part: on the left hand side for a correlation time $\tau = 0.1$ and different values of the parameter q (that indicates a departure from the Gaussian $-q = 1-$ behavior): (from top to bottom) $q = 0.25, 0.75, 1.0, 1.25$, while on the right hand side for $q = 0.75$ and different values of the correlation time: (from top to bottom) $\tau = 0.25, 0.75, 1.5$. Lower part, Monte Carlo simulation results: left side for $\tau = 0.1$ and (from top to bottom) $q = 0.25, 0.75, 1.0, 1.25$; while on the right side for $q = 0.75$ and (from top to bottom) $\tau = 0.25, 0.75, 1.5$.

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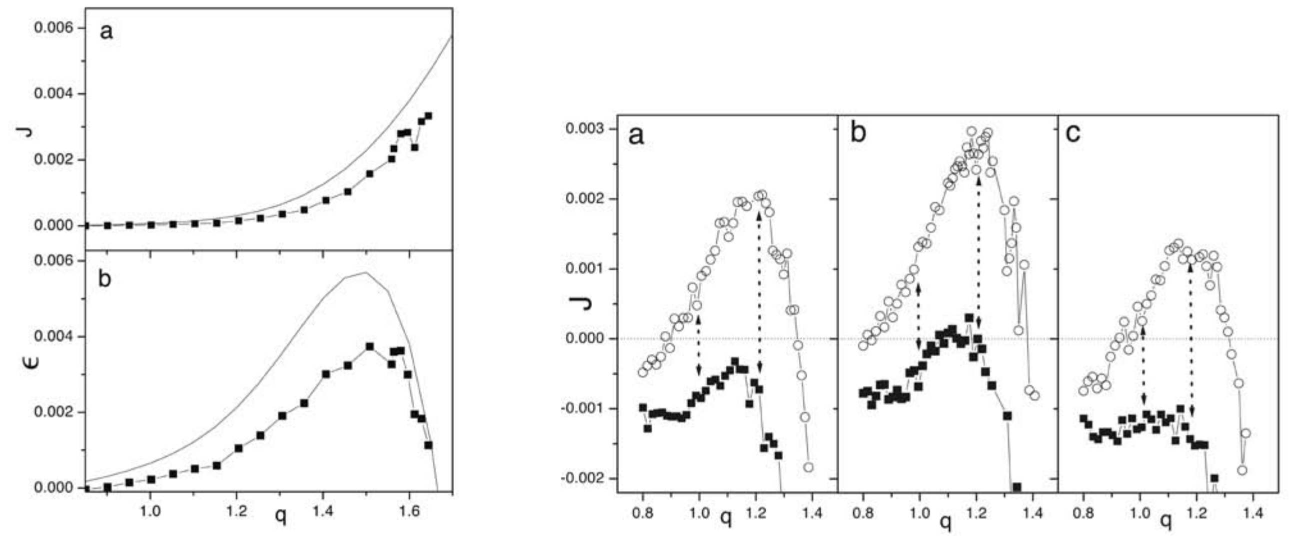

▲ **Fig. 3:** Left hand side: Current (a) and efficiency (b) for the Brownian motor described by Eq. (4), subject to the non Gaussian noise, both as functions of q. The solid line corresponds to an adiabatic approximation while the line with squares shows results from simulations. All calculations are for $m = 0$, $\gamma = 1$, $T = 0.5$, $F = 0.1$, $D = 1$ and $\tau = 100/(2\pi)$. On the right hand side: Separation of masses, results from Monte Carlo simulations for the current as a function of q for particles of masses $m = 0.5$ (hollow circles) and $m = 1.5$ (solid squares). Calculations are for three different values of the load force: $F = 0.025$ (a), $F = 0.02$ (b) and $F = 0.03$ (c). The values of the parameters were fixed at $\gamma = 2$, $T = 0.1$, $\tau = 0.75$, and $D = 0.1875$.

A region of parameters similar to the ones used in previous studies was chosen, but considering a non-zero load force, leading to a non-vanishing efficiency. As can be seen, although there is not quantitative agreement between theory and simulations, the used adiabatic approximation predicts qualitatively very well the behaviour of J (and ε) as q is varied. As shown in the figure, the current grows monotonously with q (at least for $q < 5/3$) while there is an optimal value of q (> 1) giving the maximum efficiency. This fact could be interpreted as follows: when q is increased, the width of the $Pq(\eta)$ distribution grows and high values of the non Gaussian noise become more frequent, leading to an improvement of J. Although the mean value of J increases monotonously with q, the width of $Pq(\eta)$ also grows, leading to an enhancement of the fluctuations around this mean value. This is the origin of the efficiency's decay occurring for high values of q: in this region, in spite of having a large (positive) mean value of the current for a given realization of the process, the transport of the particle towards the desired direction is far from being assured. Hence, the results indicated clearly show that the transport mechanism becomes more efficient when the stochastic forcing has a non Gaussian distribution with $q > 1$.

Regarding the situation when inertia effects are relevant (that is $m \neq 0$), taking into account the results discussed above it is reasonable to expect that non Gaussian noises might improve the capability of mass separation in ratchets. Previous work has analyzed ratchets with an Ornstein-Uhlenbeck noise as external forcing (it is worth emphasising that it corresponds to $q = 1$ in the present case), and has studied the dynamics for different values of the correlation time of the forcing, finding that there was a region of parameters where mass separation occurs. This means that the direction of the current is found to be mass–dependent: the "heavy" species moves in one direction while the "light" one does so in the opposite sense. We have analyzed the same system, but considering the case of non Gaussian forcing, and focusing on the region of parameters where (for $q = 1$) separation of masses was found. The main result was that the separation of masses indeed occurs, that happens in the absence of a load force, and that it is enhanced when a non–Gaussian noise with $q > 1$ is considered. On the right hand side of Fig. 3, part (a) the current J as function of q for $m_1 = 0.5$ and $m_2 = 1.5$ is shown. It is apparent that there is an optimum value of q that maximizes the difference of currents. This value, which is close to $q = 1.25$, is indicated with a vertical double arrow. Another double arrow indicates the separation of masses occurring for $q = 1$ (Gaussian Ornstein-Uhlenbeck forcing). It was observed that, when the value of the load force is varied, the difference between the curves remain approximately constant but both are shifted together to positive or negative values (depending on the sign of the variation of the loading). By controlling this parameter it is possible to achieve the situation shown in part (b), where, for the value of q at which the difference of currents is maximal, the heavy "species" remains static on average (has $J = 0$), while the light one has $J > 0$. In part (c) for the optimal q, the two species move in opposite directions with equal absolute velocity.

Resonant gated trapping

As indicated before, stochastic resonance has been found to play a relevant role in several biology problems. In particular, there are experiments on the measurement of the current through voltage-sensitive ion channels in cell membranes. These channels switch (randomly) between open and closed states, thus controlling the ion current. This and other related phenomena have stimulated several theoretical studies of the problem of ionic transport through biological cell membranes, using different approaches, as well as different ways of characterizing stochastic resonance in such systems. A *toy model*, sketching the behavior of an ion channel, was studied in [8]. Among other factors, the ion transport depends on the membrane electric potential (which plays the role of the barrier height) and can be stimulated by both *dc* and *ac* external fields. This included the simultaneous action of a deterministic and a stochastic external field on the trapping rate of a gated imperfect trap. The main result was that even such a simple (toy) model of a gated trapping process shows a stochastic resonance-like behavior.

The study was based on the so called *stochastic model* for reactions, generalized in order to include the internal trap's dynamics. The dynamical process consists in the opening or closing of the traps according to an external field that has two contributions, one periodic with a small amplitude, and another stochastic whose intensity is (as usual) the tuning parameter. The absorption contribution is modeled as $\sim -\gamma(t)\delta(x)\rho(x, t)$; with ρ the density of the not yet trapped particles, and $\gamma(t) = \gamma^* \theta[B \sin(\omega t) + \eta - \eta_c]$, where $\theta(x)$ – the Heaviside function – determines when the trap is open or closed: if the signal is $B \sin(\omega t) + \eta \geq \eta_c$ the trap opens, otherwise it is closed. The interesting case is when $\eta_c > B$, that is: without noise the trap is always closed. When the trap is open the particles are trapped with a probability per unit time γ^* (i.e. the open trap is "imperfect"). Finally, the colored non Gaussian noise given by Eq. (2) was used for η.

The stochastic resonance-like phenomenon was quantified by computing the amplitude of the oscillating part of the absorption current, indicated by $\Delta J(t)$. The resulting qualitative behavior was as follows: for small noise intensities the trapping current was low (as $\eta_c > B$), hence ΔJ was small too, while for a large noise intensity the deterministic (harmonic) part of the signal became irrelevant and ΔJ was again small. Hence, there was a maximum at some intermediate value of the noise. When compared against the white noise case, an increase in the system response was apparent together with a reduction in the need for tuning the noise, similarly to what was found for the "normal" stochastic resonance: the bounded character of the probability distribution function for $q < 1$ contributed positively to the rate of overcoming the threshold η_c and such a rate remained of the same order within a larger range of values than for the case of η being a white noise [5].

The dependence of the maximum of $\Delta J(t)$ on the parameter q was also analyzed, and the existence of another *resonant-like* maximum as a function of q was observed, implying that it is possible to find a region of values of q where the maximum of ΔJ reaches optimal values (*corresponding to a non Gaussian and bounded* probability distribution function), yielding the largest system response. That is, a *double stochastic resonance effect* exists as a function of both: the noise intensity and q.

Noise induced transition

A system, called the *genetic model,* that when submitted to a Gaussian white noise shows a noise induced transition, was also studied. In previous related works it was shown that, when the noise is an Ornstein-Uhlenbeck one, a re-entrance effect arose (from a disordered state to an ordered one, and finally again to a disordered state) as the noise correlation time τ was varied from 0 to ∞. The same system was studied in [9], but when it was submitted to the non Gaussian noise indicated above. The main result showed the persistence of the indicated re-entrance effect, together with a strong shift in the transition line, as q departed from $q = 1$. The transition was anticipated for $q > 1$, while it was retarded for $q < 1$. A conjecture about a possible re-entrance effect with q was shown to be false.

Final comments

The previously indicated results clearly show that the use of non Gaussian noises in many noise induced phenomena could produce significant changes in the system's response when compared to the Gaussian case. Moreover, in all cases, it was found that the system's response is enhanced or altered in a relevant way, and this occurs for values of q indicating a departure from Gaussian behavior, that is: *the optimum response* happens for $q \neq 1$: Clearly, the study of the variation in the response of other related noise induced phenomena when subject to such a kind of non Gaussian noise will be of great interest.

An extremely relevant point is related to some recent work [10] where the algebra and calculus associated with the nonextensive statistical mechanics has been studied. It is expected that the use of such a formalism could help to directly study Eq. (1), without the need to resort to Eq. (2), and also to build up a *nonextensive path integral* framework for this kind of stochastic process.

To conclude, it is worth commenting on a relevant question: how could it be possible to obtain such a form of noise from, it may be said, *first principles?* It is well known that in dynamical systems with several degrees of freedom evolving with two well separated time scales, Gaussian noises (white or colored) could be obtained through an adequate adiabatic elimination of the fast variables, and assuming some (δ or exponential) correlation properties. It can be conjectured that the form of noise used above could result from the existence of a whole *hierarchy* of time scales and, associated with it, to an adequate *hierarchical adiabatic elimination* of the faster variables. However, the proof or rejection of this conjecture requires some specific work.

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Horacio S. Wio graduated at the Instituto Balseiro, and has been visitor at several institutions in Latin America, Europa and USA. He was Professor at Instituto Balseiro, and Senior Scientist at CONICET and Centro Atómico Bariloche, where he was Head of the Theoretical Physics Division during 1992-1994, and of the Statistical Physics Group during 1986-2002. He is now at the Instituto de Física de Cantabria.

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