Foreword

It is our pleasure to welcome Jean Pierre Boon and Constantino Tsallis as guests Editors for the present Special Issue of Europhysics News on "Nonextensive Statistical Mechanics". They did a great job not only in selecting an impressive set of distinguished authors but also in writing the introductory Editorial and in each being a co-author of one of the contributions. The subject is difficult and could not go without a higher proportion of equations than usual in EPN: our thanks go to the EPN designer who had to face a heavier task than usual. It is sometimes necessary to address arduous developments to cover recent progress in Physics. This time, EPN will ask its readers to make an effort. It is always rewarding. The guests Editors were so efficient that the collected material passes largely the size of a standard EPN issue. We are grateful to the Publisher for accepting to accommodate all the articles in a single volume. It will make of this Special Issue the general reference work on "nonextensive statistical mechanics". Back to the usual mix of wide-ranging Features and News next time!

The Editors

Special issue overview Nonextensive statistical mechanics: new trends, new perspectives

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Boltzmann-Gibbs (BG) statistical mechanics is one of the monuments of contemporary physics. It establishes a remarkably useful bridge between the mechanical microscopic laws and classical thermodynamics. It does so by advancing a specific connection, $S_{BG} = -k\sum_{i=1}^{W} p_i \ln p_i$ in its discrete version, of the entropy a la Clausius with the microscopic states of the system. However, the *BG* theory is not universal. It has a delimited domain of applicability, as any other human intellectual construct. Outside this domain, its predictions can be slightly or even strongly inadequate. No surprise about that. That theory centrally addresses the very special stationary state denominated *thermal equilibrium*. This macroscopic state has remarkable and ubiquitous properties, hence its fundamental importance. The deep foundation of this state and of 27-year-old Boltzmann's famous *Stosszahlansatz* ("molecular chaos hypothesis") in 1871 lie on nonlinear dynamics, more specifically on *strong* chaos, hence mixing, hence ergodicity. However many important phenomena in natural, artificial, and even social systems do not accomodate with this simplifying hypothesis. This is particularly frequent in physical sciences as well as in biology and economics, where *non-equlibrium* stationary states are the common rule. Then, at the microscopic dynamical level, strong chaos is typically replaced by its *weak* version, when the sensitivity to the initial conditions grows not exponentially with time, but rather like a power-law.

A question then arises naturally, namely: *Is it possible to address some of these important - though anomalous in the BG sense - situations with concepts and methods similar to those of BG statistical mechanics?* Many theoretical, experimental and observational indications are nowadays available that point towards the affirmative answer. A theory which appears to satisfactorily play that role is *nonextensive statistical mechanics* and its subsequent developments. This approach, first proposed in 1988, is based on the generalization of the *BG* entropy by the expression

$$S_q = k\sum_{i=1}^{W} p_i \ln_q(1/p_i) \equiv k\frac{1-\sum_{i=1}^{W} p_i^q}{q-1}$$

with index $q \in \mathcal{R}$ and $S_1 = S_{BG}$, i.e. the *BG* theory is contained as the particular case $q = 1$ (see the Box). S_q shares with S_{BG} a variety of thermodynamically and dynamically important properties. Among these we have *concavity* (relevant for the thermodynamical stability of the system), *experimental robustness* (technically known as Lesche-stability, and relevant for the experimental reproducibility of the results), *extensivity* (relevant for having a natural matching with the entropy as introduced in classical thermodynamics), and *finiteness of the entropy production per unit time* (relevant for a variety of real situations where the system is striving to explore its microscopic phase space in order to ultimately approach some kind of stationary state). This is quite important because it is not easy to find entropic functionals that simultaneously and generically satisfy these four properties. Renyi entropy, for instance, is known to be an interesting form for characterizing multifractals. But it seems inadequate for thermodynamical purposes. Indeed, Renyi entropy satisfies concavity only in the interval $0 < q \leq 1$, and violates, for $q \neq 1$, all the other three properties mentioned above. The extensivity of S_q deserves a special mention. Indeed, if we compose

BASIC QUANTITIES
q-exponential : $\exp_q(x) \equiv [1+(1-q)\,x]^{\frac{1}{1-q}} \longrightarrow_{q\to 1} e^x$
q-logarithm : $\ln_q(x) \equiv \frac{x^{1-q}-1}{1-q} \longrightarrow_{q\to 1} \ln x$
Boltzmann-Gibbs entropy : $S_{BG} \equiv -k\sum_{i=1}^{W} p_i \ln p_i$
q-entropy : $S_q \equiv k\frac{1-\sum_{i=1}^{W} p_i^q}{q-1} = k\sum_{i=1}^{W} p_i \ln_q(1/p_i) = -k\sum_{i=1}^{W} p_i^q \ln_q p_i \longrightarrow_{q\to 1} S_{BG}$
Escort distribution : $P_i \equiv p_i^q / \sum_{j=1}^{W} p_j^q$
Ensemble q-average : $\langle A\rangle_q \equiv \sum_{i=1}^{W} A_i P_i = \sum_{i=1}^{W} A_i p_i^q / \sum_{j=1}^{W} p_j^q$

◄ **Box:** The two basic functions that appear in Nonextensive Statistical Mechanics are the q-exponential and the q- logarithm with $\ln_q(\exp_q x) = \exp_q(\ln_q x) = x$. They are simple generalizations of the usual exponential and logarithmic functions which are retrieved by performing a $|1 - q| << 1$ expansion. Similarly the q-entropy generalizes the standard Boltzmann-Gibbs entropy. The escort distribution is a generalization of the usual ensemble averaging function to which it reduces for $q = 1$.

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subsystems that are (explicitly or tacitly) probabilistically *independent,* then S_{BG} is *extensive* whereas S_q is, for $q \neq 1$, *nonextensive.* This fact led to its current denomination as "nonextensive entropy". However, if what we compose are subsystems that generate a nontrivial (strictly or asymptotically) scale-invariant system (in other words, with important global correlations), then it is generically S_q for a particular value of $q \neq 1$, *and not* S_{BG}, which is *extensive.* Asking whether the entropy of a system is or is not extensive *without indicating the composition law of its elements,* is like asking whether some body is or is not in movement *without indicating the referential with regard to which we are observing the velocity.*

The overall picture which emerges is that Clausius thermodynamical entropy is a concept which can accomodate with more than one connection with the set of probabilities of the microscopic states. S_{BG} is of course one such possibility, S_q is another one, and it seems plausible that there might be others. The specific one to be used appears to be univocally determined by the microscopic dynamics of the system. This point is quite important in practice. If the microscopic dynamics of the system is known, we can in principle determine the corresponding value of q from first principles. As it happens, this precise dynamics is most frequently unknown for many natural systems. In this case, a way out that is currently used is to check the functional forms of various properties associated with the system and then determine the appropriate values of q by fitting. This has been occasionally a point of – understandable but nevertheless mistaken – criticism against nonextensive theory, but it is in fact common practice in the analysis of many physical systems. Consider for instance the determination of the eccentricities of the orbits of the planets. *If* we knew all the initial conditions of all the masses of the planetary system and had access to a colossal computer, we could in principle, by using Newtonian mechanics, determine a priori the eccentricities of the orbits. Since we lack that (gigantic) knowledge and tool, astronomers determine those eccentricities through fitting. More explicitly, astronomers adopt the mathematical form of a Keplerian ellipse as a first approximation, and then determine the radius and eccentricity of the orbit through their observations. Analogously, there are many complex systems for which one may reasonably argue that they belong to the class that is addressed by nonextensive statistical concepts, but whose microscopic (sometimes even mesoscopic) dynamics is inaccessible. For such systems, it appears as a sensible attitude to adopt the mathematical forms that emerge in the theory, e.g. q-exponentials, and then obtain through fitting the corresponding value of q and of similar characteristic quantities.

Coming back to names that are commonly used in the literature, we have seen above that the expression "nonextensive entropy" can be misleading. Not really so the expression "nonextensive statistical mechanics". Indeed, the many-body mechanical systems that are primarily addressed within this theory include long-range interactions, i.e., interactions that are *not* integrable at infinity. Such systems clearly have a total energy which increases quicker than N, where N is the number of its microscopic elements. This is to say a total energy which indeed is nonextensive.

Acknowledgements

The present special issue of Europhysics News is dedicated to a hopefully pedagogical presentation, to the physics community, of the main ideas and results supporting the intensively explored and quickly evolving nonextensive statistical mechanics. The subjects that we have selected, have been chosen in order to provide a general picture of its present status in what concerns both its foundations and applications. It is our pleasure to gratefully acknowledge all invited authors for their enthusiastic participation.

Extensivity and entropy production

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In 1865 Clausius introduced the concept of entropy, S, in the context of classical thermodynamics. This was done, as is well known, without any reference to the microscopic world. The first connection between these two levels of understanding was proposed and initially explored one decade later by Boltzmann and then by Gibbs. One of the properties that appear naturally within the Clausius conception of entropy is the extensivity of S, i.e., its proportionality to the amount of matter involved, which we interpret, in our present microscopic understanding, as being proportional to the number N of elements of the system. The Boltzmann-Gibbs entropy $S_{BG} \equiv -k\sum_{i=1}^{W} p_i \ln p_i$ (discrete version, where W is the total number of microscopic states, with probabilities $\{p_i\}$, and where k is a positive constant, usually taken to be k_B). S_{BG} satisfies the Clausius prescription under certain conditions. For example, if the N elements (or subsystems) of the system are probabilistically independent, i.e., $p_{i_1, i_2, \ldots, i_N} = p_{i_1} p_{i_2} \ldots p_{i_N}$, we immediately verify that $S_{BG}(N) = N S_{BG}(1)$. If the correlations within the system are close to this ideal situation (e.g., local interactions), extensivity is still verified, in the sense that $S_{BG}(N) \propto N$ in the limit $N \to \infty$. There are however more complex situations (that we illustrate later on) for which S_{BG} is not extensive. The question then arises: *Is it possible, in such complex cases, to have an extensive expression for the entropy in terms of the microscopic probabilities?* The general answer to this question still eludes us. However, for an important class of systems (e.g., asymptotically scale-invariant), one such entropic connection is known, namely

$$S_q = k\frac{1-\sum_{1=1}^{W} p_i^q}{q-1} \quad (q \in \mathcal{R};\ S_1 = S_{BG})\,. \tag{1}$$

(N = 0)	1	1
(N = 1)	π_{10} π_{11}	1/2 1/2
(N = 2)	π_{20} π_{21} π_{22}	1/3 1/6 1/3
(N = 3)	π_{30} π_{31} π_{32} π_{33}	3/8 5/48 5/48 0
(N = 4)	π_{40} π_{41} π_{42} π_{43} π_{44}	2/5 3/40 1/20 0 0

▲ **Table:** *Left:* Most general set of joint probabilities for N equal and distinguishable binary subsystems for which only the number of states 1 and of states 2 matters, not their ordering. *Right:* Triangle with $\epsilon = 0.5$ and $d = 2$ constructed by modifying the Leibnitz-triangle. In general $q_{sen} = 1-(1/d)$. For $N = 5, 6, \ldots$ a full triangle emerges (on the right side) all the elements of which vanish. For any finite N, the Leibnitz rule is not exactly satisfied, but it becomes asymptotically satisfied for $N \to \infty$. See details in [3].