# Quantum computers: where do we stand?

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Quantum mechanics has had an enormous technological and societal impact. To grasp this point, it is sufficient to cite the invention of the transistor, perhaps the most remarkable among the countless other applications of quantum mechanics. It is also easy to see the enormous impact of computers on everyday life. The importance of computers is such that it is appropriate to say that we are now living in the information age. This information revolution became possible thanks to the invention of the transistor, that is, thanks to the synergy between computer science and quantum physics. Today this synergy offers completely new opportunities and promises exciting advances in both fundamental science and technological application. We are referring here to the fact that quantum mechanics can be used to process and transmit information [1,2].

Miniaturization provides us with an intuitive way of understanding why, in the near future, quantum laws will become important for computation. The electronics industry for computers grows hand-in-hand with the decrease in size of integrated circuits. This miniaturization is necessary to increase computational power, that is, the number of floating-point operations per second (flops) a computer can perform. In the 1950's, electronic computers based on vacuum-tube technology were capable of performing approximately $10^3$ floating-point operations per second, while nowadays there exist supercomputers whose power is greater than 10 teraflops ($10^{13}$ flops). As we have remarked, this enormous growth of computational power has been made possible owing to progress in miniaturization, which may be quantified empirically in Moore's law. This law is the result of a remarkable observation made by Gordon Moore in 1965: the number of transistors on a single integrated-circuit chip doubles approximately every 18 24 months. This exponential growth has not yet saturated and Moore's law is still valid. At the present time the limit is approximately $10^8$ transistors per chip and the typical size of circuit components is of the order of 100 nanometres. Extrapolating Moore's law, one would estimate that around the year 2020 we shall reach the atomic size for storing a single bit of information. At that point, quantum effects will become unavoidably dominant.

One should be aware that, besides quantum effects, other factors could bring Moore's law to an end. In the first place, there are economic considerations. Indeed, the cost of building fabrication facilities to manufacture chips has also increased exponentially with time. Nevertheless, it is important to understand the ultimate limitations set by quantum mechanics. Even though we might overcome economic barriers by means of technological breakthroughs, quantum physics sets fundamental limitations on the size of the circuit components. The first question under debate is whether it would be more convenient to push the silicon-based transistor to its physical limits or instead to develop alternative devices, such as quantum dots, single-electron transistors or molecular switches. A common feature of all these devices is that they are at the nanometre length scale, and therefore quantum effects play a crucial role.

So far, we have talked about quantum switches that could substitute silicon-based transistors and possibly be connected together to execute classical algorithms based on Boolean logic. In this perspective, quantum effects are simply unavoidable corrections that must be taken into account owing to the nanometre size of the switches. *A quantum computer represents a radically different challenge:* the aim is to build a machine based on quantum logic, that is, a machine that can process the information and perform logic operations in agreement with the laws of quantum mechanics.

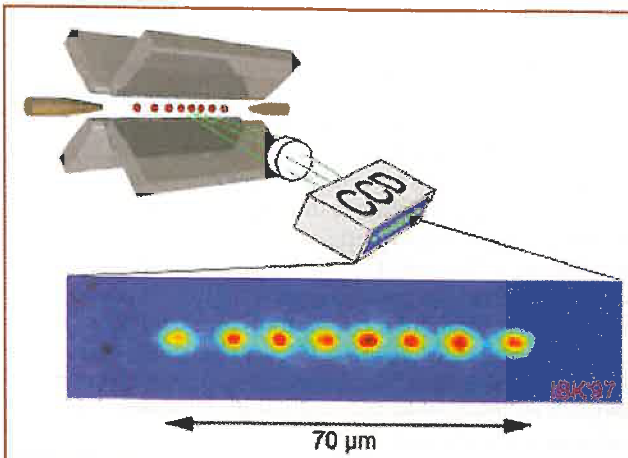
## Quantum logic

The elementary unit of quantum information is the *qubit* (the quantum counterpart of the classical bit) and a quantum computer may be viewed as a many-qubit system. Physically, a qubit is a two-level system, like the two spin states of a spin - $\frac{1}{2}$ particle, the vertical and horizontal polarization states of a single photon or two states of an atom.

A classical bit is a system that can exist in two distinct states, which are used to represent 0 and 1, that is, a single binary digit. The only possible operations (gates) in such a system are the identity ($0 \to 0, 1 \to 1$) and NOT ($0 \to 1, 1 \to 0$). In contrast, a quantum bit (qubit) is a two-level quantum system, described by a two-dimensional complex Hilbert space. In this space, one may choose a pair of normalized and mutually orthogonal quantum states, called $|0\rangle$ and $|1\rangle$, to represent the values 0 and 1 of a classical bit. These two states form a computational basis. From the *superposition principle*, any state of the qubit may be written as

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle, \quad (1)$$

where the amplitudes $\alpha$ and $\beta$ are complex numbers, constrained by the normalization condition $|\alpha|^2 + |\beta|^2 = 1$.




▲ **Fig. 1:** Schematic drawing of a string of qubits (ions) in a linear trap. The ions are confined by a combination of static and oscillating electric fields. In order to detect the ions, they are illuminated with a suitable laser beam: part of the fluorescence photons emitted by the ions are collected by a lens and sent to a CCD camera. At the bottom, an image of ions trapped in an experiment is shown. (Courtesy of Rainer Blatt, University of Innsbruck)



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The collection of $n$ qubits is known as a *quantum register* of size $n$. Its wave function resides in a $2^n$-dimensional complex Hilbert space. While the state of an $n$-bit classical computer is described in binary notation by an integer $k \in \{0,1,...,2^n-1\}$,

$$k = k_{n-1}\, 2^{n-1} + \cdots + k_1\, 2 + k_0, \quad (2)$$

with $k_0, k_1, \ldots, k_{n-1} \in \{0,1\}$ binary digits, the state of an $n$-qubit quantum computer is

$$|\psi\rangle = \sum_{k=0}^{2^n-1} c_k\, |k\rangle, \quad (3)$$

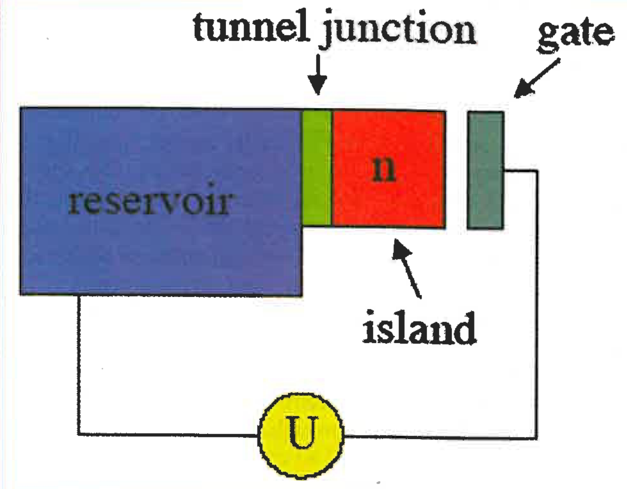
where $|k\rangle \equiv |k_{n-1}\rangle \cdots |k_1\rangle\, |k_0\rangle$, with $|k_j\rangle$ state of the $j$-th qubit, and

$$\sum_{k=0}^{2^n-1} |c_k|^2 = 1,$$

The superposition principle is clearly visible in Eq. (3): while $n$ classical bits can store only a single integer $k$, the $n$-qubit quantum register can be prepared in the corresponding state $|k\rangle$ of the computational basis, but also in a superposition. We stress that the number of states of the computational basis in this superposition can be as large as $2^n$, which grows exponentially with the number of qubits. The superposition principle opens up new possibilities for computation. When we perform a computation on a classical computer, different inputs require separate runs. In contrast, a quantum computer can perform a computation for exponentially many inputs on a single run. This huge parallelism is the basis of the power of quantum computation.

It is also important to point out the role of *entanglement* for the power of quantum computation, as compared to any classical computation. Entanglement is the most spectacular and counter-intuitive manifestation of quantum mechanics, observed in composite quantum systems: it signifies the existence of non-local correlations between measurements performed on well-separated particles. After two classical systems have interacted, they are in well-defined individual states. In contrast, after two quantum particles have interacted, in general, they can no longer be described independently of each other. There will be purely quantum correlations between two such particles, independently of their spatial separation. Examples of two-qubit entangled states are the four states of the so-called Bell basis, $|\phi^{\pm}\rangle = \frac{1}{2}(|00\rangle \pm |11\rangle)$ and $|\psi^{\pm}\rangle = \frac{1}{2}(|01\rangle \pm |10\rangle)$. The measure of the polarization state of one qubit will instantaneously affect the state of the other qubit, whatever their distance is. There is no entanglement in classical physics. Therefore, in order to represent the superposition of $2^n$ levels by means of classical waves, these levels must belong to the same system. Indeed, classical states of separate systems can never be superposed. Thus, to represent the generic $n$-qubit state (3) by classical waves we need a single system with $2^n$ levels. If $\Delta$ is the typical energy separation between two consecutive levels, the amount of energy required for this computation is given by $\Delta 2^n$. Hence, the amount of physical resources needed for the computation grows exponentially with $n$. In contrast, due to entanglement, in quantum physics a general superposition of $2^n$ levels may be represented by means of $n$ qubits. Thus, the amount of physical resources (energy) grows only linearly with $n$.

In conclusion, due to superposition and entanglement, a quantum computer could, in principle, lead to an exponential speed up with respect to classical computation. The next question is how to implement a quantum computation. For this purpose, we must be able to control the evolution in time of the many-qubit state describing the quantum computer. As far as the coupling to the environment may be neglected, this evolution is unitary and governed by the Schrödinger equation. It is well known that, on a classical computer, a small set of elementary logic gates allows the implementation of any complex computation. This is very important: it means that, when we change the problem, we do not need to modify our computer hardware. Fortunately, the same property remains valid for a quantum computer. It turns out that each unitary transformation acting on a many-qubit system can be decomposed into unitary quantum gates acting on a single qubit and a suitable quantum gate acting on two qubits, for instance the controlled-NOT (CNOT) gate. The CNOT is a two-qubit gate, defined as follows: it turns $|00\rangle$ into $|00\rangle$, $|01\rangle$ into $|01\rangle$, $|10\rangle$ into $|11\rangle$ and $|11\rangle$ into $|10\rangle$. As in the classical XOR gate, the CNOT gate flips the state of the second (target) qubit if the first (control) qubit is in the state $|1\rangle$ and does nothing if the first qubit is in the state $|0\rangle$. It is easy to see that CNOT can generate entangled states. For example, if we apply CNOT to the non-entangled state $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|0\rangle$, we obtain the Bell state $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$.




▲ Fig. 2: Schematic drawing of a Josephson-junction qubit in its simplest design: a small superconducting island with $n$ excess Cooper pairs (relative to some reference state) is connected by a tunnel junction with capacitance $C_J$ and Josephson coupling energy $E_J$ to a superconducting reservoir. The junction is biased by a gate voltage $U$ with gate capacitance $C_g$

## Quantum algorithms

As we have seen, the power of quantum computation is due to the inherent *quantum parallelism* associated with the superposition principle. In simple terms, a quantum computer can process a large number of classical inputs in a single run. However, it is not an easy task to extract useful information from the output state. The problem is that this information is, in a sense, hidden. Any quantum computation ends up with a projective measurement in the computational basis. The output of the measurement process is inherently probabilistic and the probabilities of the different possible outputs are set by the basic postulates of quantum mechanics. However, there exist quantum algorithms that efficiently extract useful information.

In 1994, Peter Shor proposed a quantum algorithm that efficiently solves the prime-factorization problem [3]: given a composite odd positive integer N, find its prime factors. This is a central problem in computer science and it is conjectured, though not proven, that for a classical computer it is computationally difficult to find the prime factors. Shor's algorithm efficiently solves the integer factorization problem in $O((n^2\log n\log \log n))$ elementary quantum gates, where $n=\log N$ is the number of bits necessary to code the input $N$. Therefore it provides an exponential improvement in speed with respect to any known classical algorithm. Indeed, the best classical algorithm, the number field sieve, requires $\exp(O(n^{1/3}(\log n)^{2/3}))$ operations. It is worth mentioning that there are cryptographic systems, such as RSA, that are used extensively today and that are based on the conjecture that no efficient algorithms exist for solving the prime factorization problem. Hence Shor's algorithm, if implemented on a large scale quantum computer, would break the RSA cryptosystem.

Other quantum algorithms have been developed. In particular, Grover has shown that quantum computers can also be useful for solving the problem of searching for a marked item in an unstructured database of $N=2^n$ items [4]. The best we can do with a classical computer is to go through the database, until we find the solution. This requires $O(N)$ operations. In contrast, the same problem can be solved by a quantum computer in $O(\sqrt{N})$ operations. In this case, the gain with respect to classical computation is quadratic.

A third relevant class of quantum algorithms is the simulation of physical systems. It is well known that the simulation of quantum many-body problems on a classical computer is a difficult task as the size of the Hilbert space grows exponentially with the number of particles. For instance, if we wish to simulate a chain of $n$ spin - $\frac{1}{2}$ particles, the size of the Hilbert space is $2^n$. Namely, the state of this system is determined by $2^n$ complex numbers. As observed by Feynman in the 1980's, the growth in memory requirement is only linear on a quantum computer, which is itself a many-body quantum system. For example, to simulate $n$ spin - $\frac{1}{2}$ particles we only need n qubits. Therefore, a quantum computer operating with only a few tens of qubits can outperform a classical computer. Of course, this is only true if we can find efficient quantum algorithms to extract useful information from the quantum computer. Quite interestingly, it has been shown that a quantum computer can be useful not only for the investigation of the properties of many-body quantum systems, but also for the study of the quantum ad classical dynamics of complex single-particle systems (for a recent review see, e.g., [5]).

## First experimental implementations

The great challenge of quantum computation is to experimentally realize a quantum computer. Many requirements must be fulfilled in order to achieve this imposing objective. We require a collection of two-level quantum systems that can be prepared, manipulated and measured at will. That is, our purpose is to be able to control and measure the state of a many-qubit quantum system. A useful quantum computer must be *scalable* since we need a rather large number of qubits to perform non-trivial computations. In other words, we need the quantum analogue of the integrated circuits of a classical computer. Qubits must interact in a controlled way if we wish to be able to implement a universal set of quantum gates. Furthermore, we must be able to control the evolution of a large number of qubits for the time necessary to perform many quantum gates. Given the generality of the requirements to build a quantum computer, many physical systems might be good candidates

In liquid state Nuclear Magnetic Resonance (NMR) quantum processors [2], the quantum hardware consists of a liquid containing a large number (of order $10^{18}$) of molecules of a given type, placed in a strong static magnetic field. A qubit is the spin of a nucleus in a molecule and quantum gates are implemented by means of resonant oscillating magnetic fields (Rabi pulses), that is, NMR techniques are used. Quantum information exchange between nuclei inside a molecule is based on spin-spin interactions (chemical bonds) between neighbouring atoms. The molecules are prepared in thermal equilibrium at room temperature. It is important to stress that in liquid-state NMR the spin state of a single nucleus is neither prepared nor measured. On the contrary, we measure the average spin state of the $\sim 10^{18}$ molecules contained in the solution. With NMR experiments, it has been possible to experimentally demonstrate several quantum algorithms, including Grover's algorithm, the quantum Fourier transform and the Shor's algorithm, using from three- to seven-qubit molecules. Unfortunately, liquid-state NMR quantum computing is not scalable since the measured signal drops exponentially with the number of qubits in a molecule.

Using cavity quantum electrodynamics (QED) techniques [6], it has been possible to realize experiments in which a single atom interacts with a single mode or a few modes of the electromagnetic field inside a cavity. The two states of a qubit can be represented by the polarization states of a single photon or by two excited states of an atom. Cavity QED techniques have allowed the implementation of one and two-qubit gates and have been particularly successful in demonstrating basic features of quantum mechanics, such as entanglement, or in exploring the transition from the quantum world to classical physics.

Several other proposals have been put forward to build a quantum computer, including quantum optics approaches, cold atoms in optical lattices and solid-state systems such as quantum dots and spin in semiconductors. It is too early to say which route will be the most suitable to build a scalable quantum hardware. Due to space limitations, we limit ourselves to discuss in more detail two implementations for which major advances have recently occurred: cold ions in a trap and superconducting circuits.

## Ion traps

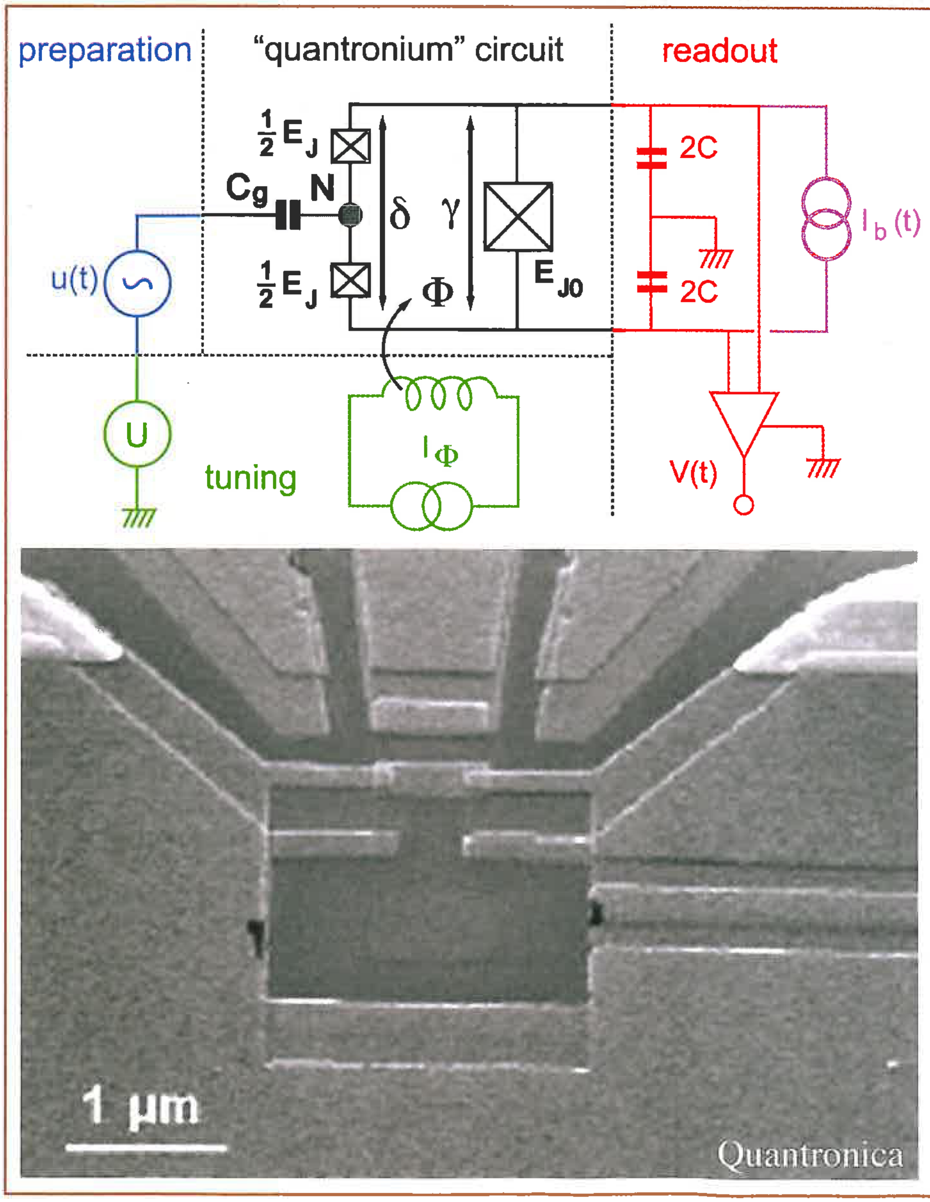
The quantum hardware is as follows: a string of ions is confined by a combination of static and oscillating electric fields in a linear trap (known as a Paul trap, see Fig.1). A qubit is a single ion and two long-lived states of the ion correspond to the two states of the qubit. The linear array of ions held in the trap is the quantum register. The initialization of all the qubits in the state $|0\rangle$ is possible by means of optical-pumping techniques: When an ion is in a state different from $|0\rangle$, it absorbs a photon and then decays, this process being repeated until each ion reaches the $|0\rangle$ state. After a quantum computation, the state of each ion can be measured using *quantum jump* detection: each ion is illuminated with laser light of polarization and frequency such as it absorbs and then reemits photonsonly if it is in the state $|1\rangle$. In contrast, if it is in the state $|0\rangle$ the laser frequency is out of resonance and does not induce any transition. Thus, the detection of fluorescence indicates that the ion was in the state $|1\rangle$.

Single-qubit gates are obtained by addressing individual ions with laser pulses of appropriate frequency, intensity and duration. The interactions between qubits, which are necessary to implement controlled two-qubit operations, are mediated by the collective vibrational motion of the trapped string of ions. To implement the two-qubit CNOT gate, Cirac and Zoller proposed the following scheme. The quantum state of the control qubit (ion) is mapped onto the vibrational state of the whole string (known as *bus-qubit*), with the use of laser beams focused on that ion. A gate operation can then be performed between the bus qubit and the target ion. We should stress that this is possible because also the target qubit participates to the collective vibrational motion. As a result, the effect of a laser beam on the target qubit depends on the state of the bus-qubit. Finally, this state is mapped back onto the control ion. Note that the preparation of the ground state of the bus-mode is nowadays possible with great accuracy using laser cooling techniques.

The Cirac-Zoller CNOT gate was realized by the Innsbruck group [7], using two $^{40}Ca^{+}$ ions held in a linear trap and individually addressed using laser beams. A generic single-qubit state is encoded in a superposition of the ground state $S_{1/2}$ and the metastable state $D_{5/2}$ (whose lifetime is approximately 1 s). More recently, scientists at NIST, Boulder and at Innsbruck were able to implement *quantum teleportation* between a pair of trapped ions [8,9]. Teleportation exploits entanglement and provides a means to transport quantum information (a quantum state) from one location to another, without transfer of the physical system that carries the quantum information. This possibility could be of practical interest for quantum computation, for example in the transfer of quantum information between different units of a quantum computer.

Sources of errors in ion-trap quantum computation are the heating due to stochastically fluctuating electric fields, the ambient magnetic field fluctuations and the laser frequency noise. At present, the implementation of the CNOT gate by a sequence of 8 laser light pulses requires approximately 500 $\mu$s, while the decoherence time scale is of the order of 1 ms. Here the termdecoherence (or loss of quantum coherence) denotes the corruption of the quantum information stored in the quantum computer, due to the unavoidable coupling of the quantum computer to the surrounding environment [10]. Using ions less susceptible to environmental influences, it seems probable that in the next few years it will become possible to apply tens of quantum gates to a few ions without loosing quantum coherence.

The scaling to large qubit numbers is envisaged by using arrays of interconnected ions traps. The communication between the traps could be achieved by photon interconnection or by moving ions from one trap to another. In the first case, the state of a qubit would be transferred from an ion in a trap to a photon and then from the photon to a second ion in another trap. In the latter case, ion qubits would be moved from one trap to another by application of suitable electric fields. It seems that there are no fundamental physical obstacles against these proposals, but a significant technological challenge remains.




◄ **Fig. 3:** Left: Schema of a superconducting circuit, nicknamed "quantronium", that behaves as a two-level system. The circuit consists of a superconducting island (black dot) delimited by two small Josephson junctions (crossed boxes) in a superconducting loop. The loop also includes a third, larger Josephson junctions. $E_J$ and $E_{J0}$ denote the Josephson energies of the Cooper pair box and of the large junction, respectively. The number $N$ of Cooper pairs in the island and the superconducting phases δ and γ are the degrees of freedom of the circuit. A current $I_\Phi$ applied to a coil produces a flux Φ in the circuit loop and is used to tune the quantum energy levels. Microwave pulses $u(t)$ are applied to the gate to prepare arbitrary states of the qubit. These states are readout by applying a current pulse $I_b(t)$ to the large junction and by monitoring the voltage $V(t)$ across it. Right: Scanning electron micrograph of a sample made of aluminum and aluminum oxide. The gate electrode is at the top and the island is below the gate. The large junction is the wide rectangle at the right side. (Courtesy of Daniel Esteve, CEA, Saclay)

## Superconducting circuits

Several proposals have been put forward to build a solid-state quantum computer. This is not surprising, since solid-state physics has developed over the years a sophisticated technology, creating artificial structures and devices on nanoscale. Solid-state physics is at the basis of the development of classical computer technology and therefore the scalability problem would find a natural solution in a solid state quantum computer. Indeed, such a quantum computer could benefit from the fabrication techniques of microelectronics.

Recently there has been very remarkable experimental progress using superconducting microelectronic circuits to construct artificial two-level systems [11]. In superconductors, pairs of electrons are bound together to form objects of charge twice the electron charge, called Cooper pairs. Electrostatic potentials can confine the Cooper pairs in a "box" of micron size. In a Josephson junction a Cooper pair box, known as the island, is connected by a thin insulator (tunnel junction) to a superconducting reservoir (see Fig.2). Cooper pairs can move from the island to the reservoir and vice versa by quantum tunneling effect. They enter the island one by one when a control gate electrode, capacitively coupled to the island, is varied. The island has discrete quantum states and, under appropriate experimental conditions, the two lowest energy states $|0\rangle$ and $|1\rangle$ form a two-level system suitable for a qubit. An improved Cooper pair box circuit acting as a qubit is shown in Fig.3. By applying microwave pulses to the gate electrode, this qubit can be prepared in any coherent superposition $\alpha|0\rangle + \beta|1\rangle$. The manipulation of one-qubit states is possible: a microwave resonant pulse of duration $\tau$ induces controlled Rabi oscillations between the states $|0\rangle$ and $|1\rangle$. If $\tau$ is appropriate, the NOT gate ($|0\rangle \rightarrow |1\rangle, |1\rangle \rightarrow |0\rangle$) is implemented. A Ramsey fringe experiment has also allowed to measure the decoherence time scale $t_d \approx 0.5\ \mu s$ for this circuit [12]. This time is much longer than the time required to implement a single-qubit gate, so that an arbitrary evolution of the two-level system can be implemented with a series of microwave pulses. Note that the time for a single qubit operation can be made as short as 2 ns. More recently, a two-qubit gate was operated using a pair of capacitively coupled superconducting qubits [13].

## Outlook

To summarize, the main question under discussion is: is it possible to build a useful quantum computer that could outperform existing classical computers in important computational tasks? And, if so, when? The difficulties are huge. Besides the problem of decoherence, we should also remark on the difficulty of finding new and efficient quantum algorithms. We know that the integer-factoring problem can be solved efficiently on a quantum computer, but we do not know the answer to the following fundamental question: What class of problems could be simulated efficiently on a quantum computer? Quantum computers open up fascinating prospects, but it does not seem likely that they will become a reality with practical applications in a few years. How long might it take to develop the required technology? Even though unexpected technological breakthroughs are, in principle, always possible, one should remember the enormous effort that was necessary in order to develop the technology of classical computers.

Nevertheless, even the first, modest, demonstrative experiments are remarkable, not only for quantum computation but also for testing the theoretical principles of quantum mechanics. Since quantum mechanics is a particularly counter-intuitive theory, we should at the very least expect that experiments and theoretical studies on quantum computation will provide us with a better understanding of quantum mechanics. Moreover, such research stimulates the control of individual quantum systems (atoms, electrons, photons *etc.*). We stress that this is not a mere laboratory curiosity, but has interesting technological applications. For instance, it is now possible to realize single-ion clocks that are more precise than standard atomic clocks. Other foreseen applications are the use of entangled states to improve the resolution of optical lithography and interferometric measurements.

Quantum mechanics also provides a unique contribution to cryptography: it enables two communicating parties to detect whether the transmitted message has been intercepted by an eavesdropper. This is not possible in the realm of classical physics as it is always possible, in principle, to copy classical information without changing the original message. In contrast, in quantum mechanics the measurement process, in general, disturbs the system for fundamental reasons: this is a consequence of the Heisenberg uncertainty principle. Experimental advances in the field of quantum cryptography are impressive [14] and quantum-cryptographic protocols have been demonstrated, using optical fibres, over distances of a few tens of kilometres at rates of the order of a thousand bits per second. Furthermore, free-space quantum cryptography has been demonstrated over distances up to several kilometres. In the near future, therefore, quantum cryptography could well be the first quantum-information protocol to find commercial applications.

To conclude, the time when a quantum computer will be on the desk in our office is uncertain. What is certain is that an exciting and very promising field of investigation has been opened. Finally, let us quote Schrödinger [Brit. J. Phil. Sci. 3, 233 (1952)]: *"We never experiment with just one electron or atom or (small) molecule. In thought-experiments we sometimes assume that we do; this invariably entails ridiculous consequences ... we are not experimenting with single particles, any more than we can raise Ichthyosauria in the zoo"*. It is absolutely remarkable that only fifty years later experiments on single electrons, atoms and molecules are routinely performed in laboratories all over the world.

## References

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