Teaching mathematics to physicists in the UK – *FLAP* and PPLATO

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In the late 1980s a problem began to emerge in the UK with mathematics teaching for undergraduate physics. A broader 16-19 curriculum in schools had encouraged more flexible study patterns, threatening the traditional pairing of A-Level physics and A-Level mathematics. Increasingly, students were studying physics without mathematics and although these students were not usually intending physics degrees the syllabuses for physics and mathematics began to evolve separately. Physics was taught with less mathematics and mathematics was taught with less reference to physics. This decoupling of the two subjects produced an undergraduate physics intake that was less comfortable with using mathematics in physics, even those with good grades in both subjects. Physics departments had to restore the relationship between physics and mathematics expected of a professional physicist. The problem was exacerbated by the common practice in universities of having service mathematics taught to physicists by mathematics departments. This tended to continue the decoupling rather than to restore the link. The realisation that mathematics, like other skills needed by a physicist, had to be securely embedded within the main subject teaching began to gain favour in the early 1990s. The changing patterns of A-level study also encouraged more flexible entry routes into physics degree courses for those without traditional A-level backgrounds in physics and mathematics. This led to the introduction of Foundation or Access year programmes as a precursor to three year BSc Physics programmes.

In 1992, following an initiative by the Institute of Physics in 1989, the project *FLAP* (Flexible Learning Approach to Physics) began [1]. *FLAP* was led by the University of Reading and the Open University as a consortium project on behalf of the university sector. Over 5 years it received £780K of funding from the UK Higher Education Funding Councils. It produced a high quality supported self-study resource covering first year and foundation year physics and its associated mathematics. This generous support allowed the production of extensive new text materials, designed for the purpose and unlike any pre-existing textbooks in physics or mathematics. It was described by some as a "breath of fresh air". It allowed physics departments to create courses to their own design and specification in response to a diversifying and changing intake. Key features were its linking of physics and mathematics and its flexibility. It was first delivered in 1995.

◂ **Photo:** Teaching with *FLAP* and *Hyperflap*.

Although *FLAP* was a response to a UK university problem, its flexibility and the global nature of the problem addressed extended its impact to schools and internationally. The resource and its evaluation have been described in detail elsewhere [2]. The factors that gave birth to *FLAP* still exist today and its approach remains relevent. In this article we give a brief description of the resource and its approach. We then describe how, in 2003, it is being incorporated into a new suite of digital resources under the banner of a newly funded project PPLATO – Promoting Physics Learning And Teaching Opportunities.

The Flexible Learning Approach to Physics

FLAP consists of text, audio, video and computer materials. The text consists of 83 free-standing modules in two parallel strands of physics and mathematics (See Appendix). Module titles show the scope of the resource. In mathematics, the range is from simple algebra (e.g. the expansion of brackets in Module M1.1) to the time-dependent Schrödinger equation (Module M6.4). In physics, the range is from the definition of SI units (Module P1.1) and linear motion (Module P2.1) to the angular momentum eigenstates of atomic hydrogen (Module P11.3). This range covers the physics and mathematics needed in the first year of a UK physics degree course and in a year's full-time study prior to this. The mathematics strand has also been published by John Wiley as two mathematics textbooks [3-4]. The text element of *FLAP* has much in common with textbooks in that it can be used to support lectures, tutorials, workshops and laboratory teaching, but there are several important differences from a conventional textbook:

- It has a completely new modular text, written specifically to cope with diversity and to reinforce the relationship between physics and its associated mathematics.
- It uses a common approach to physics and mathematics, presenting mathematics thoroughly, yet within a physical context where possible and with due regard to dimensions and units.
- It is more flexible in use, both for students and course designers. Each free-standing module specifies pre-requisite knowledge, measures student background via formative tests and provides differentiated study routes accordingly. This means that modules have no fixed order of study, quite unlike the chapters of a standard textbook, and study patterns are set by course designers and are automatically customised to the individual student.
- Its scope and depth is more extensive—its size is equivalent to about five major textbooks, two in mathematics and three in physics, but the modular structure maintains portability.
- It is sufficiently detailed to form a primary teaching vehicle, so it may be used in self-study or to replace some teaching contact time, allowing more individual and targeted support.
- It may be photocopied by an institution for its own students, allowing customised textbooks to be produced at a fraction of the cost of a normal textbook. Students do not have to purchase unwanted material or the latest edition of standard texts. The only penalty paid for being photocopiable is the removal of colour in the printed material.
- It trains students to become independent learners by using supported self-study to engage them in active learning and by encouraging reading and time management skills.
- It develops the students' knowledge and skills base, adopting a problem-solving approach with embedded questions designed to develop cognitive skills, raise conceptual understanding and challenge pre-conceptions.

- It gives the student more ownership and control of the learning through the differentiated routes through the material.

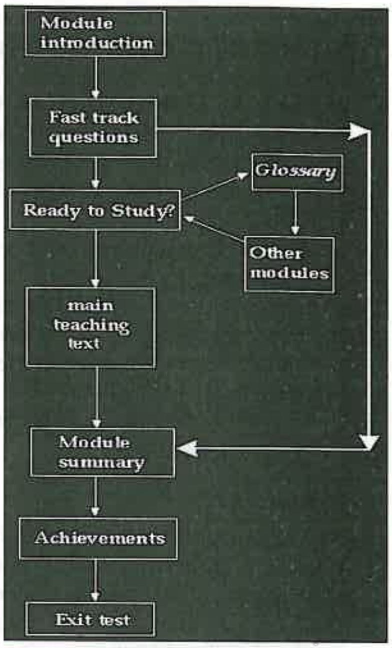
The scale of the *FLAP* resource is best illustrated by its full contents as summarized below. It contains material equivalent to about 200 lecture hours or 500 study hours and includes:

- 83 *FLAP* modules as single-sided A4 monochrome photocopy card masters (about 2250 pp).
- 1 *Index*, *Student Guide* and *Maths Handbook* (also as photocopy masters).
- Photocopying license for an institution to use to make copies for its own students.
- 1 ring bound browsing copy of all 83 *FLAP* modules.
- 1 ring bound copy of the *Glossary* (about 2500 entries), *Scientific Biographies*, *Maths Handbook* and *Index*. The *Glossary* is a relational glossary, combining the functions of index, dictionary and thesaurus, not only listing definitions but also showing how they relate to one another across the full range of the package. The *Glossary* is also the main routing document in *FLAP*. Modules contain no explicit reference to other modules and all such links are made through the *Glossary*. This feature maintains module independence and maximises flexibility.
- *Tutor Guide Part 1*, including product description, module outlines and guidance on use.
- *Tutor Guide Part 2*, is a question bank of about 1700 extra questions with fully worked answers. Questions and solutions are on computer disk, allowing convenient cutting and pasting for assessment production.
- A *Hyper-glossary* and *Index* on disk (about 2500 entries and 15000 hyper-links). This is an HTML document with about 3000 embedded graphic links and 15000 intra-glossary links. It includes an index of terms and biographies of many of the scientists and mathematicians who have contributed most to the development of the field.
- A *CAL package* on disk, including interactive simulations on *Electric fields*, *Bubble chamber*, *Gas simulation*, *Orbital motion*, and a *Forces tutorial*.
- A *CAT-FLAP package* on disk. This computer-administered self-assessment diagnostic tool contains over 400 multiple choice questions and solutions from all the modules. It also contains three diagnostic access tests, two for physics and one for mathematics. A printed report can be generated for tutor discussion and tutors may add their own questions and solutions.
- 8 C60 audio-cassette tapes, including interactive tutorials
- 4 E30 video tapes, including animations and demonstrations

Module structure

The internal structure of the modules is one of the most powerful innovations of *FLAP*. It is this that allows flexible, differentiated access for students. Students measure their backgrounds for the topics to be introduced and can plan their study time most efficiently. They may move speedily over familiar ground, reviewing it as necessary, but spend more time building new knowledge and skills. The text has a programmed approach, with interactive decisions controlling the route, as shown in the flow chart in Figure 1.

Each module begins with a diagnostic section entitled *Opening items*, which sets the scene, introduces the topics and invites students to assess their background knowledge. The material may be mostly old knowledge, mostly new knowledge or a reasonable extension of old knowledge. There are *Fast track* questions to test whether it is old knowledge and *Ready to study* questions to test whether more preparation is needed. Solutions are given for all these questions to guide students in their choice of route. The *Fast track* route leads quickly to the *Exit test* at the end of the.module, where the wisdom of this choice of route can be self-tested. Students who need help with the *Ready to study* questions are directed through the *Glossary* to other *FLAP* modules. Students who determine that they are ready for the material move on to the main teaching text of the module. Field trials show that students quickly develop the skills to use this diagnostic front-end and to make best use of their study time.



▲ **Fig. 1:** The structure of a *FLAP* module.

The main teaching text of each module is written in a clear, student friendly, interactive style, laced with short in-text questions and longer self-assessment questions to test and reinforce the learning. Students are encouraged to tackle these questions as they come to them in the text and the solutions often contain teaching comments which are essential for subsequent parts of the main text. Important terms are highlighted and flagged in the margins, major equations are boxed and asides or explanations of details are given in marginal notes. Questions are often of the type which confront pre-conceptions and encourage conceptual thinking.

After the main text there is a section entitled *Closing items*. This contains a detailed but concise *Module summary*, followed by a list of the *Achievements*, or learning outcomes for the module. These are written in operational terms, explain what students should be able to *do*, and identify what is required for assessment purposes. The *Closing items* concludes with an *Exit test* to test the learning outcomes. All routes pass through the *Closing items* since it provides valuable feedback and revision. Detailed solutions to all questions are at the end of the module, so students can assess their own progress. Evaluation has shown that an average module is equivalent in content to about 2 or 3 lecture hours and requires 5 to 6 study hours by the standard route.

Using *FLAP*

FLAP is a very large resource, not a course, and how it is used is determined by the individual teacher. The flexible structure allows *FLAP* to be incorporated easily within existing teaching programmes or to be used to design new teaching. In deciding whether to use *FLAP* the important decision is to assess the extent to which the teacher wishes to encourage independent learning, to give students more control over the learning and to see the purpose of teaching as the support of learning. A likely consequence of using *FLAP* is that the teacher lectures less and discusses more. The student listens less but reads and discusses more. For both parties this creates more active engagement with each other and with the learning process. This approach is generally welcomed by both parties, but not universally so. Students and teachers can be beguiled by the apparent efficiency of the lecture as a learning tool. Experience suggests it is highly optimistic to equate speaking and hearing with learning. Effective learning is active and effective teaching must accommodate a variety of learning styles [5-7]. Within this general picture *FLAP* has been used successfully in many situations, such as those below.

- Pre-course work for entry to a degree programme (taught or self-study)
- Diagnostic testing on entry to a degree programme or within such a programme
- Consolidation of background on entry to a programme
- Support for main-stream lecture courses
- Replacement of a conventional lecture course by a *FLAP*-based course having fewer lectures
- Creation of a complete supported self-study programme (e.g. as a Foundation Year)

Evaluation

FLAP has been evaluated extensively since it was first used at the University of Reading in developmental testing in 1993. Overall comments from all these evaluations is that when used as part of a teaching programme *FLAP* brings benefits to the student, the teacher and the institution.

Staff point to benefits from improved student effort, study discipline, organisational skills and motivation and in the teacher's ability to deal with class diversity through more focussed contact time. There are the benefits from embedding mathematics within the physics. The extensive question bank significantly reduces staff effort for testing and diagnostics. Teaching assistants make a more effective contribution through access to the same teaching materials as the students.

Students are generally enthusiastic about the *FLAP* learning style, recognize the benefits of active learning and welcome their increased ownership of the learning. They respond with increased motivation, commitment and success. They gain more self-confidence, interact better with each other and with staff and feel more confident about their learning and understanding. Reading skills are improved and students find the 'fast track' route through the modules is an excellent revision tool. Overall, students welcome *FLAP* self-study, in conjunction with some lectures, workshops or tutorials. However, active learning is not universally appreciated. A minority of students prefer to remain passive, since it places fewer demands on them. Initially, this group may be quite vocal and the general view may take a while to emerge from the noise.

e-FLAP

From 1996 the *FLAP* project has been ongoing as a self-funding operation. Five new modules, covering relativity, particle physics, vector calculus, matrices and determinants are about to be released. An electronic copy of the original modules, *e-FLAP*, has been produced on CD ROM. This may be licensed either for single users, or for institutional use on up to 10 machines or on a local area network. *e-FLAP* allows users to have quick reference to any part of the text resource or to produce their own printed copies of the modules or parts of the modules.

Hyperflap

Hyperflap is a further development in progress. This contains the full *e-FLAP* text but is designed for screen use and has hyper-links within modules, between each module and the *Hyperglossary* and thereby to other modules. *Hyperflap* has several innovative features to enhance its use in teaching and learning. The text is broken down into small segments that are viewed without scrolling. Each segment also includes all required equations, figures or tables to which it refers. All hyper-links have been selected by hand on educational grounds, not generated automatically by software. *Hyperflap* will eventually become a multimedia resource, with audio, video and simulations embedded. The scale and nature of the resource will make it unique.

The project is prepared to release developmental evaluation copies of *Hyperflap* to a limited number of institutions in conjunction with their use of *e-FLAP* or the paper version of *FLAP*. Interested institutions should contact the author.

PPLATO PROMOTING PHYSICS LEARNING & TEACHING OPPORTUNITIES

Promoting Physics Learning And Teaching Opportunities — PPLATO

In November 2002 *FLAP* and *Hyperflap* became part of a larger project, PPLATO, again funded through the Funding Councils. This is a Consortium project led by the University of Reading with Brunel University, the Open University, the University of Newcastle, the University of Plymouth and the University of Salford. It addresses the twin problems of teaching mathematics to physics undergraduates and widening participation in undergraduate physics. It will produce flexible resources for face-to-face or on-line delivery and for use within an institution in an accredited programme or for self-study. A particular remit is to develop a Foundation Programme suitable for physicists and engineers. It will survey current practice in the sector and build on previous successful developments and experiences. Project outcomes are expected to be improved student competence in mathematics and physics and wider participation and improved retention in undergraduate physics. New resources will include flexible materials for teaching, diagnostics, assessment and tutorial support, with effective strategies for their use. Institutions wishing to take part in this undertaking and to have access to the developing resources should contact the author.

References

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Appendix

FLAP physics modules

P1 Measurement
P1.1 Introducing measurement
P1.2 Errors and uncertainty
P1.3 Graphs and measurements

P2 Mechanics
P2.1 Introducing motion
P2.2 Projectile motion
P2.3 Forces
P2.4 Work and energy
P2.5 Momentum and collisions
P2.6 Circular motion
P2.7 Rotational mechanics
P2.8 Angular momentum

P3 Fields
P3.1 Introducing fields
P3.2 Gravitation and orbits
P3.3 Electric charge, field and potential

P4 Electricity and magnetism
P4.1 DC circuits and currents
P4.2 Introducing magnetism
P4.3 Electromagnetic force
P4.4 Electromagnetic induction.
P4.5 Energy in electric and magnetic fields

P5 Vibrations and waves
P5.1 Simple harmonic motion
P5.2 Energy, damping and resonance
P5.3 Forced vibrations and resonance
P5.4 AC circuits and electrical oscillations
P5.5 The mathematics of oscillations
P5.6 Introducing waves
P5.7 Sound - a wave phenomenon

P6 Light and optics
P6.1 Light - a wave phenomenon
P6.2 Rays and geometrical optics
P6.3 Optical elements
P6.4 Optical instruments

P7 Heat and properties of matter
P7.1 The atomic basis of matter
P7.2 Temperature, pressure and the ideal gas law
P7.3 Internal energy heat and energy transfer
P7.4 Specific heat, latent heat and entropy
P7.5 Kinetic theory - an example of microscopic modelling
P7.6 Mechanical properties of matter

P8 Atoms and molecules
P8.1 Introducing atoms
P8.2 Atomic spectra and hydrogen
P8.3 Multi-electron atoms
P8.4 Periodic Table and chemical bonding

P9 Nuclei, particles and relativity
P9.1 Introducing atomic nuclei
P9.2 Radioactive decay
P9.3 Fission, fusion and radiation hazards

P10 Principles of quantum physics
P10.1 A particle model for light
P10.2 A wave model for matter
P10.3 Wave functions
P10.4 The Schrödinger equation

P11 Applications of quantum physics
P11.1 Reflection and transmission at steps and barriers
P11.2 The quantum harmonic oscillator
P11.3 Schrödinger's model of hydrogen
P11.4 Quantum physics of solids

FLAP mathematics modules

M1 Algebra, functions and equations
M1.1 Arithmetic and algebra
M1.2 Numbers, units and physical quantities
M1.3 Functions and graphs
M1.4 Solving equations
M1.5 Exponential and logarithmic functions
M1.6 Trigonometric functions
M1.7 Series expansions and approximations

M2 Vectors and geometry
M2.1 Introducing geometry
M2.2 Introducing co-ordinate geometry
M2.3 Conic sections
M2.4 Introducing scalars and vectors
M2.5 Working with vectors
M2.6 The scalar product of vectors
M2.7 The vector product of vectors

M3 Complex numbers
M3.1 Introducing complex numbers
M3.2 Polar representation of complex numbers
M3.3 Complex algebra and de Moivre's theorem

M4 Differentiation
M4.1 Introducing differentiation
M4.2 Basic differentiation
M4.3 Further differentiation
M4.4 Stationary points and graphing
M4.5 Taylor expansions & polynomial approximations
M4.6 Hyperbolic functions and differentiation

M5 Integration
M5.1 Introducing integration
M5.2 Basic integration
M5.3 Techniques of integration
M5.4 Applications of integration
M5.5 Further integration

M6 Differential equations
M6.1 Introducing differential equations
M6.2 Solving first order differential equations
M6.3 Solving second order differential equations
M6.4 Waves and partial differential equations

About the author

Mike Tinker is a Senior Lecturer in Physics at the University of Reading. His research interests are biophysics and physics education. In physics education Mike was one of the two central academics involved in the development of the Flexible Learning Approach to Physics (*FLAP*) resource for the UK Higher Education sector. He is a Fellow of the Institute of Physics and in 2002 was the joint winner of their Bragg Medal and Prize for Physics Education. In 2002 he was awarded £250K from the Higher Education Funding Councils to lead a consortium project PPLATO (Promoting Physics Learning and Teaching Opportunities). He is married with two children, both now fully free in an unsuspecting world.