How many dimensions to our Universe?

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Even though this idea of extra spatial dimension may seem borrowed to the world of science fiction, physicists have grown used over the last century to the idea that there might be some compact dimensions. Of course not the three dimensions that we are used to, but some new dimensions of such a microscopic size that we would not be aware of their existence, unless we probed microphysics.

The first such attempt came from T. Kaluza [1] and O. Klein [2] in the 20's[1]. Their ideas were based on the following analogy: in general relativity, distances depend locally on the gravitational potential; one may thus imagine new dimensions such that the generalised distance depends also on the electromagnetic potential. This may lead to a unified theory of gravity and electromagnetism and, as such, immediately attracted the attention of Einstein [3].

More explicitly, the special theory of relativity of 1905 writes a invariant distance element in spacetime as

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2.$$

In the general relativity theory of 1917, gravitational potentials curve spacetime:

$$ds^2 = -(1-2\Phi_g + 2\beta\Phi_g^2)\, dt^2 + (1-2\gamma\Phi_g)\ (dx^2 + dy^2 + dz^2),$$

where Φ_g is the gravitational potential and β, γ are post-Newtonian parameters, in the early formulation of Eddington (1922). Kaluza and Klein introduce a fifth dimension measured by the coordinate x_5. Electromagnetic potentials "curve" this extra spatial dimension:

$$ds^2 = -(1-2\Phi_g)dt^2 + (1-2\gamma\Phi_g)(dx^2 + dy^2 + dz^2) - \Phi_e dt dx_5 + \mathbf{A}.\mathbf{dx}\, dx_5 + dx_5^2,$$

where Φ_e and $\mathbf{A}$ are the scalar and vector electromagnetic potentials.

This idea has been revived later in the context of string theory. The fundamental objects in string theory are not pointlike but have one dimension in space: they are microscopic strings, closed or open. This property is believed to cure one of the problems of gravity at the quantum level, the appearance of infinities related with the small distance behaviour (high energy or ultraviolet regime). But this is believed to occur only if the symmetries of the string theory are fully respected at the quantum level, which happens only for a specific number of spacetime dimensions[2].

It is usually believed that the extra spatial dimensions are compact and microscopic: an upper value of 10^{-16} m on their size comes from the fact that physics has been tested in high-energy colliders to this distance and that no sign of their existence has been found. We will see that this has to be somewhat mitigated: high energy physics experiments use the electroweak and strong interactions to test particules. Thus, in the case where extra dimensions would be only accessible to gravitational interactions, the high energy limits do not apply [4]. Indeed, until recently gravity was only tested down to the millimiter range, which thus provides the limit for the size of extra dimensions in this case, a macroscopic scale!

Let us pause a second to get a closer look at the type of universe that would emerge in this case: since non-gravitational interactions (as well as matter) do not "feel" the extra dimension, this means that our ordinary world of quarks, leptons and gauge interactions is localised on a 4-dimensional surface[3] (described by 3 spatial and 1 time coordinate) which is plunged into the higher-dimensional universe. Such a surface is called a brane, or more precisely a 3-brane (3 spatial dimensions): the term brane obviously refers to a membrane (strictly speaking a 2-brane). Our observable world is therefore "glued" to the brane and only gravity can probe the higher-dimensional universe outside the brane. Such a set-up had already been encountered in the context of string theory from which the term *brane* is borrowed: there the branes appear as the surfaces described by the end of the open strings [5]. The extra dimensions then need not be finite in size since our senses, as well as our optical or electromagnetic devices, only test the 4 usual dimensions. In what follows, we will refer to such a set-up as a braneworld and talk of Kaluza-Klein extra dimensions in the case where they can be probed non-gravitationally.

How would one identify the existence of extra dimensions?

The first test that one could think of is to check how the gravitational force decreases with the distance. Indeed, the law of variation with the distance is obviously related to the dimensionality of space. Because a sphere in 3-dimensional space (4-dimensional spacetime) has a surface which varies as the radius squared, the distant effect of any point source (whether the water projected by a sprinkler, the electric force of a pointlike charge or the gravitational attraction of a mass) decreases with inverse distance squared. This is summarised by the famous law of gravitational attraction
between two masses m_1 and m_2 distant by r; $G_{(4)}$ is Newton's constant.

$$F(r) = G_{(4)}\ \frac{m_1 m_2}{r^2} \qquad (1)$$

A sphere in (3+D)-dimensional space ((4+D)-dimensional spacetime) has a surface which varies as its radius to the power 2+D; we thus expect a gravitational force which decreases as $r^{-(2+D)}$. This should in principle be enough to discard the possibility of extra dimensions.

However, one should be careful in the case of compact dimensions when the distance r is large compared with the size L of the compact dimension(s). Let us discuss first in more details what happens in the case of a single compact dimension. We modelise such a universe by the infinite torus of Figure 1 (a): the infinite dimension represents any of the 3 standard infinite dimensions that we observe, whereas a compact dimension is visualised by the circle of length $L = 2\pi R$. We consider two masses m_1 and m_2 separated by a distance r on this torus. A gravitational field line may join them directly, or may make one (or more) turns round the torus. Hence (see Figure 1 (b) where the torus is now represented by a series of strips with proper identification) the mass m_1 feels the effect of mass m_2 and all its *images*. If r is much larger than L then these images form a continuous line. In the case of D compact dimensions, one obtains a D-dimensional continuum of

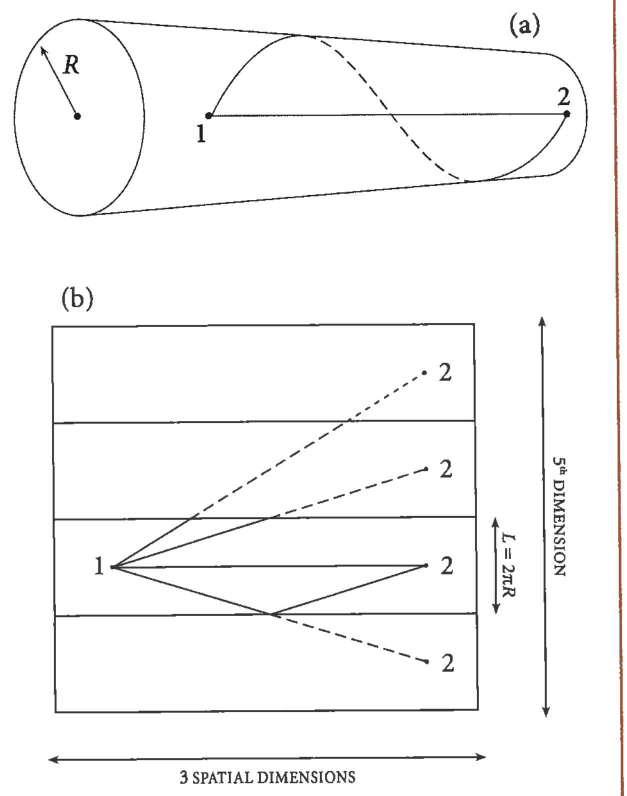

Fig. 1: (a) Infinite torus (b) Same torus represented by strips with opposite sides identified

masses. Then the gravitational force exerted by this continuum on m_1 reads

$$F(r) = G_{(4+D)} \frac{m_1 m_2}{r^2 L^D} \qquad (2)$$

which has the standard form (1) under the identification

$$G_{(4)} = \frac{G_{(4+D)}}{L^D}. \qquad (3)$$

Of course, when distances become of the order of or smaller than L, one recovers the law of higher-dimensional gravity in $r^{-(2+D)}$. Thus one still expects deviations from the standard law at small enough distances.

There is another way of identifying extra dimensions which leads to interesting signatures in particle physics. It rests on the fact that one (or more) compact dimension may be assimilated to a finite box. It is well-known that standing waves in a box have wavelengths λ such that the size $L \equiv 2\pi R$ of the box is a multiple of λ: the wave number k satisfies $k = 2\pi / \lambda = n / R, n \in Z$ and the energy is quantized: $E = \hbar ck$. We also expect the same phenomenon to occur in our higher-dimensional box. Since there is a duality at a fundamental level between particles and waves, we are expecting particles with a mass spectrum characteristic of standing waves, i.e. quantized in units of $1/R$. These oscillation modes are called Kaluza-Klein modes and they are actively searched for in high energy colliders.

Let us be a little more explicit in the case of a single extra dimension. The standard formula for the energy of a relativistic particle of 3-momentum p and mass m_0, $E^2 = \mathbf{p}^2 c^2 + m_0^2 c^4$, reads with a fifth dimension

$$E^2 = \mathbf{p}^2 c^2 + p_5^2 c^2 + m_0^2 c^4$$

where p_5 is the momentum in the fifth spacetime dimension, quantized as discussed above: $p_5 = \hbar k_5 = n\hbar/R$. Thus, in the centre of mass ($\mathbf{p} = \mathbf{0}$), one obtains the following energy spectrum:

$$E^2 = \left[m_0^2 + \frac{n^2 \hbar^2}{R^2 c^2} \right] c^4.$$

Hence, a 5-dimensional field is identified in 4 dimensions to a tower of particles regularly spaced in mass-squared, the mass gap being given by the inverse of the compact dimension size. This can be put differently: if the world is indeed 5-dimensional with a fifth dimension of finite size, one expects to find besides the electron a tower of electron-like states with exactly the same properties (spin, charge,...) except the mass: the first state lies in a mass range which is all the higher as the size of the fifth dimension is smaller. These are the Kaluza-Klein modes of the electron. Because of their characteristic spectrum, finding them would be a dramatic signature that we are living in a higher-dimensional universe.

What does experiment tell us at this point? Since no deviation from the law of gravitation has been observed and no Kaluza-Klein mode has been detected, we may only put limits on the size R of the extra dimensions. We have to consider two different cases:

- if the extra dimension is felt by non-gravitational interactions, that is if Kaluza-Klein modes have electromagnetic, strong or weak interactions, they could be found in high energy colliders[6]. Their non-observation at the highest energy presently observable gives a lower limit on their mass and an upper limit on R:

$$\frac{\hbar c}{R} > 1 \text{ TeV} \leftrightarrow R < \frac{\hbar c}{1 \text{ TeV}} \sim 10^{-4} \text{ fm}$$

- if Kaluza-Klein modes only have gravitational interactions, i.e. if we are in the braneworld set up briefly discussed above, their gravitational couplings are very small and they could easily have been missed (see however below). The only limit comes from the direct study of the law of gravitation; since it has been checked down to the millimiter range

$$R < 1 \text{ mm} \leftrightarrow \frac{\hbar c}{R} > 10^{-4} \text{ eV}.$$

Gravity and the other fundamental interactions

Gravity is the weakest of all known fundamental interactions. This can be rephrased in terms of energy scales typical of each interaction.

In the case of strong interactions, which hold together quarks in a proton, one may take as a representative scale the mass of the proton, 1 GeV/c^2: the quark masses are negligible and most of the proton mass is binding energy.

For electroweak interactions, one may take the mass of intermediate vector bosons which mediate weak interactions, typically 100 GeV/c^2. One could alternatively take the value of the scalar field in the vacuum, around 250 GeV.

Gravity is characterised by a dimensionful coupling: $G_{(4)} = 6.673 \times 10^{-11}$ m^3.kg^{-1}.s^{-2}. One may turn this into a mass scale if one uses the Planck constant $\hbar$ and the speed of light c. This is the Planck mass:

$$M_{Pl} \equiv \sqrt{\frac{\hbar c}{G_{(4)}}} \sim 1.22 \times 10^{19}\ \mathrm{GeV}/c^2 \qquad (4)$$

(around 10^{-8} kg). The presence of the Planck constant indicates that the Planck scale gives the energy scale where *quantum* effects start being important in gravity.

There are therefore some 17 orders of magnitude between the fundamental scale of gravity and those of all known gauge interactions. This induces some delicate problems of fine-tuning when one considers the effect of gravity quantum fluctuations on standard "low energy" parameters (in particular the Higgs particle mass).

Extra dimensions may bring a new twist to this question. Indeed, one should take as the fundamental coupling describing gravitational interactions the higher dimensional Newton's constant $G_{(4+D)}$, not the 4-dimensional one $G_{(4)}$. One may turn this higher-dimensional coupling into a fundamental mass scale M_f, just as in (4):

$$G_{(4+D)} \equiv \frac{(\hbar c)^{D+1}}{c^{2D} M_f^{D+2}}.$$

Then (3) gives

$$M_{Pl}^2 = M_f^{D+2}\left(\frac{Lc}{\hbar}\right)^D$$

Thus, the larger the compact dimension is, the smaller the fundamental scale could be. For example, $D = 2$ millimetre size dimensions would give a scale M_f in the TeV ballpark. How would this be detected experimentally?

Experimental determination of the size of compact dimensions

In very high energy proton-proton collisions, such as at the future CERN Large Hadron Collider (LHC), the constituents of the protons (quarks and gluons) collide. A possible final state is made of a quark and a graviton: the quark hadronizes into a jet of particles and the graviton escapes into the extra dimensions. Thus the signature is a jet plus missing energy. If M_f is in the TeV range, the signal due to extra dimensions may overcome the Standard Model background for large transverse jet energy.

One issue which has been discussed at length recently is the exciting possibility of producing microscopic black holes at colliders, in the case where dimensions are large enough to allow the fundamental higher-dimensional gravity scale M_f to be in the TeV range.

On the side of astrophysics, a strong constraint comes from supernovae. After a supernova explosion, cooling occurs by release of energy mainly through neutrinos and gravitational waves. If higher dimensions are probed by gravity, more phase space is accessible to gravitational waves and cooling is enhanced. This puts lower limits on M_f typically in the 50 TeV range, which, in the simplest models, does not favour experimental signatures at colliders.

Finally, a large effort is invested in trying to improve limits on the validity of the law of gravitation. Sophisticated systems have been conceived to try to reach a limit of a few microns [7]. One of the problems is to disentangle a novel effect due to extra dimensions from the Casimir effect due to quantum fluctuations in the region between the two test masses.

Cosmology of the brane world

Besides shooting gravitons at the extra dimensions using powerful high energy colliders, there is a priori a quieter way of testing these extra dimensions which is looking at the stars and observing the evolution of our own Universe. Indeed, since gravity is changed in a drastic way (number of space dimensions), one may expect that the cosmological evolution of the Universe is changed.

Let us consider a toy model of a 4-dimensional brane universe plunged into a 5-dimensional universe; matter and gauge interactions (and thus galaxies, photons and so on) are localised on the brane, whereas the full fifth dimension (the *bulk*) is accessible only to gravitons (see Figure 2).

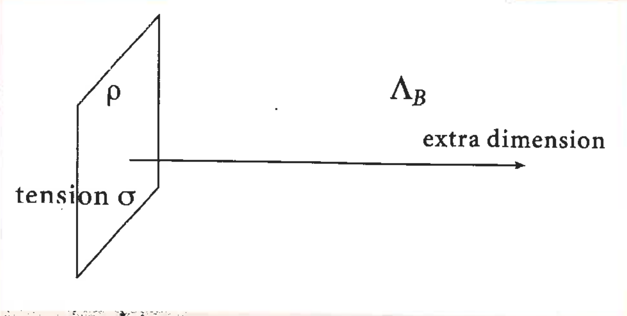

Fig. 2: Brane world set up with a single extra dimension

We are certainly used to the fact that our Universe is curved, at least locally by any gravitational field produced by a mass. This is the notion of intrinsic curvature, which can be "observed" by parallel transporting a vector along a closed curve and checking that it has changed once it has returned to its starting position. But there is also the notion of extrinsic curvature which corresponds to our naive understanding of "curved": the way a sheet of paper is bent for example. There is therefore a fundamental difference between a 4-dimensional universe and a 4-dimensional brane universe plunged into a 5-dimensional one: in the latter case, the way the brane universe is "bent" inside the bulk has some physical consequences.

One of them is of cosmological nature. The rate of expansion of the universe is measured by the Hubble parameter H. In a standard 4-dimensional universe, H^2 varies linearly with the total energy density ρ: this is the Friedmann equation. It follows that, in a radiation-dominated universe, such as our own at the time of nucleosynthesis, the cosmic scale factor (which measures the expansion of distances in the Universe) varies with time as $t^{1/2}$. In the 4-dimensional brane universe plunged in a higher-dimensional bulk that we consider, H^2 varies as ρ^2, the *square* of the energy density on the brane [8]. This would give a slower expansion (cosmic scale factor varying as $t^{1/4}$ in a radiation-dominated universe), in contradiction with was is observed from nucleosynthesis onwards. The solution is to have a constant piece in the brane energy density: this vacuum energy is interpreted as the tension σ of the brane. Then H^2 is proportional to $(\sigma + \rho)^2$: the energy density decreases with time and at late time, r being small, one recovers a linear behaviour in ρ. On the other hand, the ρ^2 term is important in the very primordial universe.

An important issue is the one of the cosmological constant. It is well-known in 4 dimensions that this constant is nothing but the vacuum energy: this is the source of the notorious cosmological problem: this vacuum energy is expected to be of the order of the fundamental scales in the microscopic theory, and this exceeds the observational constraint by many orders of magnitude.

In the brane set up that we consider, the cosmological constant λ observed on the brane receives two contributions: as we just saw, one quadratic in the brane tension (i.e. brane vacuum energy), and one linear in the 5-dimensional bulk vacuum energy Λ_B:

$$\lambda = \frac{\sigma^2}{36\,M_f^2} + \frac{\Lambda_B}{6\,M_f^3} \quad .$$

A vanishing (or very small) cosmological constant thus requires an adjustment of the two vacuum energies: this is the standard fine tuning problem. If we allow for such a fine tuning, we are in for a big bonus: we may allow the extra dimension to be infinite. Indeed, if the brane tension is positive[4], one finds a solution of the Einstein equations for which the 5-dimensional geometry is "warped". And this solution has the following property: among the Kaluza-Klein modes of the 5-dimensional graviton, there is a massless mode which is localised on the brane [9]. It is interpreted as the 4-dimensional graviton. Because of the localisation, 4-dimensional gravity becomes rapidly negligible as one goes away from the brane. It must be said that, even though the extra dimension is infinite, its volume remains finite because of the warped geometry. This model, the Randall-Sundrum model [10], has generated a flurry of activity in the last couple of years.

Of course, many other aspects of the cosmology of such brane worlds have been investigated and it goes beyond the scope of this article to review them. Let us just mention the activity going on in order to obtain definite predictions for the fluctuations in the cosmic microwave background. The difficulty comes here from the fact that the 4-dimensional brane does not form a closed system: it is for example subject to bulk mode excitations, such as gravitational waves.

It remains to be seen whether this general new perspective will provide us with solutions to long standing problems. One may lose some of the successes of the standard 4-dimensional approach, for example gauge coupling unification (more accurately, unification appears more contrived in higher-dimensional models) or some of its guiding principles (renormalisability). It is thus important to see what one gains in the long run. At this point, extra dimensions provide the ground for exciting new ideas which are (or should be) substantiated by a consistent quantum framework, string theory. A complementary approach between high energy physics, astrophysics and cosmology should provide us with some ways to test experimentally these ideas.

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Footnotes

1 with precursors such as H. Weyl and G. Nordström.

2 More precisely, the one-dimensional string covers in its motion a 2-dimensional surface, the world-sheet. String theory can be described as a theory on this 2-dimensional surface: if s labels points of the string and t time, then the position of any string point in spacetime is $x\mu(s, t)$; each coordinate $x\mu$ may thus be understood as a field on the worldsheet. This is why the dimensionality of spacetime, i.e. the number of such coordinates, plays a key role.

3 or a set of such surfaces.

4 as is probably required by stability requirements.

Offshore wind technology ready for application

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The technology for the construction and operation of offshore windfarms is ready for large-scale application. Companies in the fields of engineering and services are preparing to take part. This can be seen in the conclusions of the project Concerted Action on Offshore Wind Energy in Europe (CA-OWEE) of the European Union, in which seventeen parties from thirteen European countries have brought together knowledge on this subject from all over Europe. The Wind Energy section at TU Delft, which co-ordinated the project, has published a final report on the internet: www.offshorewindenergy.org.

The project Concerted Action on Offshore Wind Energy in Europe (CA-OWEE), was funded by the European Commission to stimulate the development of offshore wind-energy into an important energy source. Now that the technology is viable, the most important challenges lie in the reduction of costs, the building up of experience and confidence in the building and maintenance of large wind-parks, the connection of these parks to existing electricity networks and the consequences for the landscape and birds. The authors make suggestions for where further research should be focused.

The European Commission's project was focused on large-scale exploitation of offshore wind by wind-turbines with a large capacity and high scores for performance, sustainability, availability and reliability. The EU would like wind-turbines that are friendlier for the environment and for which the costs of installation and production are lower than that of current units.

Currently, the largest European offshore wind-turbine park is that at Middelgrunden, several kilometres off the Copenhagen shoreline, in Denmark, with a capacity of 40 MW. This year in Horns Rev, on the western coast of Denmark, a park with a 160 MW capacity will be built and a 100 MW wind-park is planned for construction next year at Egmond off the Dutch coast along with many other locations across Europe. Experts expect that by the end of this decade wind-parks will be built at sea with a total capacity of thousands of megawatts, comparable to that of several coal-fuelled power plants and enough to supply millions of homes. Sweden, Denmark, Germany the Netherlands, Belgium, Great Britain and Ireland have advanced plans for such parks on their shores.

In the project Concerted Action on Offshore Wind Energy in Europe, partners from many fields worked together: a public utility, windfarm developers, advisors, research institutes, universities, consultants, an offshore engineering company and a certification body; this is reflected in the broad range of subjects examined.

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