Radiation pressure induced EPR paradox

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The debate between Einstein and Bohr about the interpretation of Quantum Mechanics, initiated at the end of the twenties and formalized in the Einstein, Podolski and Rosen paper of 1935 [1], left both contenders with his own opinion. As is well known, Einstein and coworkers showed that, in a system formed by two spatially separated but physically correlated particles A and B, the possibility of determining indirectly (at the experimenter's choice) the value of one or the other of two canonically conjugate variables (e.g. **x** and **p**) of B by measuring the corresponding variable of A implied one of these two alternatives: (a) the two incompatible (according to QM) variables of each particle, contrary to the fundamental uncertainty established by the Heisenberg principle, possess simultaneously well defined, even if unknowable, values; (b) the result of the measurement performed on A is instantaneously transmitted to B whose corresponding variable, uncertain until that moment, acquires the value required by the correlation's constraint. Einstein rejected (b) and deduced from (a) that QM was an incomplete theory, while Bohr eliminated the problem by denying the possibility of speaking of the values of the variables of B not directly measured. Since no means seemed to exist to decide who was right, because the issue was a counterfactual statement, the question remained open for almost thirty years.

It was John Bell [2] who, in 1964, found a way to test whether Einstein was right or wrong. Instead of discussing about the unknowable values of unmeasured quantities he suggested to compare the values of measured quantities. Bell showed in fact that, when the variables of the two particles are only partially correlated, their experimentally measurable correlation coefficient is different if the incompatible variables of each particle actually do possess well defined simultaneous values, as Einstein believed, or not. In the former case a set of inequalities were satisfied; in the latter, violated. The first reliable evidence that the correlation coefficient between the polarizations of two photons is indeed inconsistent with Einstein's choice was performed by Aspect, Granger and Roger in 1981.

This introduction, which will sound trivial to the readers who have already read these things many times, is useful not only in order to explain to newcomers the essence of this still open fundamental issue but also to provide the context in which the paper *Radiation Pressure Induced Einstein-Podolski-Rosen* Paradox by V. Giovannetti, S. Mancini and P. Tombesi (GMT) recently published in EPL [3] should be placed. The line of research to which the paper discussed here belongs, in fact, is not the same as Bell's. Its purpose is not to test once again whether Einstein was right or wrong. It is rather to deepen our knowledge of the various aspects of the fundamental, but counterintuitive, quantum property called *entanglement*, which is an essential ingredient of the EPR paradox.

Very briefly I recall its definition. Suppose that $\psi_A{}^r$ (r=1,2,..) is the wave function of A in a state labelled by the eigenvalue r of a variable σ_A and $\psi_B{}^r$ is the wave function of B in the corresponding state r of the correlated variable σ_B. Then the wave function Ψ for the total system, describing a state in which *either* A and B are both in state r *or* A and B are both in an other state s, is given by:

$$\Psi = c_r \psi_A{}^r \psi_B{}^r + c_s \psi_A{}^s \psi_B{}^s \qquad (1)$$

This quantum state does *not* describe a statistical ensemble of N pairs AB in which $N_r = N|c_r|^2$ pairs are in state r and $N_s = N|c_s|^2$

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pairs are in state s as Einstein maintained. It describes an ensemble in which each pair has probabilities $P_r=|c_r|^2$ and $P_s=|c_s|^2$ ($P_r+P_s=1$)for the two alternatives. The difference between these two ensembles arises when one considers the measurement of variables τ_A and τ_B which are incompatible with σ_A and σ_B respectively. The expression for the mean value of $\tau_A\tau_B$ contains in fact non vanishing interference terms of the form

$$(\psi_A^{r}{}^{*}\,\tau_A\,\psi_A^{s})(\psi_B^{r}{}^{*}\tau_B\,\psi_B^{s}) + \text{compl. conj.} \qquad (2)$$

The presence of these terms is at the origin of all the strange properties of two (or more) components systems [4, 5, 6] usually attributed to what is called the *spooky action-at-a-distance* of QM.

The proposal of GMT follows the avenue suggested by Reid and Drummond [7]. These authors consider two distinct light modes A and B in a cavity spatially separated but strongly correlated by a nondegenerate optical parametric oscillator. Each mode is characterized by two conjugate quadratures whose phase differs by $\pi/2$. These quadratures $X_A(0)$, $X_A(\pi/2)$ and $X_B(0)$, $X_B(\pi/2)$ play the role of the conjugate variables **x** and **p** of the two particles A and B of EPR. Since each mode of A is correlated with the corresponding mode of B one can infer the values of $X_A(0)$, $X_A(\pi/2)$ by measuring $X_B(0)$ or $X_B(\pi/2)$. The errors of these inferences can be quantified by the variances $\Delta^2_{inf}X_A(0)$ and $\Delta^2_{inf}X_A(\pi/2)$ between the "true" values and the "inferred" values of $X_A(0)$, $X_A(\pi/2)$. The EPR paradox arises if the correlation is high enough that one can reduce the values of the variances to an extent that

$$\Delta_{inf}X_A(0)\ \Delta_{inf}X_A(\pi/2) < 1 \qquad (3)$$

because, according to quantum mechanics, the two conjugate variables must satisfy the Heinsenberg principle

$$\Delta X_A(0)\ \Delta X_A(\pi/2) > 1 \qquad (4)$$

Of course QM predicts that, as a result of entanglement, both inequalities should be satisfied. They are not contradictory. The variances appearing in inequality (2) in fact refer to values of $X_A(0)$, $X_A(\pi/2)$ inferred by *different* measurements ($X_B(0)$ or $X_B(\pi/2)$) performed on B. The prediction (3) has been experimentally realized [8].

The interest of the GMT paper arises from the fact, up to now, the production of entangled states has been generally considered as the result of quantum dynamics at the microscopic level. These authors, instead, suggest that "entanglement can be obtained via a classical force acting on a macroscopic object." To this purpose, they "consider the radiation field having a macroscopic number of photons and impinging on a completely reflecting and oscillating mirror in an optical cavity". By studying the properties of the output field they demonstrate that the state of the radiation field can become non-classical, giving rise to the appearance of the EPR paradox on its continuous variables.

In the proposed experiment, in fact, the two input modes in the cavity interact by means of the radiation pressure force by which each of them acts on the mirror. The entanglement of the two output field quadratures arises from the reaction of the mirror displacement quantum fluctuations. The Hamiltonian assumed to describe the system allows the calculation of the left hand side of (2) as a function of the input power and the temperature. Its value turns out to be about 0.7 for values of these parameters within the possibilities of the available technology.

In conclusion, the proposed scheme achieves the goal of obtaining the entanglement of radiation fields with a macroscopic number of photons, by means of a classical ponderomotive force on a macroscopic object. This work offers therefore new prospects for the investigation of the tricky borderline between the quantum and the classical world.

References

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