Determination of the gravitational constant G by means of a beam balance

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At the end of 1999, an international committee (the CODATA) decided to increase the uncertainty of the accepted value for the gravitational constant from 128 ppm to 1500 ppm. This remarkable step of increasing the uncertainty instead of decreasing was made to reflect the discrepancies between recent experiments. These experiments were originally performed with the aim to improve the accuracy of G, since even before increasing its uncertainty the gravitational constant had a very large uncertainty compared to that of other fundamental constants. The confusion started in 1995 when three groups published their results, which span a wide range of 0.7% (see Figure 1) – for reasons which remain unknown. The device most often used for measuring G is the torsion balance of Cavendish in one of its various forms, and 1998 was the 200 anniversary of the publication of his paper entitled "Experiments to determine the density of the Earth".

In the present experiment, a completely different and conceptually very simple approach is chosen. Using one of the world's most precise beam balances, the weight difference of two test masses is changed by the gravitational force of two movable tanks filled with a liquid of known density and measured with a sensitivity of 10^{-11}. From the measured change in weight difference, G is computed. Test measurements with one ton of water have been made, and measurements with 13.5 tons of mercury are currently in progress. The idea for this experiment arose from our earlier gravitation experiment at the Gigerwald storage lake, in which the stored water was used to measure G at an effective distance of approximately 100m. This experiment set more stringent constraints on the strength and range of a conjectured fifth force.

General Principles

For the new experiment, the "lake" is brought into the laboratory. As shown in Figure 2, two test masses are suspended on separate wires such that they occupy different levels on the same vertical axis. The test masses are alternately hung on the beam balance, and their weight difference is determined. Effects that are equal for both test masses cancel, such as tidal forces and zero-point drifts of the balance. The weight difference is modulated by the gravitational force of two field masses, which are moved between two positions. In the first position, the field masses are located between the test masses, while in the second, the test masses are between the field masses. These positions are alternated many times, and the averaged change in the weight difference is the gravitational signal from which G is computed. In this way, all disturbing effects which are independent of the field mass position cancel out.

Each field mass is a cylinder with a central hole, implying that its gravitational force has extrema close to each end of the hole. Since the field masses are moved such that the test masses are at an extremum, the gravitational force acting on a test mass is to first order independent of the test mass position, reducing the required accuracy of the distances between the field and test masses. In order to achieve a 10ppm relative uncertainty in G, the distances must be known with an accuracy of 0.1mm. This is contrast to the accuracy of approximately 0.001 mm required without this extremum. To take full advantage of the extremum effect, the dimension of the field masses, mainly the inner diameter, must be measured very accurately. But the diameter of a hole in a solid object is easy to measure compared to the distance between two bodies, one of which is hanging on a thin wire.

A liquid is employed for the field masses, since for large volumes the density of liquids is more homogeneous than that of typical solids, and mercury is used because of its high and well-known density. Measurements made with water allow an important consistency check, and the effect of the tanks can be measured with empty tanks.

Experimental Setup

The experiment is located in a 4.5m deep pit inside an experimental building at the Paul-Scherrer-Institute in Villigen, Switzerland. This pit has thick concrete

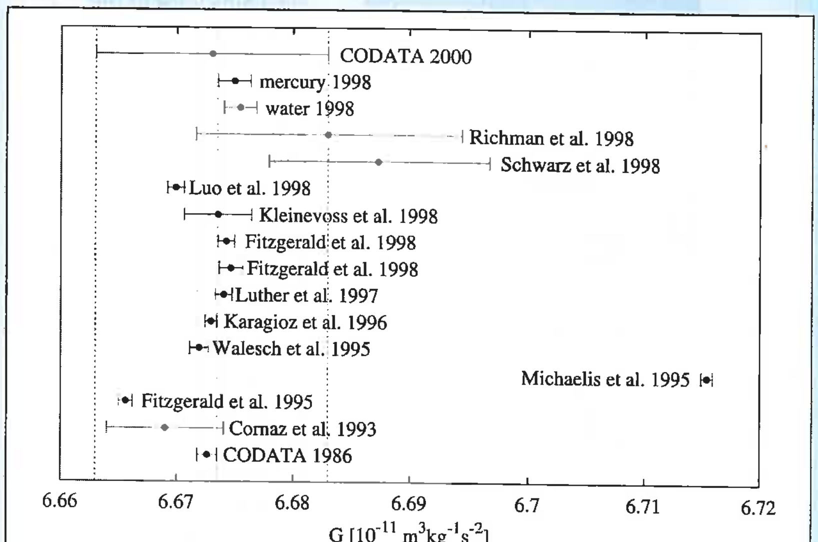

Fig 1 Recent measurements of the gravitational constant. The data points labelled "mercury" and "water" are the preliminary results from our experiment. The measurement made by Cornaz et al. is a result from our previous experiment at the Gigerwald storage lake, and the point labelled "CODATA 2000" is the presently accepted value.

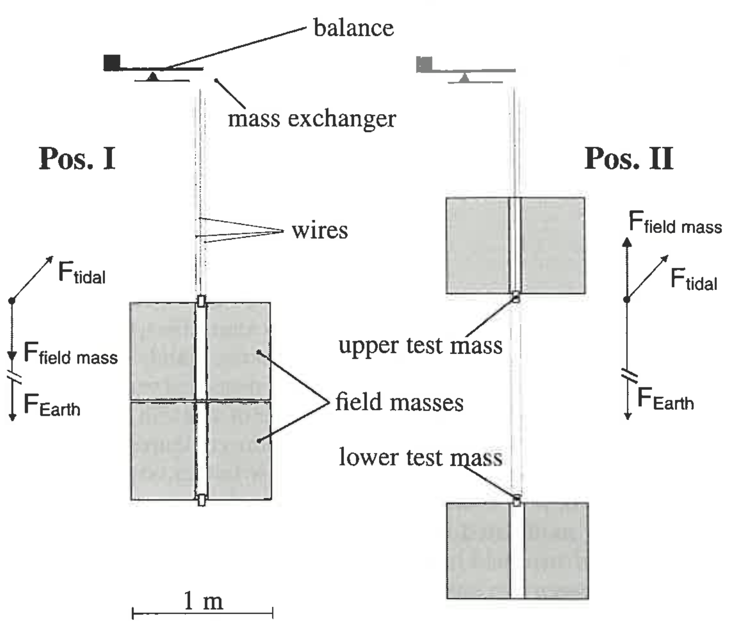

Fig 2 Schematic view of the mass arrangement. The test masses are alternately connected to the balance by means of the "mass exchanger" and their weight difference is determined. The field masses are moved between the two positions I and II. The forces on the upper test mass due to the Earth, the field masses and tidal forces are plotted for the two field-mass positions on the left and right of the figure.

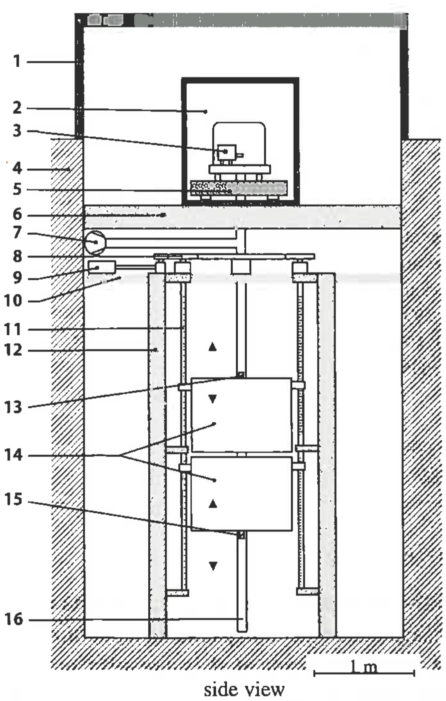

Fig 3 Side view of the experimental set-up.
Legend:
1: enclosure
2: thermally insulated chamber
3: balance inside the vacuum chamber
4: concrete walls of the pit
5: granite plate
6: steel girders supporting the balance
7: vacuum pumps
8: gearbox
9: motor
10: working platform,
11: spindle
12: steel pillar
13: upper test mass
14: field masses
15: lower test mass
16: vacuum tube

walls and good mechanical and thermal stability. A schematic view of the experiment is shown in Figure 3. The experiment extends over two rooms, separated by an insulating working platform. The upper room contains the beam balance, the electronics and the auxiliary equipment, and the lower part houses the field masses. Both parts are thermally and mechanically insulated from each other. Special care is taken to maintain a high temperature stability, each room having its own temperature stabilization system.

The balance is situated on a granite plate, which in turn is supported by two steel girders fixed to the walls of the pit. In order to avoid errors due to convection and buoyancy, the balance is operated in vacuum at a pressure of 10^{-4} Pa. At atmospheric pressure, the temperature stability would have to be in the mK-range in order to reduce to a tolerable level an apparent mass change due to thermally driven convection currents. The vacuum system consists of a 0.5m diameter chamber housing the balance and a long tube enclosing the test masses. In order to minimize magnetic forces, the 1kg test masses are made of copper.

The tanks employed for the field masses are made of stainless steel, each one 700mm high with a volume of 500 liters. The corresponding 6.8 tons of mercury result in a gravitational signal of approximately 0.8mg. To obtain the same signal with water instead of mercury, more than 2000 tons of water would be necessary. For the movements of the tanks, a robust tower was built, with the tanks supported by ball-bearing nuts on three spindles with left- and right-hand threads. The spindles are coupled by a gearbox and driven by a synchronous motor. With this construction, a positioning precision of better than 0.01mm is achieved.

The balance is a modified version of a commercial product of Mettler-Toledo (AT1006 Comparator), specifically designed for the high precision comparison of 1kg standard masses. Such balances are needed by the bureaus of standards, since the kilogram is still defined relative to a standard object. For the puposes of the experiment, the resolution of the balance was improved from 1μg to 0.1μg. Of considerable importance for the high-precision comparison of weights are a temperature stability in the mK-range and a careful exchange of the masses.

Measurements and Results

The measuring procedure is fully automated and computer-controlled. A measuring cycle consists of a determination of the weight difference in both field-mass positions and takes about 8 hours. The balance is calibrated every four hours using a 1g standard mass.

Initial tests of the experimental setup were made with empty tanks and with each tank filled with 500kg water. After a measuring time with water of 20 days, a gravitational signal of approximately 110μg was measured with a statistical uncertainty of 9 ng (one standard deviation). To determine G, the gravitational force from the field masses on the test masses was computed by numerical integration. The results with full and empty tanks were found to be consistent to within the statistical uncertainty. The result of this test is

$$G_{water} = (6.6754 \pm 0.0005_{stat} \pm 0.0014_{sys}) \times 10^{-11}\ m^3\,kg^{-1}\,s^{-2}$$

The first error is the statistical uncertainty and the second the systematic uncertainty.

During winter of 1997/1998 the tanks were filled with mercury. The mercury was purchased in 400 flasks, and the total mass of mercury in the tanks was determined, by weighing the full and the empty flasks, to be 13 520.635(27)kg. The amplitude of the gravitational signal is now 785μg, and a portion of the data for the modulated weight difference is plotted in Figure 4. During the measurements, a variation in the value of G of approximately (80 ppm was found, which is at present not satisfactorily understood. This variation appeared on a time scale of weeks, but so far no systematic behind it as been found. A preliminary result is plotted in Figure 1, in comparison with recent published measurements. The measurements made with water- and with mercury-filled tanks are consistent within their assigned uncertainties of 220 ppm, which are believed to be dominated by a nonlinearity of the balance. So far, only an upper limit for this nonlinearity has been estimated, but a precise experiment to investigate the linearity of the balance is in preparation. It should be mentioned that a nonlinearity would affect both of the G-values and their uncertainties in the same way. This is the reason why the measurement made with mercury has not yet resulted in a smaller uncertainty than the water measurement. This result, which differs from the presently accepted value by approximately two of our standard deviations, represents the current state of our experiment. Additional improvements and investigations will be made, and we are optimistic of reaching the design accuracy of the experiment of 10 ppm.

On the occasion of the 200th anniversary of the Cavendish experiment, a meeting was held in November, 1998, in London. At this meeting the preliminary results of several experiments, with uncertainties between 2000 and 100 ppm, were presented (see further readings). They are all in rough agreement, and their mean tended to exceed the at this time accepted value (see CODATA 1986 in Figure 1). Due to the variety of measurement techniques used, each with its own particular systematic effects, it is clear that progress in the accuracy and the confidence in the value of G is possible.

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Further Reading

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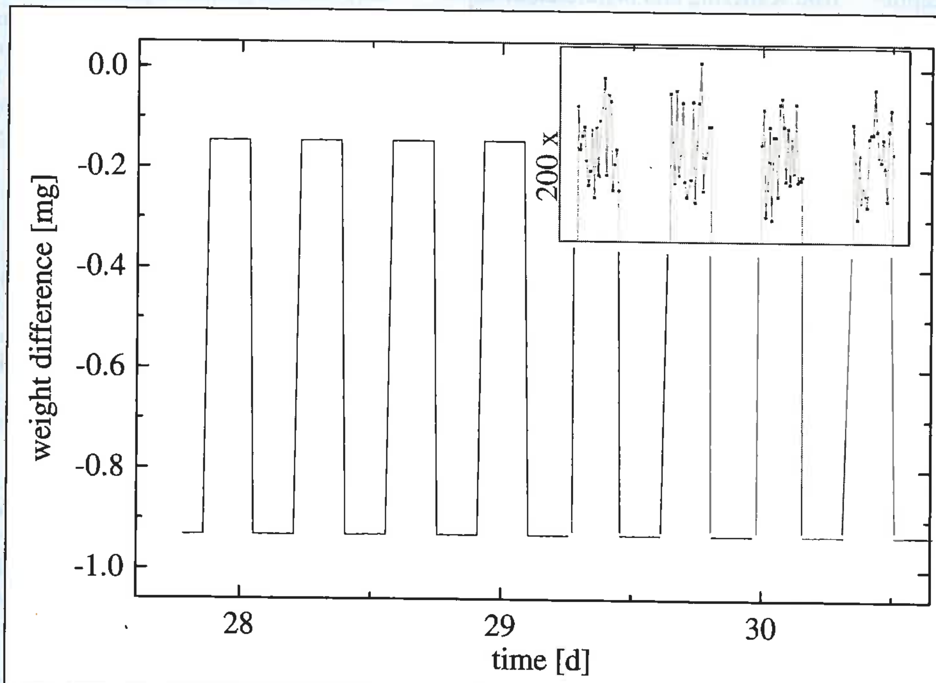

Fig 4 Modulation of the weight difference due to the gravitational force of the mercury-filled tanks. The inset shows the weight difference enlarged 200-fold for the field masses in position I (tanks together).

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