

systematic way. These control parameters could be used, for instance, in order to change the vesicle shape in a cyclic fashion. Such a cycle in shape space would represent a colloidal machine which performs mechanical work on the micrometer scale. A simple example is provided by a "swimming" cycle which should lead to self propulsion at low Reynolds numbers.

Flexible membranes also undergo thermally-excited shape fluctuations in order to increase their configurational entropy. The spectrum of these fluctuations covers many wavelengths between the nanometer and the micrometer scale. On the latter scale, these fluctuations correspond to smooth changes in the membrane curvature and are governed by the bending rigidity. On the nanometer scale, the membrane should exhibit thermally-excited protrusions, *ie* relative displacements of individual molecules which increase the roughness of the membrane/water interfaces.

We have recently shown that both types of fluctuations can be simultaneously studied in computer simulations using membrane models with molecular resolution. Our data provide strong evidence that the concept of bending elasticity is already meaningful on length scales which are only somewhat larger than the bilayer thickness. It was even possible to estimate the bending rigidity from the fluctuation spectrum. Somewhat surprisingly, the numerical value of the bending rigidity turns out to be consistent with a relatively simple relation in which the bending rigidity is expressed in terms of the lateral compressibility modulus and the bilayer thickness. It is highly desirable to study this relation for more complex membranes with several species of amphiphiles.

All phenomena discussed here are accessible to experiment. Thus, the self-organization of membranes provides many opportunities for fruitful interaction between theory and experiment.

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#### Further reading

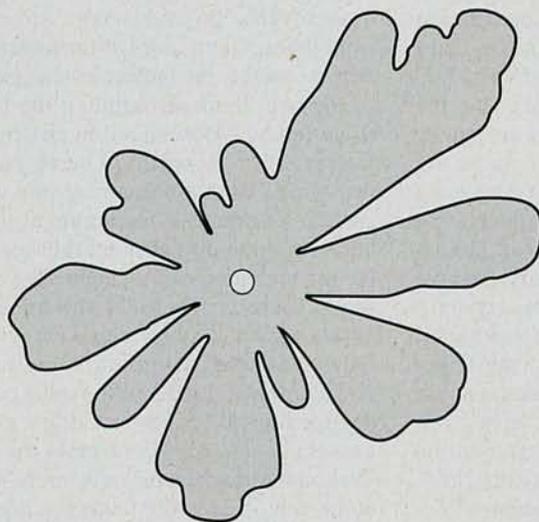
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The study of viscous fingering—the tentacle-like flow of one fluid in another—began at the bottom of an oil well

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## Controlling Viscous Fingering



**Saffman-Taylor instability (viscous fingering) for a polymer solution in a radial cell: Air (in grey) displaces the solution and is injected through the central hole**

**T**wo-phase flow in porous media occurs in a large number of practical situations, arguably the most important of which is oil recovery. Petroleum is generally found in porous rocks; the crude oil can be extracted from a well because it is pushed out by the high pressure in the reservoir. However, in the process of the recovery the pressure decreases rapidly and the flow of oil stops. Injection of another fluid—water or carbon dioxide gas injected below or above the oil layer respectively—allows extracting more oil from the well. As far back as 1740 in Sweden "running water was used to produce crude oil from galleries cut into the rocks bearing strata of tar and sand". In the USA this so-called "water flooding" was only recently (around 1940) recognized to be "the most efficient method". Before the second world war, it was believed that water could drown the reservoir and in many states injection of water was prohibited by law.

In such water-flooding operations, however, a hydrodynamic instability can develop at the interface between the water and the crude oil. This instability arises generically when a less viscous fluid (water or gas) pushes a more viscous one (oil) in a porous medium. As a result of the instability "fingers" of the less viscous fluid grow in the more viscous one. These

fingers become narrower as the flow through the reservoir increases. This instability limits the output of a well, because if the flux becomes too high, the fingers may reach the entrance of the well rapidly, and mainly water or gas instead of oil will be recovered.

In 1956 Sir Geoffrey Taylor visited a small oil company called the "Humble Oil Company" and became interested in this problem. In a seminal paper, written jointly with P.G. Saffman, they considered an idealised version of the problem. As experiments in porous media are not easy, since they are not transparent, Taylor studied the interfacial instability of a two-phase flow in a Hele-Shaw cell: two parallel glass plates separated by a small gap  $b$ . This set-up models the flow in a porous medium: the same equation (Darcy's law) determines the relation between the velocity and pressure gradient in both porous media and in the Hele-Shaw cell. The average velocity of the fluid is proportional to the gradient of the pressure over the viscosity  $\eta$ . The coefficient of proportionality is (minus) the permeability for the porous medium and  $-b^2/12$  in the Hele-Shaw cell.

Indeed, in the Hele-Shaw cell the formation of "fingers" of the less viscous fluid into the more viscous fluid was ob-

served. Ever since, this instability has been known as the Saffman-Taylor (or viscous fingering) instability. Because of its relative simplicity, and its potential applications, it has received much attention and has become an archetype of pattern-forming systems.

The original experiments of Sir Geoffrey Taylor were performed in a linear Hele-Shaw cell, *ie* in a thin rectangular channel. In this case, only one stable finger propagates steadily. The width of this finger results from a competition between two forces. On the one hand, the capillary forces tend to maximise the radius of curvature at the tip of the finger (for the same reason that soap bubbles are spherical) and so tend to widen the finger. On the other hand, the viscous forces seek the smallest resistance for the propagating finger, and thus tend to narrow it. The ratio of the viscous to the capillary forces is given by the dimensionless capillary number. Experiments with Newtonian fluids show that the width of the finger relative to the width of the channel is a universal function of the capillary number plus a geometrical factor accounting for the width and the height of the channel. The relative width is one at low velocity of the fingers (capillary forces dominate) and tends to  $1/2$  at high velocity (viscous forces dominate). While the shape of the observed fingers, at least for small capillary number, was well predicted by Saffman and Taylor, the understanding of the observation that at high velocities still half of the channel is occupied by the finger, turned out to be a very hard problem that was solved only recently. One of the original motivations of Saffman and Taylor for studying this hydrodynamic instability was to improve the efficiency of oil recovery. The more stable and larger are the fingers, the more important can be the delivery of a well.

In our laboratory, we probe the modification of the Saffman Taylor instability when so-called "complex fluids" are used, which are dilute solutions of surfactants and or polymers. Surfactants allow one to modify the capillary forces while polymers allow for a modification of the viscous forces; these are the two forces that govern the instability. In recent years, we (and a number of other groups) have shown that the Saffman-Taylor instability is modified drastically in presence of such complex fluids.

In a first experiment, heptane (the less viscous fluid) pushes water (the more viscous fluid) in the presence of an ionic surfactant (AOT) in a linear Hele-Shaw channel. Experiments show that the relative

width of a finger, at high velocity settles at a plateau value significantly above the classical value of  $1/2$ . A numerical treatment of the complete non-linear problem allowed us to model and understand the effect of surfactant addition. Surfactant molecules form a dense film at the interface between the two fluids and decrease the surface tension (and thus the capillary forces) by several orders of magnitude. When the finger propagates, two effects modify the surface tension at its tip: the flow advects the surfactant molecules from the tip to the sides of the finger. Second, as surfactant molecules are present in the bulk fluid surrounding the finger, these can be adsorbed at the unsaturated interface. The competition between these two effects leads to an anisotropy of the surface tension: it is maximum at the tip and minimum on the sides; this accounts for the widening of the finger. The larger the anisotropy, the wider the finger becomes, which can be verified experimentally by varying, for instance, the surfactant concentration. Another observation confirming this idea is that adding a small amount of salt, which decreases the equilibrium interfacial tension by more than a factor ten, widens the fingers at high velocity. The effect is so large that at the "optimal salinity" (ultralow interfacial tension) the finger occupies the whole channel at high velocity.

In the second experiment, a modification of the viscous forces was obtained by employing dilute solutions of long polymers (molecular mass of a few million) in water. In our experiments, the polymer solution is the more viscous fluid while air is the less viscous one. With polymer solutions, different non-Newtonian effects are possible; which one dominates depends on the type of polymer used: rigid (or rod-like) polymers such as Xanthane or flexible polymers such as polyethyleneoxide.

The effect of rigid polymers most likely finds its origin in a non-Newtonian shear viscosity. Solutions of these polymers are shear thinning, which means that the viscosity decreases with the shear rate, *ie* the velocity. A direct consequence is that an effect on the Saffman-Taylor instability is observed only for polymer concentrations high enough to increase the shear viscosity of the water. Experiments show that, for this case, the relative width of the fingers at high velocity is smaller than  $1/2$ . Qualitatively this is easy to understand: the shear rate in the Hele-Shaw cell is highest near the tip of the propagating finger. As a higher shear rate implies a lower (local) viscosity, this implies that the forward di-

rection (the direction of propagation) is a direction of "easy growth" for the finger, leading to narrow fingers.

The behaviour of solutions of flexible polymers turned out to be completely different. The width of the fingers at high velocity tends to a value larger than  $1/2$ , the opposite of what was observed for the rigid polymers. Additionally, the effects are observed for polymer concentrations that are so small that the shear viscosity is indistinguishable from that of water. The origin of the enlargement of the fingers is neither the surface tension, as was the case for the surfactants (the surface tension of the polymer solution is not very different from that of water), nor the shear viscosity, as was the case for rigid polymer solutions.

It is likely that the visco-elastic properties of flexible polymers are at the origin of the effects. When deformed, the macromolecules in solution behave as springs. Macroscopically, this means that the solution shows a strong resistance to elongational deformations. This is exactly what happens near the tip of the fingers, where the elongation rate of the liquid is maximum. The consequence of this completely different non-Newtonian effect is that the lubrication layer that remains between the propagating finger and the walls of the Hele-Shaw cell is much thicker than for a Newtonian fluid, and the finger is enlarged.

In conclusion, the Saffman-Taylor instability is drastically modified by the properties of complex fluids. Three different causes for the modifications have been identified so far: a dynamic (and anisotropic) surface tension, a non-Newtonian shear viscosity, and an elongational viscosity. The first and the third process lead to much wider fingers, and addition of more (or less) of the additive allows one to control the finger width for a given finger velocity. This, in a way, was the original motivation of Saffman and Taylor for studying this hydrodynamic instability: wider fingers lead to a more efficient displacement of the crude oil in the reservoir, and should thus improve recovery rates.

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#### Further reading

*Viscous Fingering and Related Instabilities in Complex Fluids* by Daniel Bonn, Hamid Kellay and Jacques Meunier *Philosophical Magazine* B78 131 (1998)