Moving Pictures

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Spatiotemporal dynamics of complex systems

Since the appearance of the modern approach to the study of dynamical systems, chaotic dynamics has become a source of fascination for researchers and for outsiders. It has many applied and fundamental implications, from the behaviour of a pendulum to the prediction of the weather. The appealing aesthetics of strange attractors or fractal basin boundaries illustrates the delicate interplay between order and disorder that has become a paradigm of complexity in nature.

When nonlinear phenomena take place in spatially extended media, evolution can be irregular not only in time but also in space. Intriguing patterns emerge and evolve, challenging us to identify their essential features. An extreme example of this situation is turbulence in fluids. Simplified models and controlled experiments in a variety of physical systems are currently aimed at understanding the appearance and the characteristics of this *spatiotemporal chaos*.

Particularly robust and general features are those related to topological properties. For example, dislocations in solids display strong similarities with defects in roll patterns of convecting fluids, and defect formation in textures of superfluid ^{3}He is presently taken as a condensed-matter analogue of the processes that occurred during the phase transitions in the early universe. These widely different systems are related because of the underlying symmetries of the fields involved. Topological structures are also present in spatiotemporally chaotic systems, playing an organizing role in their dynamical evolution.

This pictorial illustrates a variety of nonlinear phenomena occurring in spa-

Opposite page

Fig 1 Spatiotemporal intermittency in oscillating vector fields in one dimension. The moduli of the two components of the complex vector order parameter are shown, coded in colours, as a function of space and time

Fig 2 Breaking rotational symmetry changes the behaviour of the two-order parameter components, which remain remarkably correlated, however, and isolates particle-like objects

A Spin in the Real World

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A snapshot of spatiotemporal spiral chaos

As an example of complex spatiotemporal behaviour in natural patterns, a small section of a snapshot shadowgraph picture of Rayleigh-Bénard convection in a gas SF_6 near its gas-liquid critical point is shown **right**. Due to closeness to the critical point the convection occurs in a very thin layer of the fluid (130 µm) that allows the experiment to exhibit an enormous ratio between the horizontal and vertical sizes in the convection cell. This ratio turns out to be a crucial parameter in pattern selection and stability. Thanks to the large cell aspect ratio, a new type of spatiotemporal disordered state, coined spiral chaos, has recently been experimentally discovered. Moreover, it was found in regions of the stability diagram, where a roll pattern was theoretically predicted to be stable.

The image here was obtained by the shadowgraph technique which allows us to visualize a horizontal variation in the field of refractive index resulting from the convection flow. The pattern clearly demonstrates the presence of various topological defects in the structure, such as dislocations, grain boundaries, spirals and targets. These defects and their dynamics are building blocks of pattern complexity, and determine the spatiotemporal chaotic behaviour of the system.

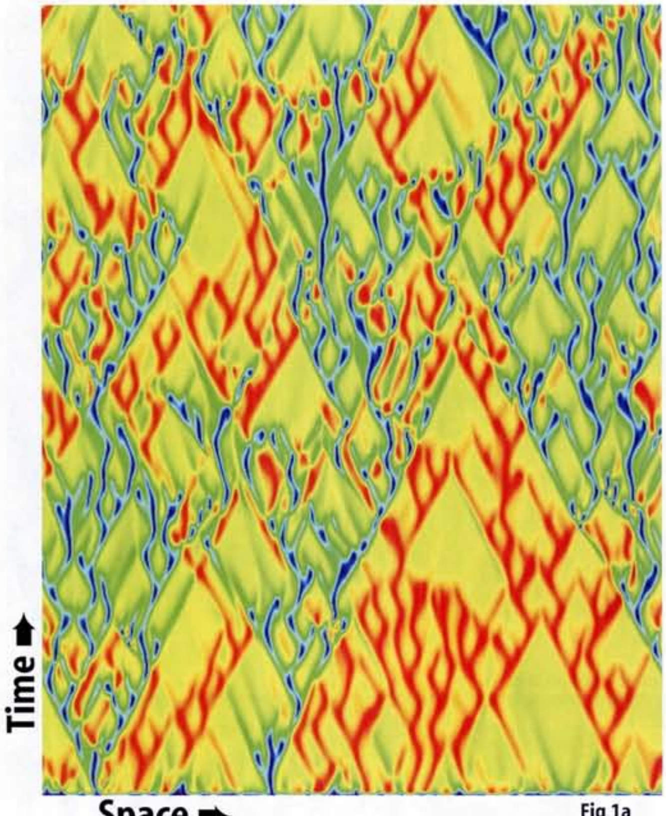

Fig 1a

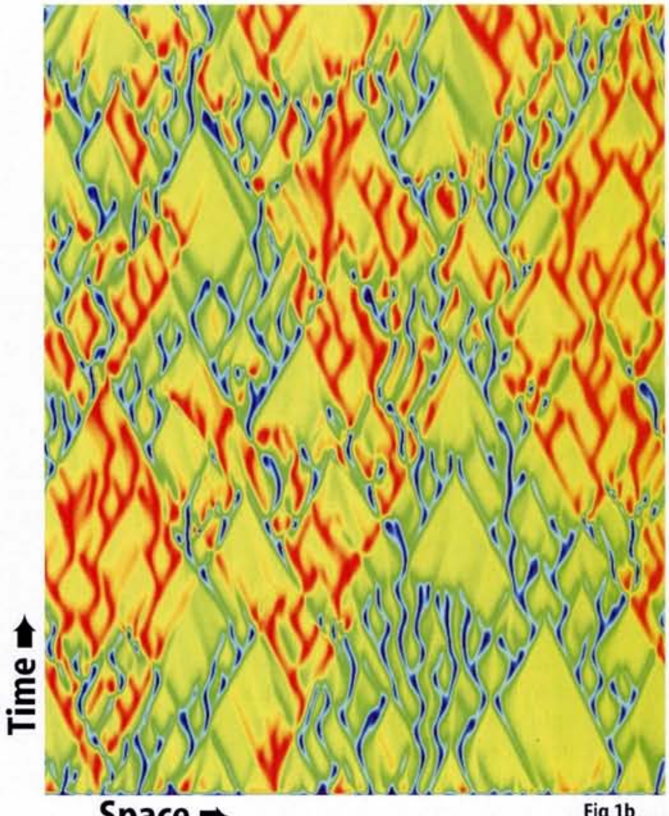

Fig 1b

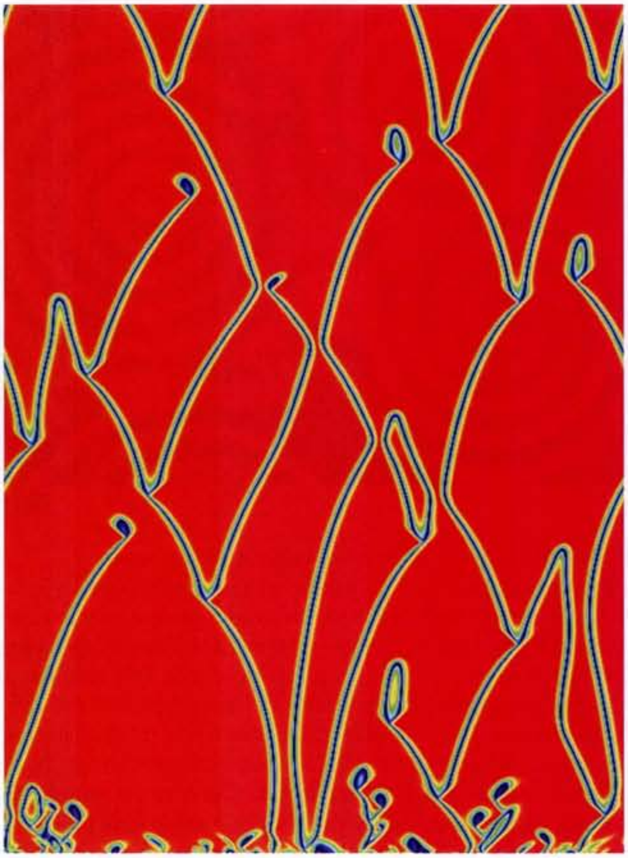

Fig 2a

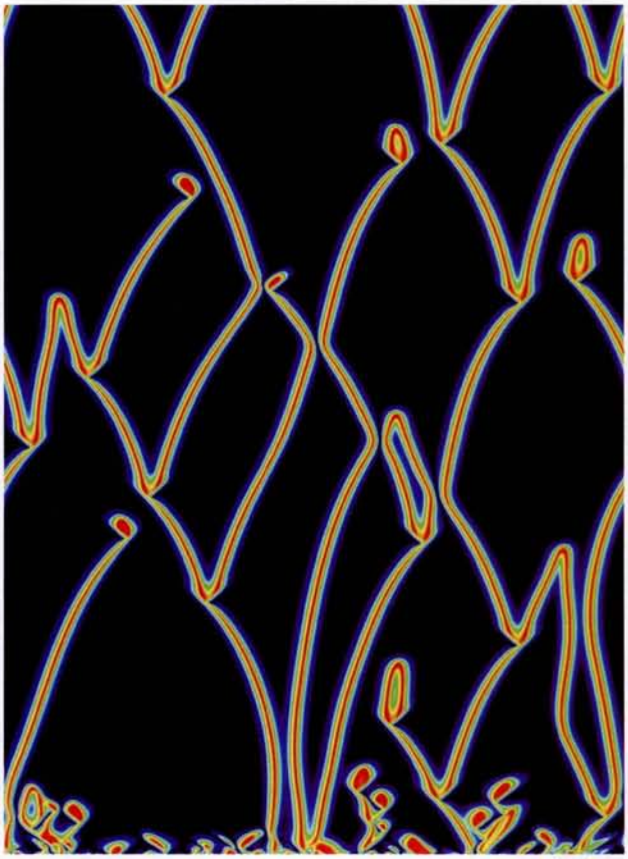

Fig 2b

tiotemporally chaotic systems by means of computer simulations of a model system, the Vector Complex Ginzburg-Landau equation (*see box opposite*). The model describes generic features of the dynamics of vector fields when they are close to instabilities leading to nonlinear temporal oscillations. A physical example of such a situation is the onset of laser emission. The oscillating vector field is in this case the electric field inside a laser resonator.

Counter propagating thermal waves in convecting mixtures of fluids and the wave-function of two-component Bose-Einstein condensates are other systems described by this or related equations.

Two broken symmetries are present in this class of problems. The appearance of oscillations breaks time-translation invariance, and the direction in which the vector field oscillates breaks rotational symmetry. As a consequence, a natural order parameter characterizing the broken symmetries is a vector complex field. At any given point, the *phase*, *modulus*, and *orientation* of the complex vector would give, respectively, the phase, amplitude, and direction of the oscillation. In nonlinear optics and lasers, the time-dependent vector orientation gives the polarization of light and coupling between different spatial points is provided by the phenomenon of diffraction. Here, we will only consider the situations of a two-component vector field extended in one- and two-dimensional oscillating media.

One type of behaviour found in the one-dimensional case is shown in *figures* 1a and 1b. At each coordinate of space and time the amplitudes of oscillation of the two independent components of the order parameter have been represented by different colours. Red pulses of strong oscillation and blue holes where the oscillation is nearly absent evolve in an intertwined spatiotemporally chaotic network. Remarkably enough, there are portions of the pictures in which the colour is rather homogeneous, identifying space-time regions of nearly periodic and smooth oscillation. This coexistence of ordered states with bursts of strongly chaotic behaviour has been named spatiotemporal intermittency, a term borrowed from fluid dynamics. The phenomenon is a quite common kind of spatiotemporal chaos and appears in a variety of models including partial differential equations, coupled maps, and cellular automata.

amplitude

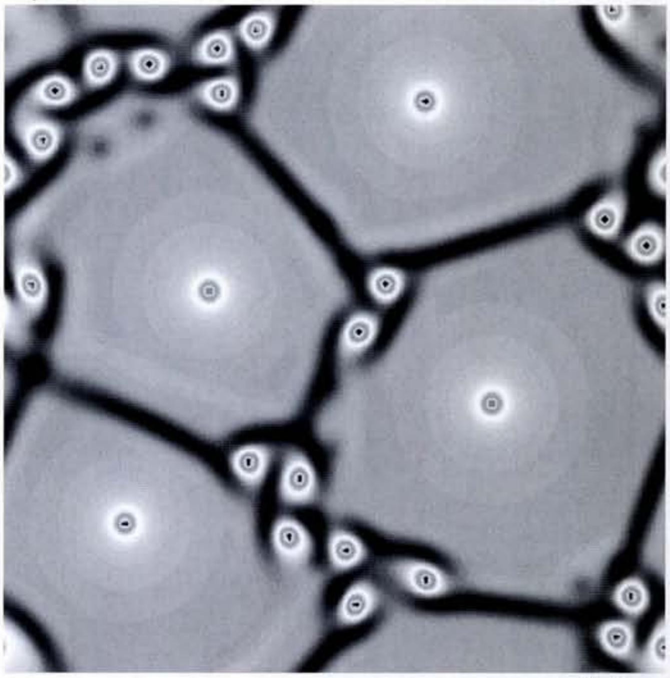

Fig 3a

sum of two phases

Fig 3b

Fig 3 A field configuration containing different kinds of vector topological defects for an oscillating medium in two spatial dimensions. Shown are one of the amplitudes **Fig 3a** and the sum of the two phases **Fig 3b** at a particular time

An interesting peculiarity of a vector order parameter is that the two components of the vector field can be interpreted as two chaotic locally coupled fields. This is a natural set-up with which to study phenomena related to the synchronization of chaotic oscillators, an area presently under intense study. The correlated behaviour of the two chaotic fields is manifest in *figures* 1a and 1b: red pulses in one of the components are associated with blue pulses in the other, and *vice versa*.

Changes of symmetry strongly influence the dynamics of spatiotemporally chaotic systems, as in many other physical contexts. *Figures* 2a and 2b show the amplitudes of two components of the order parameter when rotational symmetry has been explicitly broken by adding an extra term to the equations. Non-chaotic regions become much larger and the elementary constituents of the chaotic bursts are isolated and clearly seen as localized particle-like objects. In optical applications the symmetry breaking term is related to birefringence of the medium, and the localized structures are pulses of light in a polarization state orthogonal to that of the background. The existence of pulse-like objects and other localized structures is common in spatially extended nonlinear media. They are the precursors of the topological defects that appear in higher dimensions. Their interactions, including splitting and annihilation, frequently contain clues for the understanding of spatiotemporally chaotic states.

Fields extended in two dimensions present qualitatively new features. The most important ones arise from topological restrictions. *Figure* 3a shows the modulus of one of the components of the order parameter, in a snapshot taken at a particular time. It is clear that the field is organized in domains of rather homogeneous oscillation states. Each domain however contains a dot, which is a point at which the oscillation amplitude vanishes. Other dots are also present at domain walls. Inspection of the phase (*figure* 3b—shown here is the sum of the two phases associated with the two components) reveals that these dots are different kinds of topological defects, since they are phase singularities. The defects inside domains are singularities in the two components of the vector field simultaneously. In the phase representation they appear either as two-armed spirals or as targets. Defects confined to the domain walls are singularities of one of the two components of the vector field and they appear as truncated simple spirals. The singular character of the defects is clearly seen, for example, at the centre of the two-armed phase spirals, where different phase values characterized by different colours meet. The oscillation phase is thus ill-defined and this is the reason why the oscillation amplitude vanishes there. Based on the values of the two independent phases in a closed path around the defects, two kinds of topological charges can be assigned to each of them, thereby giving a classification of these defects. Topological constraints enforce charge conservation rules that greatly influence the dynamics.

Such field configurations as the one shown in *figure* 3a can freeze in a glassy state with slow dynamics or follow a spatiotemporally chaotic evolution. In any case, dynamics is essentially governed by the interactions of the topological defects that behave as particle-like objects.

Defects, pulses, spiral waves, and intermittency are beautiful phenomena appearing in a variety of situations. Frequently they provide basic units with which more complex behaviour is built up. Their systematic study within simplified models as the one considered here or in specifically designed experiments is a needed step for the general understanding of spatiotemporally chaotic

Past Issue
See also Critical Phenomena in Hydrodynamics, by M. Assenheimer and V. Steinberg, in *Europhysics News* July/August 1996

The Vector Complex Ginzburg-Landau Equation (VCGLE)

$$\frac{\partial \mathbf{A}}{\partial t} = \mu\mathbf{A} + \alpha\nabla^2\mathbf{A} + \beta[\,(\mathbf{A}\,.\,\mathbf{A}^*)\mathbf{A} + \delta(\mathbf{A}\,.\,\mathbf{A})\mathbf{A}^*]$$

is an order parameter or amplitude equation for a vector complex order parameter $\mathbf{A} = (\,A_x(\mathbf{r},t)\,, A_y(\mathbf{r},t)\,)$ describing generic features of the dynamics of vector fields close to oscillatory instabilities (Hopf bifurcation). It can be derived on symmetry grounds requiring invariance under time and space translations, space inversion and rotations. The dot in the equation denotes scalar product and the star indicates complex conjugation.

The VCGLE can be written as two coupled-scalar equations for the two components:

$$A_\pm = \frac{A_x \pm A_y}{\sqrt{2}}$$

In optical systems, A_+ and A_- correspond to the amplitudes of the circularly polarized components of light.

Appropriate values of the parameters have to be derived for each specific physical model: μ is a measure of the distance to the instability threshold; the parameter α comes from diffraction and diffusion phenomena, and β is associated with saturation and nonlinear frequency shifts. The strength of the coupling between the two components of the field is measured by δ. In general α, β, and δ are complex numbers.

The VCGLE has two interesting limits. In the limit of conservative dynamics ($\mu = 0$, α and β are purely imaginary and δ real) one obtains a vector form of the Nonlinear Schrödinger equation. When α, β, and δ are real the equation describes purely relaxational dynamics in a potential related to the one originally written by V.L. Ginzburg and L. D. Landau to describe superconductivity.

Further reading

D. Walgraef Spatiotemporal Pattern Formation, with Examples in Physics, Chemistry and Materials Science; Springer Verlag (1996)
general background on nonlinear dynamics in extended systems

L. Pismen Vortices in Nonlinear Fields; Oxford University Press (1998)
focuses more on the subject of this pictorial than the above book does

editors L.A. Lugiato and M.S. El Naschie Nonlinear Optical Structures, Chaos, Solitons and Fractals 4 (1994)
many optical applications can be found in this special issue

L. Gil Vector order parameter for an unpolarized laser and its vectorial topological defects *Phys. Rev. Lett.* 70 (1993) page 162

M. San Miguel Phase instabilities in the laser vector complex Ginzburg-Landau equation *Phys. Rev. Lett.* 75 (1995) page 425

A. Amengual *et al.* Synchronization of spatiotemporal chaos: the regime of coupled spatiotemporal intermittency *Phys. Rev. Lett.* **78** (1997) page 4379