

# Critical Fields in High $T_c$ Superconductors

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**Using microwave methods one can determine upper critical fields which lie beyond the field of the laboratory magnet, and avoid ambiguities associated with the broadened transitions in high  $T_c$  superconductors.**

Upper critical field  $B_{c2}$  is one of the fundamental parameters which characterize a type-II superconductor. The common definition for  $B_{c2}$  is that the superconducting state persists until the applied field  $\mu_0 H_a$  reaches some value  $B_{c2}$ , when the transition to the normal state occurs. But this is not the only significance of the upper critical field. Namely, at fields  $B < B_{c2}$ , the magnetic flux penetrates the superconductor in a quantised form. Each flux quantum  $\Phi_0$  is carried by a superconducting current vortex whose core contains normal electrons. The radius of the vortex is given by the coherence length  $\xi$ , which itself is a very important physical quantity. There is a simple relationship between  $B_{c2}$  and  $\xi$  ( $B_{c2} = \Phi_0 / 2\pi\xi^2$ ) so that, in practice,  $\xi$  is evaluated from the measurement of  $B_{c2}$ . Of course,  $\xi$  is a field independent quantity since at all fields  $B < B_{c2}$  the vortices have the same radius, only the number of vortices increases with B. The field at which the vortices of size  $\xi$  reach maximum close packing is  $B_{c2}$ . Clearly, the determination of  $B_{c2}$  is of far more general importance than just the knowledge of the field value where the normal state is restored. When the temperature dependence of  $\xi(T)$  is determined, one can compare it with the predictions of various theoretical models, gaining insight into the nature of the superconducting state.

In classical low temperature superconductors the upper critical field could be determined with great precision by resistance measurements, because the transition to the normal state was always sharp. When the transitions were detected at different temperatures, one obtained the temperature dependence of  $B_{c2}$ . Usually,  $B_{c2}(T)$  increased linearly from zero value at  $T_c$  and saturated at low temperatures to a maximum value  $B_{c2}(0)$ .

## Broadened Transitions

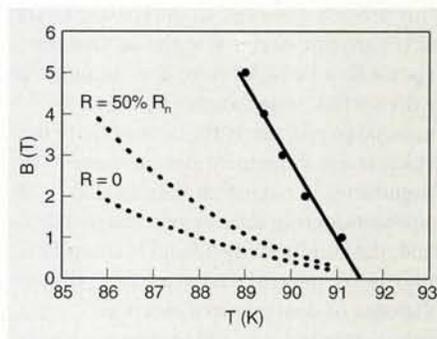
Early measurements on high  $T_c$  superconductors revealed that  $B_{c2}(T)$  increased rapidly to the limit of even the most powerful magnets when the temperature was lowered by only a few degrees below  $T_c$ . At lower temperatures, the external field was simply not high enough to drive the sample to the normal state so that  $B_{c2}$  could not be measured. There was also another difference with respect to classical superconductors. The resistive transitions in external magnetic fields, which could be observed near  $T_c$  in high- $T_c$  superconductors, appeared to be significantly broadened. Therefore, the question arose as to which criterion should be used for the normal to superconducting transition, and the determination of  $B_{c2}$ . It was found that regardless of the criterion used, e.g. zero resistance, 50% of the normal state  $R_n$ , or other, the resulting curves of  $B_{c2}(T)$  had an unusual positive curvature below  $T_c$  (Fig. 1).

The ambiguity of the criterion for  $B_{c2}$ , and the puzzle about the curvature was solved by the analysis of the measured reversible magnetization. The values of  $B_{c2}$  obtained in this analysis, appeared higher than those determined by any criterion in resistivity measurements, and showed linear dependence below  $T_c$  (Fig. 1). It was realized that the unusual behaviour of the resistance curves was due to the magnetic

flux motion near  $T_c$ , which causes some dissipation even in the superconducting state. This phenomenon is enhanced in high- $T_c$  superconductors and gives rise to broadened transitions. The zero resistance criterion was associated with the irreversibility line  $B_{irr}(T)$ , i.e. the line below which there is practically no flux motion, while other criteria  $0 < R < R_n$  had no precise meaning, but represented various degrees of flux line mobility [1].

## Scaling Properties

The upper critical field can also be extracted from the analysis of the superconducting fluctuations. According to the general theory of phase transitions, the order parameter fluctuates near  $T_c$ . This means that above  $T_c$  superconducting droplets appear in the predominantly normal medium, and decay after a short lifetime. Below  $T_c$ , normal droplets appear and decay in the predominantly superconducting medium. When fluctuations contribute to various thermodynamic and transport quantities (magnetization, specific heat, fluctuation conductivity) one expects the so-called scaling properties. For a given quantity, a set of experimental curves as functions of field and temperature can be made to collapse into a universal curve by some rescaling of the quantities on the axes. The scaling law results from a theoretical model of fluctuations [2,3]. The upper critical field  $B_{c2}$  enters the relevant expressions so that its value can be determined as that which results in the optimum scaling of the given set of experimental curves. In high  $T_c$  superconductors the fluctuations are greatly enhanced and the scaling procedure works very well. It was found [4-6] that the values of  $B_{c2}(T)$  lay on a



**Fig. 1.** Upper critical field  $B_{c2}(T)$  in single crystal  $\text{YBa}_2\text{Cu}_3\text{O}_{7-x}$  (data taken from Ref. [1]). The linear extrapolation of the reversible magnetization yields data points on the straight line. Resistance measurements with various criteria result in much lower field values lying on the lines with a positive curvature.

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straight line below  $T_c$  and the slope  $dB_{c2}/dT$  agreed with those determined previously from linear extrapolation of the reversible magnetization (Fig. 1). Thus the scaling procedure of various thermodynamic and transport quantities emerged as a reliable method for the determination of  $B_{c2}(T)$ .

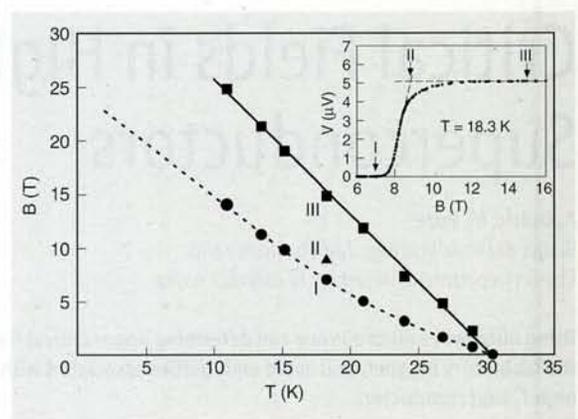
Scaling has recently shown advantages even in the  $K_xBa_{1-x}BiO_3$  superconductor which apparently has sharp resistive transitions [7]. With zero resistance criterion, one obtains the irreversibility line with a positive curvature (Fig. 2). The crossection of the linear extrapolations of the normal state and resistive transition section, which might look as a reasonable criterion for the determination of  $B_{c2}$ , turns out to be misleading too. Therefore, one can analyse the upper tail of the resistance curve (inset of Fig. 2) which is dominated by the fluctuation conductivity. The scaling of this quantity yields much higher values of  $B_{c2}$ , lying on a straight line. Such values could not be detected directly on the resistance curves. Similar behaviour was also observed in overdoped  $Tl_2Ba_2CuO_6$  [8], Sr deficient  $Bi_2Sr_2CuO_y$  [9] and  $Bi_2Sr_2CaCu_2O_8$  [10], but no fluctuation scaling was attempted.

Although the scaling procedure has an obvious advantage, it keeps the same limitations on the determination of  $B_{c2}$  within the fields accessible by the laboratory magnet, and this is not enough for high  $T_c$  superconductors.

### Microwave Method

An alternative method for the determination of  $B_{c2}$  has emerged from the temperature and field dependent microwave absorption. When a normal conducting sample is exposed to a microwave field, current is induced on the surface and within the penetration depth (skin effect). This process gives rise to microwave losses which are proportional to the surface resistance  $R_s = \text{Re}(Z_s)$ , where  $Z_s = (i\mu_0\omega/\sigma)^{1/2}$  is the surface impedance. The only material parameter is the conductivity  $\sigma$ , which is a real quantity at microwave frequencies in normal metals. In the superconducting state in zero magnetic field, the conductivity becomes complex  $\sigma = \sigma_1 - \sigma_2$ , the real part expressing the response of uncondensed electrons (quasiparticles), and the imaginary part that of superconducting pairs. With the superconductor in the mixed state, the situation is more complicated. The microwave current drives vortex oscillations and yields a contribution to the microwave dissipation. The total

**Fig. 2.** Upper critical field  $B_{c2}(T)$  in  $K_xBa_{1-x}BiO_3$  (data taken from Ref. [7]). Zero resistance criterion (position I in the inset), yields lower curved line. Linear extrapolation to the normal state (position II in the inset) does not alter the result significantly. Only the fluctuation scaling procedure yields  $B_{c2}$  on a straight line. This value is also marked on the resistivity curve (position III in the inset).



response of the superconductor to the microwave field is expressed by the effective conductivity [11]

$$\frac{1}{\delta_{eff}} = \frac{1-b}{(1-b)\sigma + b\sigma_n} + \frac{b}{\sigma_n} \frac{v}{v_f} \quad (1)$$

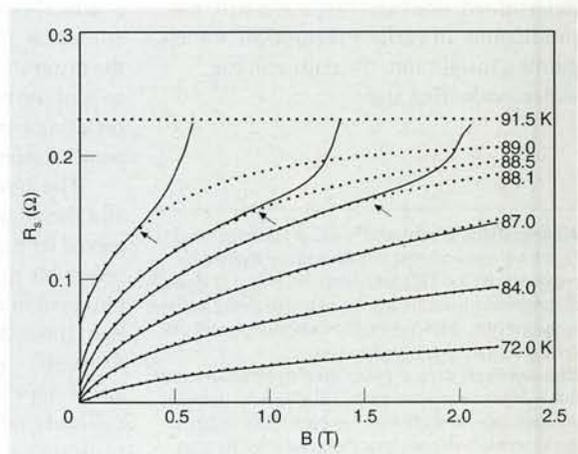
where  $b=B/B_{c2}$  is the reduced field,  $\sigma_n$  the normal conductivity, and the ratio  $v/v_f$  is the vortex mobility factor, which attains unity in the flux flow regime. With the effective conductivity used in the expression for the surface impedance  $Z_s$ , one obtains the analytical expression which can be used to make fits to the experimental curves. One of the fit parameters is the upper critical field  $B_{c2}$ , which can thus be determined.

It is important to note that this method does not require the knowledge of a complete experimental curve from  $B=0$  up to  $B_{c2}$ , but only some initial fraction of it suffices. An example of  $YBa_2Cu_3O_{7-\delta}$  single crystal is shown in Fig. 3 [12].

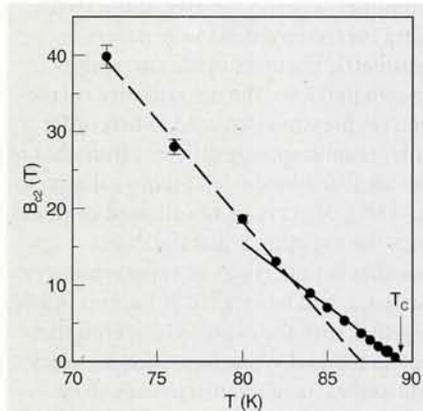
At lower temperatures  $B_{c2}$  is clearly beyond the field of the laboratory magnet used in the experiment (2.25 T). Yet, the fit

of the theoretical expression to the available experimental data suffices to determine  $B_{c2}$  as high as 40 T (Fig. 4).

The situation at temperatures closer to  $T_c$  is different. The low field segments of the experimental curves can be fitted by the theoretical expression using  $\sigma_{eff}$ . These segments suffice to determine  $B_{c2}$  values. Indeed, the extensions of the theoretical curves rise further to the normal state and show where  $B_{c2}$  is. On the contrary, the experimental curves do not rise to the normal state because of the fluctuations which dominate at higher fields. The high field segments of the experimental curves can also be used to determine  $B_{c2}$ . Namely, in this region, the total conductivity is the sum of normal and fluctuation contributions. One can use the scaling procedure for the fluctuation conductivity [13], which yields the values of  $B_{c2}$  as fit parameters. Thus, each experimental curve near  $T_c$  can be used to determine the  $B_{c2}$  value twice, from low field and high field segments, in two different procedures. It is most reassuring that both procedures yield the same  $B_{c2}$  value, thus demonstrating the consistency of the method.



**Fig. 3.** Experimental microwave surface resistance  $R_s$  in a single crystal  $YBa_2Cu_3O_{7-\delta}$  (dots) and theoretical fits (solid lines) using mean field  $\sigma_{eff}$  in the mixed state. The arrows indicate the crossover from the mixed state to the fluctuation region.



**Fig. 4.** Upper critical field in a single crystal  $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$  determined from the fits in Fig. 3. Note that  $B_{c2}$  could be determined up to 40 T even though the experimental  $R_s$  was measured only up to 2.25 T.

The temperature at which the coherence length diverges ( $B_{c2} \rightarrow 0$ ) is defined as  $T_c$  (Fig. 4). At about 83 K, one observes a change in the slope of  $B_{c2}(T)$  which may result from the crossover from three to two dimensional superconductivity.

## Outlook

The determination of the upper critical field, or equivalently the coherence length, has fundamental importance for the study of the nature of the superconducting state. It has recently been shown theoretically that for the gap anisotropy with the symmetry of the pairing state  $d_{x^2-y^2}$  one should have an anisotropy of  $B_{c2}$  in the  $ab$  plane (copper-oxide plane) which has fourfold symmetry near  $T_c$  [14]. Also, at lower temperatures, the temperature dependences of  $s$ -wave and  $d$ -wave superconductors are expected to be different [15].

The microwave technique promises to provide solid experimental data to answer the above questions. Yet, some associated problems have to be solved beforehand. Vortex motion driven by the microwave current is not completely in the flux flow regime so that the mobility factor  $v/v_f$ , which can be temperature and field dependent, should be precisely determined [16]. The anisotropy study would require measurements with several different field and current geometries, e.g. dc field in the  $ab$  plane, and the current along the  $c$  axis, or in the  $ab$  plane

perpendicular to the dc field, or in the  $ab$  plane parallel to the dc field. This will increase the number of experimentally determined parameters and eliminate the uncertainties. Thus, one may expect that the experiments underway in many laboratories will promote exciting discoveries in the field.

## References

- [1] U. Welp *et al.*, *Phys. Rev. Lett.* **62** (1989) 1908.
- [2] S. Ullach and A. T. Dorsey, *Phys. Rev. Lett.* **65** (1990) 2066; *Phys. Rev. B* **44** (1991) 262.
- [3] M.B. Salamon *et al.*, *Phys. Rev. B* **47** (1993) 5520.
- [4] U. Welp *et al.*, *Phys. Rev. Lett.* **67** (1991) 3180.
- [5] N. Overend *et al.*, *Phys. Rev. Lett.* **72** (1994) 3238.
- [6] S.W. Pierson *et al.*, *Phys. Rev. Lett.* **74** (1995) 1887; M.A. Howson *et al.*, *Phys. Rev. Lett.* **74** (1995) 1888, E74 (1995) 43.
- [7] T. Klein *et al.*, *Phys. Rev. B* **53** (1996) 9337.
- [8] A.P. Mackenzie *et al.*, *Phys. Rev. Lett.* **71** (1993) 1238.
- [9] M.S. Ososky *et al.*, *Phys. Rev. Lett.* **71** (1993) 2315.
- [10] A.S. Alexandrov *et al.*, *Phys. Rev. Lett.* **76** (1996) 983.
- [11] A. Dulcic and M. Pozek, *Fizika A2* (1993) 43; A. Dulcic and M. Pozek, *Physica C* **218** (1993) 449.
- [12] I. Ukrainczyk and A. Dulcic, *Europhys. Lett.* **28** (1994) 199.
- [13] I. Ukrainczyk and A. Dulcic, *Phys. Rev. B* **51** (1995) 6788.
- [14] K. Takanaka and K. Kuboya, *Phys. Rev. Lett.* **75** (1995) 323.
- [15] H. Won and K. Maki, *Europhys. Lett.* **30** (1995) 421.
- [16] M. Golosovsky *et al.*, *Supercond. Sci. Technol.* **9** (1996) 1.



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