Applying Permanent Magnets

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Permanent magnets have been known from antiquity. In China, natural magnets carved from lodestone were used for geomancy; archaic bonded magnets made from powdered lodestone and cocks' blood are intriguingly related to a precursor of chess. The compass, clearly described in a Chinese text by Shen Kua dating from about 1088, was the first magnetic device to change the world. It launched the magnetics industry.

Understanding progressed from Gilbert's qualitative description of the dipolar field in *de Magnete*, published in 1600, to the quantitative formulation of electromagnetism by Maxwell and the description of hysteresis by Ewing in the 19th century. Yet up to the beginning of the 20th century there was practically no significant improvement in permanent magnet materials, only the discovery that a steel bar or needle could be bent into the form of a horseshoe to concentrate the magnetic flux somewhat.

The problem that has been successfully overcome in recent times is the metastability of the state of any fully-magnetized block of ferromagnetic material more than a few microns in size. Modern "hard" magnetic materials have a coercivity which exceeds their magnetization and can be produced in blocks of any desired shape and size. So industrial applications of permanent magnets have burgeoned in the latter part of 20th century.

Several Categories of Applications

The unique feature of permanent magnets is their indefinite ability to deliver magnetic flux into a region of space known as the airgap with no continuous expenditure of energy. The applications may be classified according to the nature of the flux and nature of the working point of the magnet [1].

A *steady homogeneous field* may be used to generate torque Γ or align existing magnetic moments $\boldsymbol{m}$ since $\Gamma = \boldsymbol{m} \wedge \boldsymbol{B}$ where $\boldsymbol{B}$ is the magnetic induction (in Tesla). Charged particles moving through the field with velocity v and charge q are deflected by the Lorentz force $F = qv \wedge \boldsymbol{B}$, which causes them to move in a circle in free space with the cyclotron frequency (28 GHz/T for an electron). When the electrons are confined to a conductor of length L aligned perpendicular to the field where they constitute a current I the Lorentz force leads to the familiar expression $F = BIL$. Conversely, moving a conductor through the field produces an induced emf given by Faraday's law $E = -d\Phi/dt$ where Φ is the flux threading the circuit of which the conductor forms a part.

Spatially *inhomogeneous fields* offer another series of potentially useful effects. They exert a force on a magnetic moment given by the energy gradient $\boldsymbol{F} = -\nabla(\boldsymbol{m}.\boldsymbol{B})$. They also exert nonuniform forces on charged particles, which can be used to focus ion or electron beams or to generate electromagnetic radiation as electron beams pass through the inhomogeneous field. The ability of rare-earth magnets to generate complex flux patterns with rapid spatial variation ($\nabla B > 100$ T/m) is unsurpassed by any electromagnetic device. This point can be appreciated by considering the Amperian surface current equivalent to a long magnet with $\mu_0 M \approx 1$ T, which is about 800 kA/m. Solenoids, whether resistive or superconducting, would need to be several centimetres in diameter to accommodate the requisite ampere-turns. Permanent magnet structures can be produced by assembling blocks of rare-earth or ferrite magnets in any desired orientation. The field of one magnet does not significantly perturb the magnetization of its neighbours because the longitudinal susceptibility is zero for a square hysteresis loop, and the transverse susceptibility $\mu_0 M_s / B_a$ is only of order 0.1, since the anisotropy field H_a is much greater than the magnetization (see table in the insert). Hence, for example, the directions of magnetization of two blocks of $SmCo_5$ in contact, with their easy directions perpendicular, will deviate by less

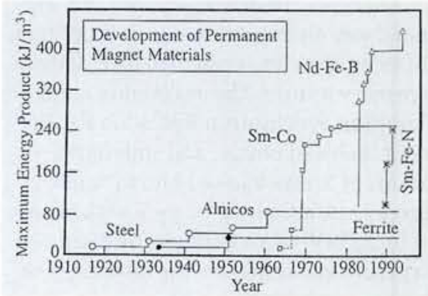

Fig. 1. Progress in the maximum energy product $(BH)_{max}$ of fully dense permanent magnet materials during the 20th century. The units are in kJ/m³.

Basics of Pemanent Magnetism

A fully-magnetized block of ferromagnetic material more than a few microns in size is in a metastable state. It is subject to an internal demagnetizing field $H_d = -NM$ oppositely directed to the magnetization M, which will destroy the magnetization unless the $M(H)$ loop exhibits sufficient coercivity. A steel magnet, for example, may have a coercivity H_c of only 10 kA/m and a magnetization of 1 MA/m. Hence the demagnetizing factor N, which depends on shape, must be less than 10^{-2} corresponding to a bar with a ratio of length to thickness of 15 (N is zero for a long needle and one for a flat plate). Modern magnets by contrast may be any shape since $H_c > M_s$, the saturation magnetization.

Applications of permanent magnets generally involve delivering flux from the magnet into an airgap. It may be shown that $\int \boldsymbol{B}.\boldsymbol{H} dV = 0$ over all space where B is the magnetic induction, so $-\int_1 \boldsymbol{B}.\boldsymbol{H} dV = \int_2 \mu_0 H^2 dV$ for a vacuum permeability μ_0. The integral 1 is over the volume V of the magnet and integral 2 is over the surrounding space. Integral 2 is identified as twice the energy stored in the field created in space surrounding the magnet, and it is maximized at a point in the second quadrant of the $B(H)$ loop of the magnet where the product $-BH$ is maximized. The maximum energy product $(BH)_{max}$ is an extrinsic property of the magnet, governed by microstructure. In the absence of an external field, the H-field in the magnet is the demagnetizing field H_d. An ideal permanent magnet has a square $M(H)$ loop and since $B = \mu_0(H + M)$, the energy product $\mu_0(-NM + M)(-NM)$ is maximized when $N = 1/2$. It then takes the value $(1/4)\mu_0 M^2$. Oriented ferrite and rare-earth magnets have near-ideal square loop shapes, so today's archetype for a permanent magnet is a cylinder with a ratio of diameter to height of about 3, so that $N \approx 1/2$. The energy product is a convenient figure of merit for a permanent magnet as it represents twice the maximum energy stored in the magnetic field created in space around a magnet of optimum shape.

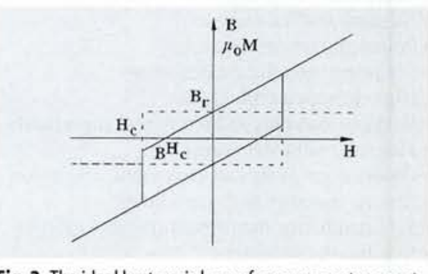

Fig. 2. The ideal hysteresis loop of a permanent magnet.

than a degree from the easy directions. A consequence of the rigidity of the magnetization is that the superposition of the induction of rare-earth permanent magnets is linear and the magnetic material is effectively transparent, behaving like vacuum with permeability μ_0. Transparency and rigidity of the magnetization greatly simplify the design of magnetic circuits.

Time-varying fields can be produced by displacing or rotating the magnets. They may induce an emf and exert forces on the resulting eddy currents in a conductor. Otherwise they may be used to switch on or measure continuously other effects of a field (*e.g.*, inducing moments in a magnetization measurement).

Certain reputed benefits of magnetic fields, such as use of magnets in acupuncture, suppression of wax formation in oil-wells or control of limescale formation from hard water are difficult to classify because the underlying mechanisms have yet to be identified. It is likely they will turn out to be related to one of the effects mentioned above.

Viewed from the standpoint of the permanent magnet, the applications can be classified as *static* or *dynamic* according to whether the working point of the magnet in the second quadrant of the hysteresis loop is fixed or moving. Its position depends on the magnitude of the *H*-field to which the magnet is subjected, so it is determined in turn by the shape of the magnet, the air gap and the fields generated by electric currents flowing nearby.

The working point is usually close to $(BH)_{max}$. It will change whenever magnets move relative to each other, when the air gap changes or if there are time-varying currents. In the former there is mechanical recoil, as the working point moves along the loop whereas in the latter the recoil is active (Fig. 3). Owing to their square loops, oriented ferrite and rare-earth magnets are particularly well-suited for dynamic applications that involve changing flux density in the magnet. Ferrites and bonded metallic magnets also minimize eddy current losses. For mechanical recoil, the air gap changes during operation from a narrow one with reluctance R_1 to a wider one with reluctance R_2, where $R = Hl_m/B$ for a magnet length l_m.

Active recoil occurs in motors and other devices where the magnets are subject to an *H*-field during operation as a result of currents in the copper windings. The field is greatest at startup, or in the stalled condition. Active recoil is represented by a displacement of the reluctance line along the *H*-axis (Fig. 3c). Provided $\mu_0 H_c$ exceeds B_r, it is possible to drive the working point momentarily into the third quadrant of the *B:H* loop without demagnetizing the magnet. The intrinsic coercivity may be as important a figure of merit as $(BH)_{max}$ in this type of application.

Permanent Magnet Materials

Up to about 1950, progress in permanent magnet materials first involved producing steels with defects which created coercivity by pinning the domain walls, and then developing the Alnicos – alloys with a two-phase microstructure where elongated Fe-Co regions are embedded in a nonmagnetic Al-Ni matrix. Coercivity is due to the shape anisotropy of the tiny Fe-Co needles. Modern magnetic materials, developed since 1950, all have a uniaxial crystal structure. They owe their coercivity to strong magnetocrystalline anisotropy, coupled with an appropriate microstructure. In hard ferrites, the anisotropy is mainly due to Fe^{3+} in five-fold coordinated trigonal sites, whereas in the rare-earth permanent magnets, Sm-Co, Nd-Fe-B, and Sm-Fe-N it is mostly due to the Sm^{3+} or Nd^{3+} ion cores. The anisotropy is represented by an anisotropy field H_a.

All the magnets may be isotropic or anisotropic, depending on whether the *c*-axes of the individual crystallites are random or aligned along a common direction. There is a considerable advantage in using aligned magnets because the net remanence of an ensemble of randomly-oriented crystallites is only $M_s/2$ so the energy product is reduced by at least a factor four compared with fully-aligned material. The magnets are also available in sintered or bonded form. Sintered magnets have almost full density and even with some misalignment of the crystallites and the presence of secondary phases which may be crucial to develop coercivity, the remanence of the magnet may approach the intrinsic spontaneous magnetization of the hard phase. Bonded magnets are a dispersion of hard magnetic powder in a polymer matrix. They may be produced in net shape by compression or injection moulding. The disadvantage of reduced magnetization due to 20-30 vol.% of polymer is offset by the ease of fabrication.

The magnets in electrical machines can be subject to temperatures in excess of 100°C.

The principal intrinsic magnetic properties (Curie temperature, saturation polarization, anisotropy field, domain wall width) of the main types of permanent magnetic materials.

Compound	T_c (°C)	$\mu_0 M_s$ (T)	$\mu_0 H_a$ (T)	δ_W (nm)
$BaFe_{12}O_{19}$	450	0.48	1.3	15.4
$SmCo_5$	720	1.05	40.0	3.7
Sm_2Co_{17}	827	1.30	6.5	8.6
$Nd_2Fe_{14}B$	312	1.61	7.6	4.2
$Sm_2Fe_{17}N_3$	476	1.54	14.0	3.6

Remanence, coercivity, temperature coefficients and maximum operating temperatures of sintered magnets.

Magnet	$\mu_0 M_r$ (T)	H_c (kA/m)	$\frac{d(\ln M_r)}{dT}$ (%/K)	$\frac{d(\ln H_c)}{dT}$ (%/K)	T_{max} (°C)
$BaFe_{12}O_{19}$	0.38	240	-0.20	-0.45	250
Alnico 5	1.28	51	-0.02	-0.03	500
$SmCo_5$	0.90	1200	-0.04	-0.20	250
Sm_2Co_{17}	1.10	520	-0.03	-0.20	350
$Nd_2Fe_{14}B$	1.35	1000	-0.13	-0.60	150

Magnetization and coercivity naturally decline as the Curie point is approached, and the temperature coefficients of these quantities around ambient temperature are listed in the table for magnets made of different materials. Not all the loss is necessarily recoverable on returning to ambient temperature. The maximum temperatures at which the materials can safely be used are also indicated in the table. The limit for $Nd_2Fe_{14}B$ is lower than for any of the others because of the relatively low Curie point and the temperature coefficients are more severe. These are difficulties to overcome by astute design.

More than half the sales volume and the overwhelming bulk of the magnets used today are hard ferrites $BaFe_{12}O_{19}$ or $SrFe_{12}O_{19}$. Alnicos are most suitable for high-temperature applications. The SmCo magnets have been superseded to a considerable extent by Nd-Fe-B. Sm-Fe-N is a comparative newcomer which is only just beginning to be used on an industrial scale.

Prospects

A further doubling of the energy product of permanent magnets to 800 kJ/m³ might be possible, but redoubling to 1600 kJ/m³ is out of the question in view of the intrinsic magnetic properties of materials that are or might be magnetically-

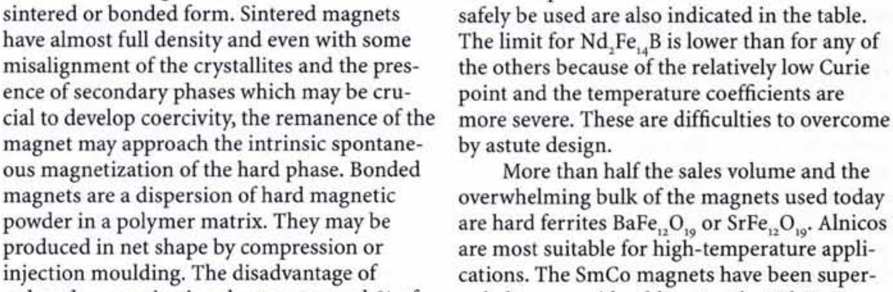

Fig. 3. Hysteresis loops showing the working point for a) a static application, b) a dynamic application with mechanical recoil; and c) a dynamic application with active recoil.

ordered at room temperature. The energy product will follow an "S"-shaped curve extending into the 21st century. Much of the future progress in hard magnetic materials is likely to be in the direction of more versatile and cost-effective grades, with better thermal characteristics or ease of fabrication or magnetization, for example. Bonded magnets will continue to increase their share of the market.

Fig. 4 vividly illustrates the influence of modern permanent magnet material on reducing the size and complexity of devices. However, the utility of modern permanent magnets has been slow to be appreciated in some areas. There are good prospects for innovative applications. One prediction is that the electromagnet is likely to be superseded for many purposes by permanent magnet variable flux sources (see below) which have the advantages of compactness and independence of power supplies or cooling requirements.

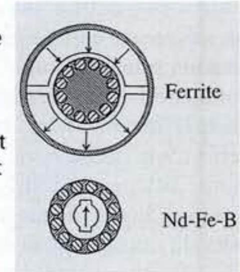

Fig. 4. Sections normal to the axes of permanent magnet motors showing the influence of permanent magnet properties on the design. Not only can the device be made smaller with high energy-product magnets, but the number of parts can be reduced. There is a trend towards moving-magnet designs coupled with miniaturization (*e.g.*, placing the magnets of a brushless DC motor on the rotor and moulding the magnets together with the shaft and gear). These have the virtues that moving magnets have low inertia and the stationary windings can be thermally heat sunk. The advantage of the magnet can be appreciated by comparing a small disk-shaped magnet with a coil having the same moment. A disk with a diameter of 8 mm and height 2 mm made of a material with $M = 1$ MA/m has $m \approx 0.1$ Am^2. The equivalent current loop, $m = IA$ would require 2000 ampere-turns.

New products such as cordless electric tools or personal stereos owe their existence to advanced permanent magnets. One may expect that other new consumer products will appear which exploit their benefits. The fact that such a large proportion of the rare-earth permanent magnet market depends on one application, voice-coil actuators for hard-disc drives, suggests not so much that it is vulnerable to changes in the shape of personal computers and home electronics as that it is capable of great expansion as a few more mass applications emerge. The electric automobile is one of the products which could transform the scale of industrial applications of permanent magnets.

References

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Variable Fields for Flux Sources

Consider first the creation of a *uniform field* within a certain region of space using permanent magnets. This problem can be approached using long cylindrical magnets with a hollow bore [1, 2]. By choosing the orientation of each segment appropriately, the fields can all add at the centre. In the transverse field design shown in Fig. 5a, the outer surface will be an equipotential provided the ratio of the radial and transverse distances $t/r = \sqrt{2} - 1$. Multiples of this field can be obtained by nesting similar structures one inside the other. Shimming [3] compensates for imperfections so these open cylinders are used for non-invasive magnetic resonance imaging in medicine where permanent magnet flux sources supply fields of order 0.3 T for whole-body scanners.

Fig. 5b shows a different design where the direction of magnetization of any segment is at 2θ from the vertical axis. The magnetic field, in polar coordinates (r, θ) due to an extended *line* dipole of moment λ is: $H_r = \lambda\cos\theta/4\pi r^2$; $H_\theta = \lambda\sin\theta/4\pi r^2$. So the magnitude of H, the quantity $(H_r^2 + H_\theta^2)^{1/2}$, is actually independent of θ ; its direction makes an angle 2θ with the orientation of the magnet. These equations indicate that all segments now contribute to create a uniform field across the airgap.

Unlike the structure of Fig 5a, the radii r_1 and r_2 can take any values without creating a stray field outside the cylinder. The device is known as a Halbach cylinder. It can be shown that the flux density in the airgap is $B = B_r \ln \varrho$ where $\varrho = r_2/r_1$ is the ratio of the outer to inner radii. In practice it is convenient to assemble the device from n trapezoidal segments (Fig. 5c). In that case, a factor $\{\sin(2\pi/n)\}/(2\pi/n)$ is included on the equation for B. The cylinders are never infinitely long; the length is typically comparable to the diameter, so the field is reduced by another factor $f(z)$,where z is the distance from the centre.

The cylindrical configurations for uniform fields may be modified to produce a variety of *inhomogeneous* fields which are particularly useful for beam control. In particular, Halbach's original cylindrical design has been used to produce a quadrupole field for focussing beams of charged particles. Higher multipole fields than dipole are obtained by having the orientation of the magnets in the ring vary as $\{(n/2) + 1\}\theta$, where $n = 2$ for a dipole field, $n = 4$ for a quadrupole and so on. The field at the centre of the quadrupole is zero, but whenever the particle beam deviates it experiences an increasing field which causes its trajectory to curve back to the centre.

To create a *uniform variable* field, two Halbach cylinders of the type shown in Fig. 5d with the same radius ratio $\varrho = r_2/r_1$ can be nested inside each other. By rotating them through an angle $\pm\alpha$ about their common axis, a variable field $2\cos\alpha(B_r \ln\varrho)$ is generated in the solid magnet. Another solution is to rotate the rods in the so-called magic mangle device (Fig. 5e). By gearing a mangle with an even number of rods so that the alternate rods rotate clockwise and anticlockwise though an angle α, the field varies as $B_{max}\cos\alpha$. Further simplification is possible with a magnetic mirror, a sheet of soft iron containing the symmetry axis which produces an inverted image of the magnets, and halves the number required.

These permanent magnet variable flux sources are compact and particularly convenient to use since they can be driven by stepping or servomotors and they have none of the high power and cooling requirements of a comparable electromagnet. For example a 7 kg Nd-Fe-B magnet of the design shown in Fig 6a can generate ±1.2 T in a 25 mm bore. Large alternating fields can be generated by rotating the magnets continuously.

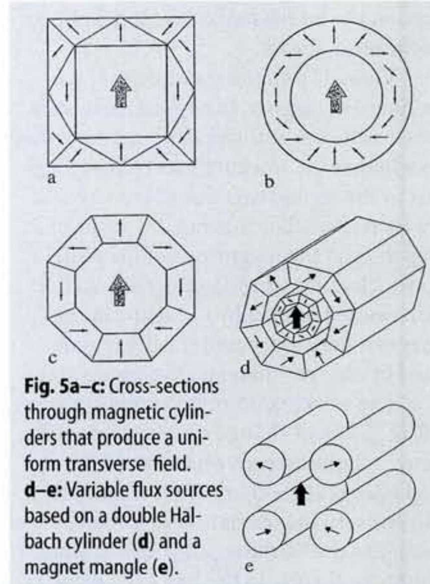

Fig. 5a–c: Cross-sections through magnetic cylinders that produce a uniform transverse field. **d–e:** Variable flux sources based on a double Halbach cylinder (**d**) and a magnet mangle (**e**).

The limit to the fields that can be conveniently generated using permanent magnets is about 2 T. This is set in part by the coercivity of the material. The vertical segments in Fig. 5d are subject to a reverse H-field equal to the field in the bore. But there is also a practical size limitation. Admitting a material existed with $B_r = 1.5$ T and $\mu_0 H_c = 5$ T, the diameter required to achieve 5 T in a 25 mm bore is 700 mm. Such a structure 400 mm high would weight about a tonne. Permanent magnet variable flux sources can be expected to displace resistive electromagnets for fields of up to about 2 T, but they cannot compete with superconducting solenoids at higher fields.