Critical Phenomena in Hydrodynamics

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Combining hydrodynamic pattern-forming instabilities with thermodynamic critical points to study fundamental problems in nonequilibrium physics is leading to many novel and unexpected results.

When a system is displaced from equilibrium by some sort of external forcing, it may display a patterned state above a certain forcing threshold. While their microscopic descriptions often differ substantially, such systems can nevertheless exhibit similar periodic or nearly periodic macroscopic spatial patterns when driven out of equilibrium. These patterns are generally associated with nonlinear effects [1].

The richness of structures in a system which extends in space over many wavelengths of the pattern (*i.e.*, a spatially extended system), is strikingly similar in systems of physical, hydrodynamic, chemical, or biological origin [1]. Typical examples of simple structures are rolls or stripes, hexagons, spirals, and target or focus patterns. When no special care is taken to prepare a particular state in a spatially extended system, patterns are more often than not complex assemblies (*i.e.*, textures) of locally simple structures such as rolls, hexagons, targets, spirals, and defects.

The dynamics by which a pattern transforms from one morphology to another, the mechanisms by which it adapts its average spatial wavelength, and

the dynamics of pattern defects are remarkably similar whatever the systems' origin. Owing to their inherently visual character, patterns therefore offer an easy and attractive way to study nonlinear nonequilibrium physics, not only qualitatively but also quantitatively.

By examining pattern formation one aims to understand:

- the universal mechanisms which govern the nonlinear spatio-temporal dynamics in various systems;
- the relation of these mechanisms to the underlying space-time symmetries (*e.g.*, the translational, rotational, inversion, and Galilean symmetries);
- the various transitions from spatially ordered (perfectly periodic in space and/or time) to disordered (*i.e.*, irregular, chaotic or turbulent) states and their possible universality.

Extensive studies of nonequilibrium pattern-forming systems during the past few decades have revealed many qualitatively new dynamic phenomena. Their relevance becomes particularly evident on considering the efforts to include them into novel theories in other fields such as superconductors and superfluids [2, 3].

Rayleigh-Bénard Convection

Although each experimental system has unique possibilities, Rayleigh-Bénard convection comprising thermal convection of a thin, horizontal fluid layer heated from below offers exceptional experimental, theoretical and numerical advantages. Well-controlled experiments with well-defined boundary conditions are easy to perform, and all relevant materials properties of both the working fluids and boundaries are readily available. Since the underlying equations are theoretically well understood, and extensive and detailed nonlinear stability analyses of an initially straight roll pattern have been performed, mainly by Busse *et al.* [4], Rayleigh-

Bénard convection is the best documented pattern-forming system (see insert).

Over the years Rayleigh-Bénard convection (RBC) has proved to be an excellent model system in which to study pattern formation. It has, nevertheless, two serious drawbacks, which are in fact common to most other systems. First, one is constrained experimentally to moderate aspect ratio cells (see inset) for ordinary working fluids. Second, the fluid properties are limited to a restricted range. Of significance here is the Prandtl number P , the dimensionless ratio of the thermal and viscous time scales, which is a measure of the relative importance of momentum *versus* energy dissipation mechanisms. Since the Prandtl number is fairly constant for a given fluid over normal ranges of temperature and pressure, different working fluids are usually required in order to modify substantially the Prandtl number.

It was realized rather early on that RBC in a gas under high pressure allows a significant reduction (by a factor of about 2 to 3) in the cell thickness without violating the Boussinesq approximation [5]. This approximation assumes that changes in the fluid properties owing to temperature variations can be neglected, except for the density variation in the driving buoyancy term [4]. Under these conditions, the fluid properties are constant and the convection problem is symmetric in the verti-

RBC: The Basics

In Rayleigh-Bénard convection (RBC), a thin horizontal fluid layer of thickness d is heated from below. The dimensionless control parameters governing the problem are the Rayleigh number R and the Prandtl number P . The Rayleigh number $R \sim \Delta T d^3$ describes the external forcing, while P represents the physical properties of the fluid. There exists a critical value for the stress across the layer, $R_c = 1708$ (and a corresponding critical temperature difference ΔT_c), at which the buoyancy force exceeds viscous and thermal dissipation. The resulting fluid motion, induced by the instability, is characterized by patterns with a typical size of the order of d and corresponding wavenumber $k \sim d^{-1}$.

Pattern dynamics and the detailed bifurcation sequence (*i.e.*, the transition from one stable state to another as the control parameter is varied) depend, however, on another two dimensionless quantities, namely the non-Boussinesq parameter Q and the aspect ratio Γ of the convection cell. Q determines the degree of deviation from the up-down symmetry of the RBC problem; Γ is the ratio of the typical horizontal and vertical dimensions of the cell. Here, the horizontal cell size is taken as the typical horizontal scale, although sometimes a size based on average inter-defect distances is considered. In the latter case Γ is dynamic, depending on both R and P . When Γ is sufficiently large, the spatial degrees of freedom define the pattern dynamics and the system is said to be spatially extended.

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cal direction with respect to the horizontal mid-plane of the cell. Thus, larger aspect ratios Γ can be achieved for a fixed cell diameter. Today's state-of-the-art experiments use gases to attain $\Gamma \approx 100$. However, $P \approx 1$ for gas convection.

RBC near the Critical Point

The disadvantages of an experimental limit to the aspect ratio and a restricted range for the Prandtl number can easily be overcome by using a fluid near its liquid-vapour critical point (see insert). The strong, and characteristic, temperature dependence of many fluid properties near the critical density ρ_c (e.g., thermal conductivity, isobaric specific heat, isobaric thermal expansion, etc.) permits one to scan P continuously from unity to practically infinity. Moreover, ΔT_c , the critical temperature difference for the onset of convection, tends asymptotically to the adiabatic temperature difference ΔT_{ad} when the thermodynamic critical point at $T = T_c$ is approached and the density equals the critical density ρ_c [6].

Convection starts when the forcing exceeds both the viscous and thermal dissipation mechanisms as well as the stable stratification caused by compressibility effects. The latter can be expressed in terms of the adiabatic temperature gradient $\Delta T_{ad}/d$, where d is the thickness of the fluid layer, and can be neglected when the size of a convection pattern is less than 0.1 m. So the adiabatic temperature difference essentially results from a balance between a destabilizing buoyancy effect and a stabilizing compressibility effect; the adiabatic gradient is roughly constant and on the order of 10^{-2} K/m.

Hence, extremely thin cells become feasible for RBC and, in consequence, exceptionally large aspect ratios. The variability of the Prandtl number permits pattern dynamics to be studied as a continuous function of P for any given value of the Rayleigh number R [7]. Moreover, using a single fluid in large aspect ratio cells one is able to study pattern formation in a substantial fraction of (R, P) space. Typical values of the experimental parameters for RBC are presented in the insert.

We have extensively exploited the advantages offered by a gas near (ρ_c, T_c) by conducting RBC experiments in pressurized SF_6 near the gas-liquid critical point. Several fundamental problems can be tackled successfully, notably:

a) Pattern formation and selection in extremely large systems over a very wide range of parameter space, and as a func-

tion of the Prandtl number in particular. b) The effect of the strong coupling between thermal fluctuations (see insert) and the order parameter responsible for a transition from one stable state to another when the control parameter is varied (i.e., so-called bifurcation to the patterned state). The order parameter is a physical quantity which has an average value of zero for a state of high symmetry (i.e., conduction) and changes to an average non-zero value at the transition to the state of lower symmetry (i.e., convection). In RBC, the order parameter corresponds to the amplitude of the fluid velocity. c) Behaviour of a strongly fluctuating hydrodynamic system when the hydrodynamic size scale is on the order of the thermal fluctuation size scale.

It is noted that RBC near the critical point is the sole experimental system in which (b) and (c) can be studied.

The last issue (c) is related to the drastic increase of the thermodynamic correlation length ξ (i.e., the characteristic length scale for thermal fluctuations). Close to the thermodynamic critical point, ξ can reach macroscopic values (see insert). If the thermodynamic correlation length becomes comparable with a typical hydrodynamic length scale of the system (e.g., the average pattern wavelength), one is dealing with a hydrodynamic system in which the concept of average physical quantities, introduced to obtain a hydrodynamic description in the first place, is no longer justified. The extremely interesting and fundamental question then

RBC Near a Critical Point

For a convection cell of height $d = 3$ mm filled with water, the onset of convection occurs at a temperature difference of $\Delta T_c = 4.5$ K. Typically, a large cell will have a diameter D of about 100 mm, leading to a radial aspect ratio of $\Gamma \approx 17$. To double the aspect ratio, two alternatives are possible. The thickness can be reduced by a factor two, but this leads to an eightfold increase in the temperature difference for the onset of convection (since $\Delta T_c \sim d^{-3}$). In this case, variations of fluid properties with height caused by their temperature dependence become a drawback (i.e., non-Boussinesq effects characterized by Q are no longer negligible). On the other hand, increasing the cell diameter by a factor two leads to excessively large time scales. The relevant time scale here is the horizontal diffusion time: $\tau_h = D^2/\kappa$ (κ is the thermal diffusivity). For a diameter $D = 100$ μm filled with water, the horizontal diffusion time $\tau_h = 19$ hours, while for a cell with $D = 3$ mm and radial aspect ratio $\Gamma = 100$, τ_h already exceeds four weeks! A practical alternative is gas convection at elevated pressures, in which aspect ratios somewhat less than 100 are routinely achieved.

These drawbacks are circumvented by taking

Fig. 1. A typical spatio-temporal chaotic RBC state at $R = 1.9R_c$ and $P = 6.6$ in a large aspect ratio convection cell (radial aspect ratio $\Gamma = 80$; thickness $d = 380$ μm). The convecting fluid is SF_6 at $T - T_c = 4.992$ K ($T_c = 318.73$ K) and density $\rho = 1.015\rho_c = 748$ kg/m^3 . The temperature difference across the layer is only 17.4 mK.

Thermodynamic Critical Phenomena

The liquid-gas critical point T_c, ρ_c, ρ_c (critical temperature, pressure and density, respectively) is a particular location in thermodynamic phase space at which most thermodynamic and kinetic fluid properties exhibit universal anomalies. These singular contributions can be described close to the point in the form of power laws of $\tau = (T - T_c)/T_c$ at $\rho = \rho_c$, with known fluid independent universal exponents.

The characteristic distance (i.e., the correlation length $\xi = \xi_0 \tau^{-\nu}$) over which local density fluctuations are felt, can, sufficiently close to the critical point, even reach macroscopic sizes (ξ_0 is of the order of the intermolecular distance and $\nu \approx 0.63$). For example, at $\tau = 10^{-6}$, $\xi \approx 1$ μm , which can be comparable with the thickness d of the fluid layer. It is precisely this appearance of a new length scale which is responsible for the universal, fluid-independent critical behaviour.

advantage of the power-law behaviour of fluid properties near the thermodynamic critical point. For example, for a thin fluid layer of SF_6 of thickness $d = 100$ μm at its critical density ($\rho_c = 0.74$ kg/m^3), the convection onset $\Delta T_c = 1.05$ K when $T_{av} - T_c = 9.6$ K, where T_{av} is the average cell temperature. The increase in ΔT_c caused by a reducing the cell height can be compensated for by approaching the critical point, since $\Delta T_c \sim (T_{av} - T_c)^{1.88}$.

Thus, a cell of $d = 50$ μm will have $\Delta T_c \approx 1.05$ when $T_{av} - T_c$ is reduced to 3.2 K. With $D = 30$ mm and $d = 50$ μm , the radial aspect ratio is already huge since $\Gamma = 300$. Nevertheless, τ_h remains within a reasonable bound of about 1 day (and with $\Gamma = 100$, $\tau_h < 3$ hours).

The up-down symmetry breaking, which comes about through the temperature dependence of the fluid properties, can be avoided even though the fluid properties are strongly temperature dependent near the critical point. This is mainly because ΔT_c tends to $\Delta T_{ad} = 0$ more rapidly than Q increases when the critical point is approached. Moreover, compared to gas convection where $P \approx 1$, the Prandtl number near T_c can be tuned from about 1 to several hundreds by simply varying the average temperature.

becomes: what kind of (hydrodynamic) framework is appropriate for describing such a situation?

Although hydrodynamic phenomena near a critical point have been discussed in detail [6], pattern formation in this regime has received only limited attention. The strong variability of the fluid properties near T_0 was first exploited at low temperatures in convecting helium gas to vary P up to 4 with $\Gamma = 57$ [8]. The first experiments to visualise pattern formation were performed by S. Fauve *et al.* They concerned the interaction between surface waves generated by a parametric instability (using an external vertical vibration as a forcing) of a liquid-vapour interface (the so-called Faraday instability) and a liquid-vapour phase transition near the thermodynamic critical point [9]. A new type of boundary layer instability close to the thermodynamic critical point has also been observed very recently by Buschhorn *et al.* [10].

Experimental Procedure

In the experiments, pressurized sulphur hexafluoride was enclosed in a stainless steel vessel with an horizontal sapphire window. The bottom of the cell comprised a nickel-coated copper disk with a thin-film heater glued underneath; a thin, annular plastic sandwiched between the disk and the window defined the side walls of the convection cell. The entire pressure vessel was enclosed in a thermostated bath held at ± 1.0 mK. The temperature difference across the convecting fluid in the cell was regulated by adjusting the heater current, while the temperature of the cell's upper surface was kept constant through the thermal coupling with the thermostated bath. A specially designed high-spatial resolution shadowgraph, with or without digital image enhancement, was used to observe the flow patterns from above. This technique is based on the different temperatures of the up- and down-flows. Since the refractive index is a function of temperature, the accompanying horizontal (periodic) modulation of the refractive index gives rise to an array of focusing and defocusing lenses which image an incoming plane wave to bright and dark regions in the image plane to allow visualization of the convection patterns. So images of the up- and down-flow structures appear as dark and bright regions, respectively.

Our experimental studies have demonstrated the advantages of using a gas near the gas-liquid critical point for inves-

Fig. 2. The transient evolution – clockwise, from top left – from a hexagonal state at $R \approx 1.5 R_0$ to a globally labyrinthine state at a slightly higher value of R . The labyrinthine state persists for at least one horizontal diffusion time. The experiment is performed at $P \sim O(100)$ very close to the critical point ($\tau \sim O(10^{-5})$) in a convection cell with $d = 19 \mu\text{m}$ and $\Gamma \approx 200$.

Fig. 3. A typical breakup of a short enclosed roll that is initiated by wavenumber adaption of the background pattern and leads to the formation of a target. Time progresses from left to right and top to bottom in units of the vertical diffusion time given by $\tau_v = d^2/\nu$ where ν is the kinematic viscosity

tigating Rayleigh-Bénard convection, especially at extremely large aspect ratios. For example, we are presently working with cells of $19 \mu\text{m}$ thickness which yield an aspect ratio close to 1000. On the other hand, the Prandtl number can be scanned over an extremely wide range, as can the parameter describing the non-Boussinesq behaviour [1, 4]. These features have already lead to the observation of numerous unexpected phenomena including:

- novel textures containing many spirals and targets (Fig. 1) [11];
- new mechanisms and scenarios for the evolution to disorder (Fig. 2) [12];
- spiral-target transitions [11];
- dislocation-core instabilities [7];
- a breakup – reconnection mechanism responsible for the creation of the extended spirals and targets (Fig. 3) [7, 11];
- the coexistence of up- and down-flow hexagons (Fig. 4) [12, 13].

Results, and the Remaining Puzzles

One of the most intriguing recent discoveries in *natural* pattern formation (*i.e.*, without externally imposed constraints on the degrees of freedom) is the recent observation of disordered spiral and target

patterns [7, 11, 14] in large-aspect ratio RBC where only rolls were known to be stable [4]. These novel states have since been accurately reproduced by numerical simulation of a generalized model [15] and by integrating the full thermally driven Navier-Stokes equations in the Boussinesq approximation [16]. Although, it has since

Fig. 4. Typical coexisting up- and down-flow hexagons together with rolls. Note that the hexagon wavelength is about 20 % larger than the roll wavelength. The experimental parameters are: $P = 4.5$, $R = 4.5 R_0$, $\Gamma = 80$, and $d = 380 \mu\text{m}$.

been established that these extended patterns are intrinsic to RBC, there is still little understanding of their dynamic behaviour and the reasons why they develop.

Our experiments have raised a number of fundamental questions concerning:

- the condition for the appearance of textures with many targets and/or spirals;
- the mechanism of the spiral-to-target and target-to-spiral transition;
- the different classes of spatio-temporal behaviour and spatio-temporal disordered states that exist in nonequilibrium systems;

- the region of stability of the newly discovered states;
- the role of long-range forces (created in RBC by horizontal pressure gradients) in pattern dynamics at high P [17];
- the possibility of observing the nonlinear interaction of thermal fluctuations with the hydrodynamic order parameter leading to a weak first-order jump in the order parameter, an effect which was predicted long ago [18].

Three new types of patterns recently observed in RBC will be discussed because they demonstrate the importance of large-scale mean flow and transitions to novel extended spatio-temporal chaotic states at Prandtl numbers greater than unity.

Target-spiral chaotic states

A major step forward in understanding the dynamics of complex patterns in RBC was the appreciation of the crucial role played by the large-scale mean-drift flow [17], a non-local phenomenon. This mean flow is, in a self-consistent fashion, driven by local roll curvature which in turn advects and distorts the pattern itself. This effect is particularly relevant for fluids with small Prandtl numbers [5, 17]. While the relative importance of mean-flow effects diminishes with increasing P , this decrease is compensated for by a non-local action over increasingly larger distances [19]. Consequently, pattern dynamics, stability and selection can be altered considerably by mean flow, even at $P > 1$, in sufficiently large aspect ratio cells, a fact that has been overlooked [1].

Thus the main ingredient which distinguishes various recent experiments from previous ones, and which leads to spiral- and target-defect chaos (the term refers to the irregular behaviour of the textures containing many spirals and targets – see Fig. 1) is the large aspect ratio. Comparison of experimental observations with numerical simulations at $P \approx 1$ shows that the large-scale flow is responsible for

spiral-defect chaos, and that deviations from up-down symmetry (non-Boussinesq effects) are irrelevant [15, 16].

The only reported theoretical attempt to clarify the occurrence and characteristics of extended patterns introduced the notion of invasive defects [20]. According to this theory, wavenumber frustration is the dominant concept for understanding the dynamics.

The invasive nature of spiral and target defects is based on the difference in the wavenumber k between that selected by a focal defect and some (average) background wavenumber. The latter wavenumber is suggested to be the one for which a dislocation defect has zero climbing velocity (i.e., velocity along the rolls). Cross and Tu [20] mentioned an unusual core instability involving axisymmetric targets transforming to non-axisymmetric perturbations which may induce a transition from a target morphology to a spiral – an observation that can be related to the experimentally observed transition from spirals to targets as a function of P [11]. It is indeed quite remarkable that simulations reproduce so well the experimentally observed patterns as a function of the coupling to the mean flow [16, 20].

Since an established theoretical framework is lacking, the obvious experimental first-step is to map out the stability region of the novel extended patterns in (R, P, k) space. An extensive study of the detailed mechanism(s) responsible for the creation of these patterns would follow (an example can be seen in Fig. 3). Currently, we are verifying whether or not the background wavenumber suggested in Cross and Tu's theory is related to the wavenumber that is relevant for dislocation defect dynamics.

Up- and down-flow hexagons

Another unexpected observation occurs at the upper stability limit of the extended patterns. Still in the region in which straight rolls were considered to be stable, hexagonal patterns appear *via* a core instability of spirals and targets, even for negligible values of Q . Although, the appearance of hexagons is usually related to a deviation from the Boussinesq approximation, here they definitely occur for an up-down symmetric fluid layer. Their most striking feature, however, is the fact that both up- and down-flow hexagons coexist simultaneously. Moreover, the hexagons introduce a new wavelength which differs substantially from the roll wavelength (see Fig. 4).

Dewel *et al.* [21] recently suggested a model in which a $k = 0$ mode (suggested to be of thermodynamic origin) couples to a basic bifurcation to rolls and produces the observed patterns. We suggest instead that the zero mode might be of hydrodynamic origin, i.e., due to mean flow caused by curved rolls. The appearance of hexagons is in this case related to a self-induced symmetry breaking caused by the coupling of the large-scale flow with the underlying instability. The observed ratio of roll-to-hexagon wavenumber, which agrees remarkably well with numerical simulations based on the model, strongly supports this suggestion. Further support is based on our numerical simulations [22]. As a result, we have observed both the spiral-core instability and coexisting hexagons [22]. A recent numerical stability analysis of hexagonal patterns shows that far from the onset for convection, both types of hexagons are observable and stable at $P > 1$ with a wavelength larger than that of the coexisting rolls [23].

The observation of these hexagons seems to indicate that large-scale flow is important not only for the transition to spatio-temporal complexity but also for pattern and wavenumber selection.

A new route to disorder

Another interesting observation in very thin ($\approx 20 \mu\text{m}$) RBC cells is a new type of globally isotropic disordered roll state, which may be described as labyrinthine [12]. Fig. 2 shows the evolution from a hexagonal pattern to the labyrinthine texture. This scenario differs from the hexagon-to-roll transition which is usually observed [1]. Here, owing to the significant non-Boussinesq behaviour, the hexagons appearing near the onset of convection persist up to large values of the control parameter R where the mean-flow becomes important. The flow probably modifies drastically the pattern selected, leading to the observed labyrinths.

Labyrinth patterns reveal a remarkable structural similarity to states found in various equilibrium systems with competing interactions (e.g., amphiphilic monolayers, Type-I superconductors in the intermediate state, ferromagnetic garnet films, thin layers of ferrofluids in Hele-Shaw geometries, etc.), and in chemical reaction-diffusion systems [24].

The appearance of labyrinthine patterns requires the presence of non-local interactions [25]. In the case of hydrodynamic patterns, these can be associated with mean-drift flows. So the observation

of labyrinths at large values of P is another indication of the importance of large-scale flow, contrary to its generally attributed insignificance for high- P fluids.

One of the mechanisms for the creation of labyrinths in magnetic garnet films is based on a transverse instability of ordered magnetic stripes. This instability eventually leads to the formation of a globally isotropic and disordered state via an unbinding of disclination dipoles [26]. We have not yet investigated the generation of labyrinthine patterns in RBC, but a similar scenario can be expected. Indeed, the transverse instability observed in magnetic garnet film stripes and the well-known zig-zag instability in RBC [1] are strikingly similar. In fact, the zig-zag instability in RBC is a mechanism by which a locally straight roll pattern of a wavelength larger than its optimal value adapts its wavelength by alternate lateral bending, hence the name zig-zag. Globally, however, rolls will remain straight with zig and zag excursions.

The conventional nonlinear analysis of the zig-zag instability neglects the coupling to the mean drift (i.e., only short-range interactions are considered). As a result coarsening towards larger and larger zig and zag (canted) domains at long time scales is predicted [27]. This phenomenon is similar to what happens in spinodal decomposition, and is governed by the same equation. On the other hand, including non-local interactions, such as those caused by large-scale flows, changes completely the coarsening process at long time scales [28]. From the similarity with equilibrium systems, it follows that the zig-zag instability may exhibit different dynamics at long time scales, depending on the range of the interaction between the rolls: coarsening towards zig and zag (canted) domains on the one hand, and towards ramified or labyrinthine structures on the other. We are investigating this issue both experimentally and numerically.

Interesting Perspectives

Rayleigh-Bénard convection experiments near the critical point offer major advantages in studies of pattern formation, notably the ability to work with extremely large extended systems. This has opened up major opportunities and led to several breakthroughs in understanding important problems, especially pattern selection.

The main step forward in our understanding pattern and wavelength selection is the appreciation of the crucial – and

previously neglected – role played by large-scale flow, even for high Prandtl number fluids, and particularly for transition to the novel extended spatio-temporal chaotic states. The observation of novel types of extended patterns comprising targets and spirals, coexisting up- and down-flow hexagons, and labyrinthine patterns – each of which is ubiquitous in Nature – support this claim.

It is interesting to note that the magnitude of the mean fluid flow is determined not only by the widely accepted parameter $(R - R_0)/P^2 R_0$, where R_0 is the Rayleigh number at the which buoyancy exceeds viscous and thermal dissipation, but also by the cell aspect ratio Γ . This was pointed out a long time ago, but was only recently appreciated [19]. According to a suggestion by Cross and Meiron [2, 29], the result indicates that the coarsening dynamics of a pattern following a sudden jump from below to above R_0 can be used to probe the effect of large-scale mean flow.

A linear coupling between thermally induced fluctuations and a hydrodynamic bifurcation has been investigated recently [30]. Quantitative measurements of fluctuation-induced convection rolls of random orientation below the convection onset were reported. These fluctuating patterns had been considered theoretically some time ago [18], and were expected to be unobservably weak owing to the small value of the thermal energy, relative to a typical kinetic energy for convective flow. High-resolution shadowgraph techniques have in fact allowed measurements of both the scaling and the prefactor of the amplitude of the fluctuating modes as a function of departure from the onset of convection; remarkable agreement between theory and experiment was established [30].

The coupling in the vicinity of the critical point is expected to become significantly nonlinear and may lead to a weak first-order jump in the transition, even in the Boussinesq case when there is no deviation from up-down symmetry. Preliminary experimental results indicate that this is indeed the case.

The thermodynamic correlation length becomes comparable with the cell thickness in the vicinity of a critical point, and the fluctuations of the velocity field start to be of the same order of magnitude as the mean velocity. A hydrodynamic description of Rayleigh-Bénard convection might fail altogether in this regime. So the study of Rayleigh-Bénard convection near the critical point offers an unparalleled

opportunity for investigating the dynamics of a strongly fluctuating hydrodynamic flow.

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