FRACTIONAL HALL EFFECT

Evidence Mounts For a New Picture

Rudolf Morf of the Paul Scherrer Institute, Zurich, describes how the quest for understanding the physics of half-filled Landau levels has led to a powerful, new theory for the fractional quantum Hall effect.

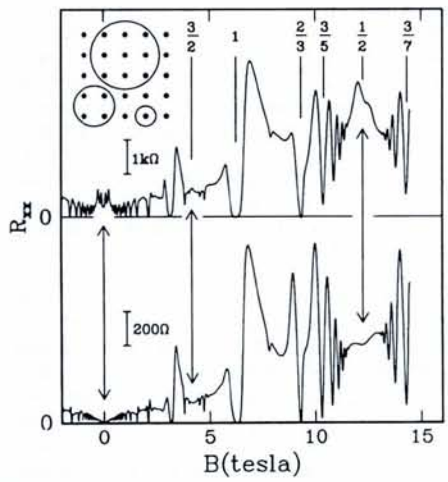
Since its discovery in 1982 by Tsui, Störmer and Gossard [1], studies of the fractional quantum Hall effect (FQHE) have continued to lead to surprising new discoveries concerning the physics of the two-dimensional electron system. As samples (GaAs/AlGaAs heterostructures) became purer and electron mobilities increased (*via* the clever use of modulation doping), plateaux in the Hall resistance were observed at filling fractions $\nu = 2/5, 3/5, 3/7, 4/7, 4/9$, *etc.* with larger and larger denominators, mainly along the "principal" sequence $\nu = p/(2p+1)$ and its particle-hole conjugate $\nu = 1-p/(2p+1)$ where p is a positive integer.

A hierarchical picture was developed [2] to describe the FQHE in 2-d electron systems. It is based on Laughlin's idea that the plateau at $\nu = 1/3$ is a consequence of the condensation of the electron system into an incompressible fluid with 1/3-charged excitations. The $\nu = p/(2p+1)$ state is thus depicted as the result of the condensation of $1/(p-1)$ charged defects of the (incompressible) parent state at $\nu = (p-1)/(2p-1)$, in an iterative way. This prompted an intriguing question: what happens at the accumulation point of this sequence, *i.e.*, at $\nu = 1/2$, which requires infinite iteration of the hierarchy and cannot thus be analyzed? Other questions such as how the gap scales with the order p of the hierarchy, and whether the hierarchy terminates at some finite p, have remained unsolved within this picture. Finally, many predicted states were not seen in experiments.

Composite Fermions Proposed

An alternative picture was proposed by Jain [3] who pointed out that the principal sequence of FQH states could be viewed as resulting from integer Hall quantization of composite fermions, *i.e.*, electrons to which an even number of flux units are attached. Explicit microscopic trial wave functions implementing this idea were shown to have very low energy. They can be regarded as natural generalizations of the famous Laughlin wave function used to describe FQHE. A real breakthrough in the understanding of the 2-d electron system near and at half-filling was made in recent theoretical work by Halperin, Lee and Read (HLR) [4]. Motivated by Jain's construction of trial states, they made use of a singular gauge transformation, which transforms the electrons into a system of particles interacting with a Chern-Simons gauge field and which attaches a flux tube containing an even number q of flux units to each electron, such that the transformed system again obeys Fermi statistics. The motivation for using the transformation is that for suitable q, the transformed system can be treated reliably in a Hartree approximation, and that methods of diagrammatic perturbation theory can be employed to study corrections to the Hartree results.

In this picture, for $q = 2$, the fractional states belonging to the principal sequence $\nu = p/(2p+1)$ are converted into a system of fermions in an effective (external + Chern-Simons) magnetic field of reduced strength such that exactly p-Landau levels are filled by the fermions. As a result, the gap of the fractional states is interpreted as a cyclotron energy of the transformed particles.



Comparison of the magnetoresistance R_{xx} of a bulk two-dimensional gas (lower panel) and the R_{xx} of an antidot superlattice of 600 nm spacing fabricated in a high-quality, modulation-doped GaAs/AlGaAs heterostructure. Note the appearance of oscillations near $B = 0$ and filling factors $\nu = 3/2$ and $1/2$. The insert illustrates commensurate cyclotron orbits encircling 1, 4 and 9 antidots. From [7].

At a filling factor $\nu = 1/2$, where the magnetic field strength is such that two units of magnetic flux can be associated to each electron, HLR realized that a gauge transformation with $q = 2$ converts the electron system into a system of fermions where the average Chern-Simons gauge field just cancels the external magnetic field and should lead to physics resembling that of the electron system in zero external magnetic field, consistent with the absence of fractional Hall quantization at half-filling (in single-layer systems).

Indeed, if the magnetic field B is measured by its deviation ΔB from its value $B_{1/2} = 4\pi\hbar c n_e/e$ for a filling fraction $\nu = 1/2$, then along the principal sequence $\nu = p/(2p+1)$, $\Delta B = (B-B_{1/2}) = [2\pi\hbar c/e]n_e/p$ for a system of electrons at fixed density n_e. This equation is identical to the one describing the location of integer quantum Hall plateaux in the vicinity of $B = 0$, except that in the present case the deviation ΔB appears instead of the field B. Moreover, as the integer plateaux in the Hall resistance can be regarded as the extreme low-temperature form of the Shubnikov-de Haas effect, which is a consequence of the Fermi surface at $B = 0$, HLR attempted to interpret the fractional Hall quantization again as the consequence of the existence of a Fermi surface for the system at $\nu = 1/2$, *i.e.*, at $\Delta B = 0$. Indeed, if fluctuations in the gauge field are ignored, and if impurity scattering is absent, a well-defined Fermi surface is expected. In addition, HLR show that even in the presence of gauge fluctuations, important features of a Fermi surface will still exist.

Mounting Evidence

Evidence from recent experiments as well as finite-system calculations in favour of the Chern-Simons composite fermion picture is mounting. In particular:

– Using samples of extreme quality (which show exceptionally well-developed FQH states at $\nu = 2/7$, 3/11, 2/9 and 3/13), Du *et al.* [5] found energy gaps along the principal sequence $\nu = p/(2p+1)$ and the particle-hole conjugate up to $p = 5$ which scale as $1/(2p+1)$, in agreement with HLR.

– Anomalies in the propagation of surface acoustic waves (SAW) at $\nu = 1/2$ in the form of resonances between the sound wave and cyclotron orbits of the charge carriers (as for "normal" electrons in a metal) have been observed [6]. The structure and wavelength dependence of these anomalies is in excellent agreement with HLR predictions.

– Similar resonances have been seen [7] in a system comprising a superlattice of antidots. In addition to the known resonances near $B = 0$ [8], resonances of the same type are observed near $\nu = 1/2$ (see figure), but here as a function $B-B_{1/2}$ as expected for composite fermions.

– Finite-system calculations [9] demonstrate that the half-filled Landau level is indeed a Fermi liquid-like state. Quantum numbers of the ground state are correctly predicted by Hund's second rule applied to composite fermions at $B = 0$. The pair-correlation function shows the type of oscillations at large distances expected for a Fermi liquid.

[1] Tsui D.C., Störmer H.L. & Gossard A.C., *Phys. Rev. Lett.* **48** (1982) 1559.
[2] For a review, see: *The Quantum Hall Effect*, Eds.: R.E. Prange & S.M. Girvin (Springer; 1990).
[3] Jain J.K., *Phys. Rev. Lett.* **63** (1989) 199; *Phys. Rev. B* **40** (1989) 8079 & **41** (1990) 7653.
[4] Halperin B.I., Lee P.A. & Read N., *Phys. Rev. B* **47** (1993) 7312.
[5] Du R.R. *et al., Phys. Rev. Lett.* **70** (1993) 2944.
[6] Willett R.L. *et al., ibid.* **71** (1993) 3846.
[7] Kang W. *et al., ibid.* **71** (1993) 3850.
[8] Weiss D. *et al., ibid.* **66** (1991) 2790.
[9] Rezayi E. & Read N., *ibid.* **72** (1994) 900.

Council Targets Partnerships

UNESCO's Physics Action Council (PAC), set up by the Agency's Director-General, F. Mayor, following the recommendations of a Consultative Meeting last June, will advise UNESCO on the shape and implementation of programmes to promote the widest possible participation of physicists. Meeting for the first time on 14-15 April in Paris, it decided to form *ad hoc* working groups that aim to open up large physics facilities to international collaboration (Chair: H. Schopper, Geneva), to widen access to literature and electronic communications (Chair: I.A. Lerch, New York), and to improve physics education (Chair: M. Konuma, Tokyo). Partnerships among national and regional societies are seen as the way to coordinate actions, and the PAC hopes to have some $90 000 available to help scientists from less-favoured countries participate in activities.