The atomic nucleus provides an unique and fascinating object for studying complexity owing to its extended compact structure, the strength of quantum effects and the delicate balance between a Coulomb force of infinite range and a short range nuclear force.

**Atomic Nuclei: Laboratories for Studying Complexity**

Y. Abe\(^1\), B.G. Giraud\(^2\), M. Ploszajczak\(^3\), E. Suraud\(^1,4\)

---

\(^1\)Yukawa Institute for Theoretical Physics, Kyoto University, Japan  
\(^2\)Service de Physique Théorique, CE-Saclay  
\(^3\)Grand Accélérateur National d’Ions Lourds (GANIL), Caen  
\(^4\)Laboratoire de Physique Quantique, Université Paul Sabatier, Toulouse

**Fundamental research on complexity will impact upon areas as diverse as biology, weather prediction, and the regulation of stock markets.** One is now beginning to discover the fascinating richness of self-organization in apparently disordered systems and the immense wealth of moderately broken symmetries emerging from large collections of individually simple systems.

As a matter of fact, the richness of phenomena shown by nuclei when \(N\) runs from a few units to a few hundreds is staggering: 20 to 30 years ago, the beauty of the shell model [1] and the collective model [2] as efficient simplifications of this many-body problem exemplified what basic science can do to make the real world intelligible. More recently, it has been found that order parameters describing the shell structure are still useful when studying very excited states such as superdeformed nuclei [3] and new kinds of giant resonances [4].

**Complex Nuclear Systems**

In drawing attention to the complexity of nuclear dynamics we shall first briefly look back to the early days of nuclear physics. We shall see that a stochastic picture of nucleonic motion inside the nucleus is almost as old as nuclear physics itself. In spite of the symmetries of the mean field in which nucleons move, the stochastic picture basically remains valid and reflects the complex nature of nuclear dynamics. Indeed, experiments performed during the 1980’s at new heavy-ion facilities all over the world have shown that this complex picture is the rule rather than an exception.

We shall also present a contemporary example of a heavy-ion reaction exhibiting typical stochastic features. The questions raised demonstrate the liveliness of this field of research (Fig. 1), while the techniques used to address the problems are of general interest outside nuclear physics.

From an experimental point of view, physicists have learned to build detectors able to keep track of thousands of projectiles. Moreover, they have gained considerable experience in analyzing large bodies of data and extracting from them relevant physical information. On the other hand, the theoretical description of complex heavy-ion reactions requires the development and simulation of microscopic, dynamical models in strongly out-of-equilibrium situations. We shall try to describe some benchmarks for present and future tasks.

**Early Stochastic Models**

Niels Bohr writing in *Nature* in 1936 is probably responsible for introducing statistical concepts into the description of a nuclear problem, namely the interaction of a neutron with a target nucleus. A statistical picture of the numerous interactions of the incident neutron with the nucleons constituting the nucleus basically reflects the complexity of the process. In the case of moderately energetic neutrons, such as those available in the mid-1930’s, the entire interaction process is divided into a sequence of events which will be called interactions (Fig. 1).

---

**Fig. 1** — The INDRA detector at GANIL features a high geometrical efficiency (~90% coverage of the 4π solid angle) and low values of the energy threshold (~1 MeV per nucleon). The multi-detector system comprises 336 detector cells of various sizes to give an approximately uniform counting rate. Their arrangement provides a mass identification of light charged particles and charge identification of fragments (up to \(Z = 30\)). Experiments designed to investigate among other topics, the onset of multifragmentation as a function of the size of the nuclear system, started in March 1993 using \(A\), \(Xe\) and \(Gd\) beams bombarding various targets.

---

Y. Abe joined the research staff of Hokkaido University, Japan, on graduating in 1984. He has been a professor at the Yukawa Institute for Theoretical Physics since 1975.

B.G. Giraud graduated from the École Polytechnique, Paris, in 1961 and has worked at the Commissariat à l’Énergie Atomique, Saclay, since 1965.

M. Ploszajczak studied in Cracow and Bonn and joined the Institute of Nuclear Physics, Cracow, in 1981 before moving to GANIL, Caen, in 1990.

E. Suraud has been a professor at the University of Toulouse since 1992. He received his Ph.D. from the University of Paris in 1984 and then worked at Grenoble University and GANIL.
The Beauty of the Shell Model

But apparent regularity was also a feature of nucleonic motion. Indeed, the introduction of the shell model in the late 1940's by Mayer and Jensen shed a new light on the motion of nucleons inside a nucleus [1]. To a good approximation, nucleonic motion could be represented as nucleons following single-particle trajectories related to the levels of an unique, common, one-body potential. The shells (degeneracies with energy gaps between them) can be understood as corresponding to classical closed orbits in the potential, and are thereby related to its symmetry properties [6]. Incidentally, this concept is also fully applicable to another finite fermion system, namely microclusters, where it is further developed to account for "supershells" observing interference between two nearby orbits. All told, the shell model suggested an underlying regularity rather than complexity — a picture that was apparently difficult to reconcile with the earlier stochastic one.

The paradox was in fact short-lived since Wigner soon realized [7] that the structure of nuclear spectra was much richer than that given by the shell model alone in quantum mechanics. By studying the spacings of compound nucleus levels he showed that the distribution of spacings was special, representing a signature of a restricted class of stochastic Hamiltonians. As a side product, his remarks gave birth to today's most widely recognized definition of quantum chaos.

Spectra of compound nucleus levels provide the best experimental data for statistical studies of quantum chaos [8]. So once again nucleonic motion is associated with stochasticity, at least at high excitation energies, thus reflecting the complex nature of nuclei.

Modern Nuclear Complexity: The Physics of Violent Collisions

The beauty of the nuclear shell model enlightened nuclear physics in the 1960's and 1970's and allowed the development of original contributions to the physics of strongly interacting many-body systems. The Hartree-Fock mean-field theory permitted the description of many ground-state nuclear properties, with the help of realistic phenomenological effective interactions. To dynamical calculations in an exactly the so-called time-dependent Hartree-Fock (TDHF) theory, however, soon suffered from deficiencies in the description of nuclear collisions as bombarding energies increased in magnitude.

The mean-field picture using the approximated mean-field dynamics was basically unable to account for the dissipation responsible for the heating of nuclei, which experimentalists were starting to observe in fusion-like reactions of heavy ions at beam energies of a few tens of MeV per nucleon. Once again, statistical concepts were taken into account in the study of many-body problems such as the determination of the equation of state for nuclear matter. This obliged nuclear physicists to depart from a mean-field picture and to accommodate the observed dissipative patterns. The existence of so-called nuclear temperatures also implied the occurrence of corresponding thermal fluctuations, thus bringing the nuclear physics community back to basic questions concerning the statistical mechanics of complex systems. Moreover, it soon turned out that many of the properties of atomic nuclei, such as the nuclear equation of state and transport properties, could in fact only be studied in strongly out-of-equilibrium processes. These processes, in turn, exhibited unique features that called for further study.

The Dynamics of Multifragmentation

The dynamics of nuclear fragmentation in heavy-ion collisions is a very good example of a situation where a discussion of both chaotic structures and complexity owing to multiparticle correlations may be especially relevant. In heavy-ion collisions at beam velocities of the order of the average velocity of nucleons in a nucleus at rest, the system turns out that many of the properties of atomic nuclei, such as the nuclear equation of state and transport properties, could in fact only be studied in strongly out-of-equilibrium processes. These processes, in turn, exhibited unique features that called for further study.

From Statistical to Non-statistical Patterns

It is well known that the flow of a classical incompressible fluid, which for low viscosities is laminar, exhibits highly chaotic and irregular behaviour at high viscosities. In this viscous regime, flow is both unstable and unpredictable: a small perturbation at a given time may rapidly lead to a strong disordered state, a phenomenon known as turbulence. In contrast, the study of quantum systems has shown that, despite their inherently complex nature, they can exhibit a wide range of behaviors, from stable to chaotic. This is especially true in the context of nuclear physics, where the complex interplay of nuclear forces and quantum mechanics leads to a rich variety of phenomena. The study of nuclear chaos has thus become a central area of research, with implications not only for our understanding of the fundamental laws of physics but also for applications in areas such as astrophysics and condensed matter physics. The beauty of the shell model, enlightened nuclear physics in the 1960's and 1970's and allowed the development of original contributions to the physics of strongly interacting many-body systems. The Hartree-Fock mean-field theory permitted the description of many ground-state nuclear properties, with the help of realistic phenomenological effective interactions. To dynamical calculations in an exactly the so-called time-dependent Hartree-Fock (TDHF) theory, however, soon suffered from deficiencies in the description of nuclear collisions as bombarding energies increased in magnitude.

The mean-field picture using the approximated mean-field dynamics was basically unable to account for the dissipation responsible for the heating of nuclei, which experimentalists were starting to observe in fusion-like reactions of heavy ions at beam energies of a few tens of MeV per nucleon. Once again, statistical concepts were taken into account in the study of many-body problems such as the determination of the equation of state for nuclear matter. This obliged nuclear physicists to depart from a mean-field picture and to accommodate the observed dissipative patterns. The existence of so-called nuclear temperatures also implied the occurrence of corresponding thermal fluctuations, thus bringing the nuclear physics community back to basic questions concerning the statistical mechanics of complex systems. Moreover, it soon turned out that many of the properties of atomic nuclei, such as the nuclear equation of state and transport properties, could in fact only be studied in strongly out-of-equilibrium processes. These processes, in turn, exhibited unique features that called for further study.

The Dynamics of Multifragmentation

The dynamics of nuclear fragmentation in heavy-ion collisions is a very good example of a situation where a discussion of both chaotic structures and complexity owing to multiparticle correlations may be especially relevant. In heavy-ion collisions at beam velocities of the order of the average velocity of nucleons in a nucleus at rest, the system turns out that many of the properties of atomic nuclei, such as the nuclear equation of state and transport properties, could in fact only be studied in strongly out-of-equilibrium processes. These processes, in turn, exhibited unique features that called for further study.

Theoretical description of multifragmentation was, and remains, tortuous.

The first step is to depart from the concept of a mean-field picture by accommodating dissipative features. This is a very challenging task at the quantum level since a generally accepted solution to the problem has not yet been found. Fortunately, the de Broglie wavelengths of nucleons inside colliding heavy ions very soon become negligibly small on increasing the bombarding energy. This allows one to rely on semi-classical descriptions which are relatively simple as compared to those formulated in a quantum framework.

During the 1980's, a nuclear version of Boltzmann's equation was used with success in the context of the semi-classical model [10]. It turned out that although a complete description of heavy-ion collisions helped greatly in understanding the basic dynamical process. In particular, it allowed one to account for, with a reasonable degree of confidence, the formation of "hot" nuclei (coherent systems of collective excitations exceeding a few MeV resulting from incoherent excitations). The dynamics of multifragment formation, however, could not be fully explained within such a semi-classical framework, which by its very nature only predicts average quantities.

The next step in the development of these microscopic theories was to extend basic kinetic equations into a stochastic regime [13]. Stochastic expansions of kinetic equations, introduced in the late 1950's, had been studied in fact during the 1960's and 70's in situations close to equilibrium. They were now developed and applied in strongly out-of-equilibrium contexts, and in situations where self-consistent mean-field effects still play an important role. Such analyses are presently the subject of many investigations, both experimental and theoretical. Combined with elaborate statistical descriptions of the decay properties of the fragments produced during collisions, they may eventually provide a microscopic theory of multifragment emission.

The relevance of this research extends beyond understanding fragmentation formation. This is because these dynamical systems offer interesting clues to the general question of the correspondence between the quantum and classical regimes. As implied above, variations of the beam energy in a certain sense amount to tuning an effective Planck constant. On the other hand, the self-consistent nature of the nuclear mean field makes the basic equations of motion non-linear, which brings us back to chaotic features. This non-linearity seems to be unavoidable as it stems from a reduction of the many-body problem to one-body dynamics.

From Statistical to Non-statistical Patterns

It is well known that the flow of a classical incompressible fluid, which for low viscosities is laminar, exhibits highly chaotic and irregular behaviour at high viscosities. In this viscous regime, flow is both unstable and unpredictable: a small perturbation at a given time may rapidly lead to a strong disordered state, a phenomenon known as turbulence.
tortion of the flow pattern. An understanding of a large number of statistical properties of the fluid can be obtained by fragmenting at different scales the turbulent structures, called “eddies”, into smaller and smaller eddies. The breaking up of these fragmenting eddies provides the principle physical mechanism by which the energy of the flow is transmitted to motion at smaller scales down to almost the molecular scale, where dissipation halts further fragmentation.

In heavy-ion collisions, one expects that hot, expanding nuclear matter may enter a region where density perturbations of finite length become unstable [12]. Inside these regions of mechanical instability, called spindal instabilities, the system is thermodynamically unstable with respect to fluctuations having small wavelengths and it is doubtful that Boltzmann-like kinetic equations, which neglect high-order correlations, continue to be valid approximations. The picture emerging from these considerations resembles that of a critical phenomenon involving a random cascade of fragmentations with unstable blobs of nuclear matter at different scales. For some specific scaling fragmentation conditions of this non-equilibrium cascade process, the distribution of fragment sizes may reach a limit reminiscent of those distributions produced by percolation [13]. This leads at once to the following questions: is the nuclear fragmentation process related to a critical phenomenon? And if so, what is the nature of this finite-size phase transition, and under which kinematic conditions of the collision is the dynamics of the fragmenting matter governed by the criticality?

At the present time, experiments with an exclusive analysis of event by event, of the fragment-size distribution are restricted to those employing nuclear emulsion techniques and only a limited number of fully resolved events are at our disposal. However, questions related to non-equilibrium cascading will soon be studied in great detail using electronic techniques based on sophisticated multidector systems now operational at GANIL (Caen, France: Fig. 1), GSI (Darmstadt, Germany) and Michigan State University (East Lansing, USA).

Existing data strongly suggest that a critical phenomenon is indeed involved. The way to identify it is by analyzing observable quantities which behave in a qualitatively different manner when a critical phenomenon is present. The next step is to compare these analyses with models, such as non-equilibrium random cascading or percolation (see insert), where the nature of the “finite size phase transition” is well understood.

Critical phenomena are usually associated with symmetry breaking and large fluctuations. In the nuclear case, since the distribution of the cluster distribution is not directly measurable it has been proposed to look for a signal of critical behaviour in the cluster size distribution [13]. In Fig. 3 we show an example of such an analysis for the dimensionless quantity \( \gamma = \gamma_0 \) plotted versus the reduced multiplicity \( m_0 \) for:

(a, upper) experimental data on the fragmentation of gold nuclei with incident energies of approximately 1 GeV/nucleon following interactions in a nuclear emulsion;

(b, lower) three-dimensional random bond percolation on a cubic lattice, in which an infinite lattice exhibits a second-order phase transition when \( m_0 \approx 0.25 \).

The fragment-size distribution is calculated in each event by the factorial moments \( M_k = \frac{1}{k!} \sum_{i=1}^{n} \frac{n!}{i!} \), where \( k > 0 \) and the sum runs over all fragment charges (size) excluding the largest. The value \( \gamma_0 \) exceeds 2 indicates a power law distribution of the fragment sizes, as expected around a critical point. The distributions plotted in Fig. 3 show that large values of \( \gamma_0 \) are found around \( m_0 \approx 0.25 \) in both the experimental data and the three-dimensional percolation analysis.

Concluding Remarks

The theories of percolation and non-equilibrium random cascading for fragmentation, as well as the exclusive analysis of the multifragment events, have helped in developing the idea that randomness and disorder associated with heavy-ion collisions may result in characteristic features that are remarkable [14, 15]. These features are not merely the result of statistical fluctuations of otherwise continuous, smooth spectra of fragments, but represent a genuine physical effect related to the scaling features of the nuclear dynamics.

One is naturally led to study distributions, such as those given by the theory of fractals, which satisfy the observed anomalous scaling laws and reflect the scale invariance of an underlying geometry or chaotic dynamics. This leads to new types of kinetic theories and a new version of the statistical mechanics of small systems in strongly out-of-equilibrium situations, with the possibility of dynamic instabilities and spontaneous pattern formation.

Let us risk a speculation: nothing prevents a priori a local density fluctuation in a soup of nucleons from propagating and even self-replicating within the nucleons. The challenge is to understand the dynamic evolution of these fluctuations at the microscopic level. Further development of stochastic extensions of kinetic equations may offer a method to attack this fundamental problem.

There is probably a long way to go before all these problems can be correctly formulated and solved. But even at the present stage of this development, it may be helpful to discuss the atomic nucleus as an intrinsically dynamic object. Its extended compact structure, the strength of quantum effects and the delicate balance of a Coulomb force of infinite range and a short-range nuclear force make the atomic nucleus an unique and fascinating object for research on complexity. Theoretical and experimental work is now helping to explain nuclear collisions in regimes where classical limits and residual quantum mechanisms compete and collective and isolated degrees of freedom...
coexist, with many cases of interference, intermittency, stability, and instability.

A conceptual framework which could transfer the experience gained in nuclear physics to other fields is still missing. Interdisciplinary experts are not so numerous, and there remains much to be done until solutions to many-body problems in nuclear physics could be useful in biology, sociology or economics. Nonetheless, it can be safely claimed that atomic nuclei, and more specifically nuclear collisions, provide a framework of reference for the study of complexity that is unparalleled in the richness of its phenomenology. It would be a serious mistake to overlook this opportunity.


More Light on Random Number Generators

Considerable activity has been triggered by recent results of Monte Carlo calculations showing that random number generators considered good were giving the wrong answers in Ising model simulations using the highly efficient Wolff algorithm [see EN 24 (1983) 24].

Until recently, the only pseudorandom number generators which had been studied extensively were those of the multiplicative congruential type; they present a well-known defect whose effect can be made arbitrarily small at the expense of some computing time, and there is no wider theory which excludes their having additional as yet unknown defects. For this reason, many Monte Carlo practitioners prefer to use random numbers generated using other algorithms which produce much longer sequences before repeating and do not have known defects of congruential generators. Unfortunately, Ferrenberg et al. [Phys. Rev. Lett. 69 (1992) 3328] showed that for certain problems these newer generators gave wrong answers. Now Martin Lüscher of DESY has studied one of these generators, the "subtract-with-borrow" algorithm of Marsaglia and Zaman known as RCARRY, and was able to show how to improve its chaotic properties.

As Lüscher says: "The characteristic feature of a chaotic dynamical system is that trajectories starting at nearby states diverge exponentially with time. Even if the evolution is locally continuous, such a system appears to behave randomly on larger time scales. One could also say that any state specified to some finite precision has an exponentially deteriorating memory of its history. Although RCARRY is strongly chaotic in this sense, it has short-term correlations which can be eliminated by a simple modification. The RCARRY algorithm works on a table of 24 floating-point numbers and produces its next number by combining two of the numbers in the table and replacing one of them by the new result. Lüscher therefore looked at it as a generator of vectors in the 24-dimensional hypercube, and considered how two neighboring vectors in this hypercube would evolve according to the algorithm. He showed that indeed nearby trajectories would diverge exponentially, on the average with a Lyapunov exponent close to one, that is the separation would increase by about a factor e every cycle of 24 numbers. After about 16 cycles, the smallest possible separation (6 x 10-8) would grow to be of order one and fill the hypercube.

In order to accelerate this "chaotic explosion", he suggested a modification of the algorithm whereby one would generate 24 numbers and then skip a certain number (at most 365) before using 24 more. This "throwing away" of random numbers is normally to be avoided since it shortens the effective period of a generator, but Lüscher also showed that if the total number generated and skipped per cycle is prime, the period is maintained. (In fact, since the basic period of RCARRY is about 1070, one can afford to shorten the period by many orders of magnitude without any danger of exhausting it.)

F. James, CERN, Geneva

ERRATA

Some mistakes crept into last month's anniversary issue of Europhysics News. The Editor apologises for the following:

- The group shown on the cover is the Conference Committee and not the Publications Committee.
- E. Gotsman from Tel Aviv University shown in the photo on p. 120 is clearly not a "Turkish politician".
- J.M. Charp was the Chairman and not the Secretary of High Energy Physics in 1981-84 (p. 126).
- Seated to the right of W. Buckel in the photo on p. 150 is L.A. Radicati and not G. Szegeti.