Doped cuprates are characterised by strong electronic correlations which infuence substantially excitation spectra and transport properties. Low-energy excitations should describe physics in the low-temperature regime. Models are complex, so only partial and mainly qualitative explanations of experimental data have emerged.

Theoretical Models for High-T_c Superconducting Materials

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Since the discovery in 1986 by Bednorz and Müller of superconductivity with high critical temperatures T_c in the copper oxide-based ceramics, an enormous amount of work has sought a better theoretical understanding of the electronic properties of these materials. We shall discuss some aspects which are nowadays much better understood than they were a few years ago, and others which remain open problems.

Normal State Properties

A theory which will eventually be able to explain superconductivity and to calculate the transition temperatures for the high-T_c materials must be based on sufficient knowledge of the normal state, especially its low-energy excitations. For example, one must know if these excitations are still in an one-to-one correspondence with those of a weakly interacting electron gas, as this is prerequisite for applying Landau's Fermi liquid theory and his concept of quasiparticles. A few examples will show that the low-energy excitations of the normal state are not trivial.

A doped Mott insulator

The quasiparticle concept has been very successful in explaining the physical properties of ordinary metals. However, the high-T_c family of materials has a number of experimental properties which differ strongly from those of conventional metals. Most importantly, La_2CuO_4, the parent compound of the high-T_c material $La_{2-x}Sr_xCuO_4$ is an antiferromagnetic semiconductor and not a metal. With La^{3+} and O^{2-} one is left with a formal valency of Cu^{2+} implying one hole per unit cell in the Cu 3d shell. One thus expects a half-filled conduction band, and therefore metallic behaviour.

The fact that the material is a semiconductor below and above the magnetic ordering temperature of approximately 300 K is evidence of strong electron correlations [1]. The electrons are so constrained by their strong Coulomb repulsions that they cannot move freely as in a metal; in other words, the system is a Mott insulator. The situation changes when additional holes are introduced by doping, for example, by replacing La^{3+} by Sr^{2+}.

Anderson localization (localization due to disorder) is expected when a Mott insulator is doped with a small number of holes or electrons owing to the presence of random impurity potentials. However, the free motion of carriers is anticipated for significant degrees of doping owing to the screening of the impurity potentials. The system then becomes hole conducting, and even superconducting provided the doping concentration is not too high. Frequency dependent conductivity as well as other transport measurements such as the Hall conductivity suggest that only the holes added by doping contribute to transport. However, photoemission data indicate a large, electron-like Fermi surface similar to those predicted by standard band-structure calculations when electron interaction effects are not included. The situation is similar for $Nd_{2-x}Ce_xCu0_4$, except that the system is electron- and not hole-doped.

Unusual resistivity variations

The most important structural element of the high-T_c materials are planes formed by Cu-O (see Fig. 1). As a consequence, conductivity is highly anisotropic: while the resistivity ρ_{ab} parallel to these planes shows metallic behaviour for sufficient doping, the resistivity perpendicular to the planes ρ_c shows semiconducting behaviour. Moreover, ρ_{ab} exhibits an unusual dependence on the temperature T. For doping concentrations giving rise to the high T_c's one finds $\rho_{ab}(T) \sim T$ above T_c, with no appreciable residual resistivity on extrapolation to zero. For higher doping concentrations, *i.e.*, in the overdoped non-superconducting regime of $La_{2-x}Sr_xCuO_4$, an almost perfect $\rho \sim T^{3/2}$ power law has been observed over the entire temperature range up to 1000 K. Such a dependence is not at all expected from scattering of lattice vibrations.

Part of the difficulty which theory faces is due to the strong coupling of the carriers to the spin degrees of freedom, *i.e.*, to the Cu d^9 configurations. This coupling is evident from the rapid suppression of antiferromagnetic long-range order upon doping in, for example, $La_{2-x}Sr_xCuO_4$ at $x \approx 0.02$. Perturbative approaches are bound to fail and more complex non-perturbative methods must be chosen. However, such methods up to now have only been studied for sufficiently simple models.

Models for the Electronic Structure

Single-band Hubbard model

Many theorists assume that the low-energy excitations of doped Cu-O planes are properly described by a single-band Hubbard model. It is defined by one orbital per site on a square lattice with a hopping matrix element t between nearest neighbours $<ij>$ and an on-site Coulomb repulsion energy U. The Hamiltonian is written as

$$H = t \sum_{<ij>\sigma} (c^+_{i\sigma} c_{j\sigma} + hc) + U \sum_i n_{i\downarrow} n_{i\uparrow} \quad (1)$$

where the $c^+_{i\sigma}$ ($c_{i\sigma}$) create (destroy) an electron with spin σ at site i and $n_i = c^+_{i\sigma} c_{i\sigma}$ are the electron number operators.

For $U/t >> 1$, the model reduces to the Heisenberg model with an antiferromagnetic exchange interaction constant $J = 4t^2/U$ between neighbouring spins $\underline{S}_i$, $\underline{S}_j$ and a hopping term for the electrons, *i.e.*,

$$H = t \sum_{<ij>\sigma} (\hat{c}^+_{i\sigma} \hat{c}_{j\sigma} + hc) + J \sum_{<ij>} \underline{S}_i \underline{S}_j \quad (2)$$

This model is called the t-J model. Typical values are $J = 0.1$ eV and $t = 0.2$–0.4 eV.

The hats on the operators $\hat{c}^+_{i\sigma}$, $\hat{c}_{j\sigma}$ indicate that not more than one electron may occupy a given site. At half-filling (1 electron per

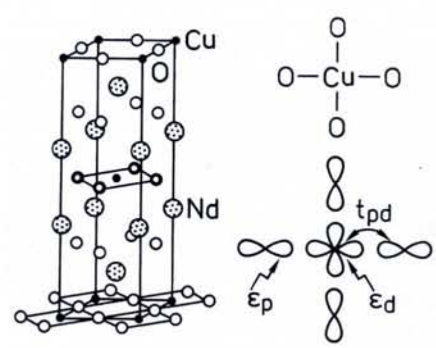

Fig. 1 — (a, left) Structure of Nd_2CuO_4, the parent compound of the electron-doped high-T_c superconductor $La_{2-x}Ce_xCuO_4$. (b, right) $3d_{x^2-y^2}$ and $2p_x$ orbitals which are used to model the Cu-O planes.

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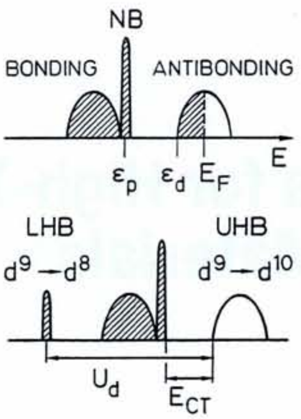

Fig. 2 — Sketch of the density of states for the Emery model at half-filling: (a, upper) neglecting the strong electron correlations; (b, lower) splitting of the anti-bonding band by correlations into upper (UHB) and lower (LHB) Hubbard bands.

site), the kinetic energy is zero, while doped holes can propagate.

The *t-J* model has survived critical examination by many groups. They based their studies on more complex models accounting for more details of the electronic structure on a larger energy scale, thus allowing comparisons with photo-emission data. The low-energy excitations of these more refined models corresponded with the excitations of the *t-J* model.

Three-band Hubbard model

Among the more complex models, the most widely studied is the three-band Hubbard or Emery model. There is evidence from both band-structure calculations and spectroscopic investigations that the generic properties of the Cu-O planes can be described solely in terms of the Cu $3d_{x^2-y^2}$ orbital and the O $2p_{x(y)}$ orbitals (see Fig. 1b). The hopping matrix element t_{pd} between the Cu and O orbitals, the on-site Coulomb repulsion U_d of two holes in a Cu $d_{x^2-y^2}$ orbital, and the difference $\varepsilon = \varepsilon_p - \varepsilon_d$ between the p and d orbital energies are the three most important parameters of the model. Because $\varepsilon \approx 3$ eV (in the hole picture), the holes in undoped La_2CuO_4 reside predominantly on the Cu sites. It is convenient to adopt the hole picture here since the relevant configurations of Cu have 1 or 0 holes in the d-shell as compared to 9 or 10 electrons. A large value of $U_d \approx 9$ eV as compared with $t_{pd} \approx 1.5$ eV means that $3d^8$ configurations, *i.e.*, $d_{x^2-y^2}$ orbitals with two holes, are strongly suppressed. This influences the magnetic properties of the ground state. For instance, holes primarily occupy O sites on doping the system with holes.

The Cu ions in undoped La_2CuO_4 fluctuate between d^9 and d^{10} with an average d count n_d of ≈ 9.3. The relatively large matrix element t_{pd} leads to a covalent splitting into bonding and antibonding bands forming the bottom and top of the p-d band complex. In a conventional band-structure picture, the Fermi level would be in the middle of the antibonding band as shown in Fig. 2a. Indeed, a standard band-structure calculation predicts La_2CuO_4 to be metallic. Because $U_d > \varepsilon > t_{pd}$, the system shows instead a charge-transfer gap E_{CT}, as indicated in Fig. 2b, where the antibonding band of predominantly $3d_{x^2-y^2}$ character splits into an upper and a lower Hubbard band with the latter being pushed below the oxygen level. Consequently, optical excitation involves the creation of a hole in a p band and of an electron in the d band ($3d^{10}$ configuration). If Fig. 2b were to represent the final answer, the model would predict that doped holes occupy the non-bonding oxygen band; it would not explain the observed strong coupling of the charge carriers with the Cu spins.

Singlet plus triplet states

This, however, is not the complete answer for the Emery model, as can be seen from the photo-emission and inverse photo-emission spectra (Fig. 3) obtained by numerical diagonalisation of the Hamiltonian for a $(CuO_2)_4$ cluster with periodic boundary conditions. Intrinsic to the model is a large exchange interaction between a charge carrier (hole) on an O site and a hole on a neighbouring Cu site (see Fig. 1b). Since $U_d > t_{pd}$, the situation resembles that of a H_2 molecule in the Heitler-London limit where the two electrons in H_2 form a singlet (S) state with an excited triplet (T) state. The same happens here with respect to two holes, whereby one hole occupies a Cu-site (d^9) and the other an orbital which involves four neighbouring O-ligands. Exchange interaction leads to the so-called Zhang-Rice singlet and triplet states. In the electron (as opposed to hole) picture the singlet band is pushed above the non-bonding states and forms the lower edge of the gap (Fig. 3a).

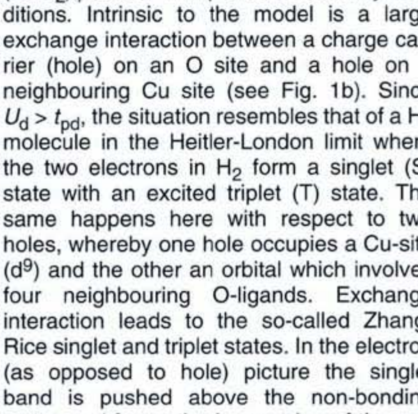

Fig. 3 — Reverse (a) and inverse (b) photo-emission spectra (PES) of the Emery model for the undoped initial state as obtained from exact diagonalisation of a small cluster [2] for the parameters $\varepsilon = 2$ and $U_d = 6$, with $t_{pd} = 1$ taken as the unit of energy. The singlet, triplet and upper Hubbard bands are indicated by S, T and UHB; NB is the non-bonding state. (c, d) represent the same calculations for 25% hole doping. Similar results can be rederived analytically by applying novel many-body techniques.

Closer inspection reveals that the singlet band separated from the upper Hubbard band by the charge-transfer gap shows essentially the same physics as a one-band Hubbard model with U of order E_{CT} and t given by the hopping matrix element for a singlet.

The most appealing experimental evidence supporting this picture comes from x-ray absorption spectroscopy. An important prediction of the Hubbard model is that the number of states in the upper Hubbard band should decrease as hole doping increases. This is precisely what the experiments indicate (Fig. 4). These results are in striking contrast to the behaviour of conventional semiconductors where the number of states in the conduction band is given by the number of atoms, and is independent of the number of holes in the valence band.

The Fermi Surface Problem

Consider La_2CuO_4 doped with a few holes. Leaving aside the possibility that the holes might be trapped by spatial inhomogeneities, we are dealing with the problem of hole motion in a quasi-two dimensional Heisenberg antiferromagnet. This topic has attracted many theorists. A moving hole generates a chain of overturned spins if the ground state is approximated by a Neél state with antiparallel spins on two sublattices. As the energy increases with the length of the path one might think that a hole cannot delocalize but instead has to remain near where it was created.

However, there exist processes which delocalise a hole. The most effective involves spin flips at neighbouring sites described by the spin-flip part $(J/2)\, S_i^+ S_j^-$ of the Hamiltonian (Eq. 2). By their action, disordered spins can pairwise re-order. The string of disordered spins dragged by a moving hole can therefore be reduced, resulting in a delocalization of the hole. The hole, together with the disordered spins in its immediate neighbourhood (the so-called spin bag), form a quasiparticle with an energy spectrum $E(\underline{k})$ having an effective bandwidth of order J which is a factor of t/U smaller than the one expected from the bare hopping matrix element t. Because $J << 8t$ we have here an example of a small energy scale generated by the strong correlations.

The spin-bag concept may be meaningful even in the paramagnetic phase because short-range antiferromagnetic order still prevails. The bag can be considered as a number of locally excited spin waves, implying that the excitation spectrum has a large incoherent part. An equivalent statement is that the Green's function

$$G(\underline{k}, \omega) = \frac{a(\underline{k})}{\omega - E(\underline{k}) + i0^+} + G_{inc}(\underline{k}, \omega) \quad (3)$$

has a quasiparticle pole with a small residue $a(\underline{k})$ and hence a large incoherent contribution G_{inc}. The dispersion $E(\underline{k})$ is determined by the energy scale J rather than by t.

As more holes are doped into the system, they fill the lowest energy states $E(\underline{k})$ of the hole band which are around the points $(\pm\pi/2, \pm\pi/2)$ of the Brillouin zone. A "small" hole Fermi surface is established with an enclosed volume given by the doping concentration. Antiferromagnetic long-range or-

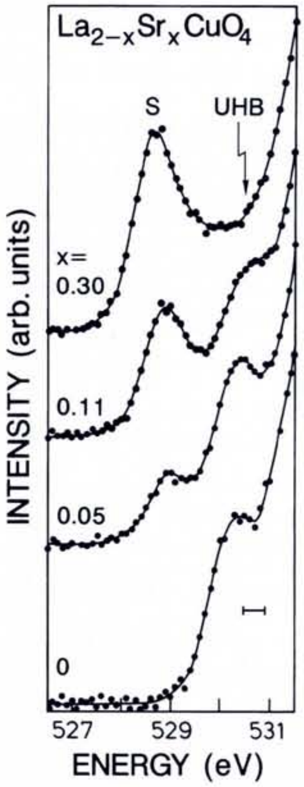

Fig. 4 — X-ray absorption spectrum of $La_{2-x}Sr_xCuO_4$ for various doping concentrations, redrawn after [3]. The peak at 530 eV is interpreted as the upper Hubbard band and decreases in intensity upon hole doping, in correspondence with Figs. 3b) and 3d). Possible energy shifts due to the O-1s core hole which is created in this experiment are expected to be small.

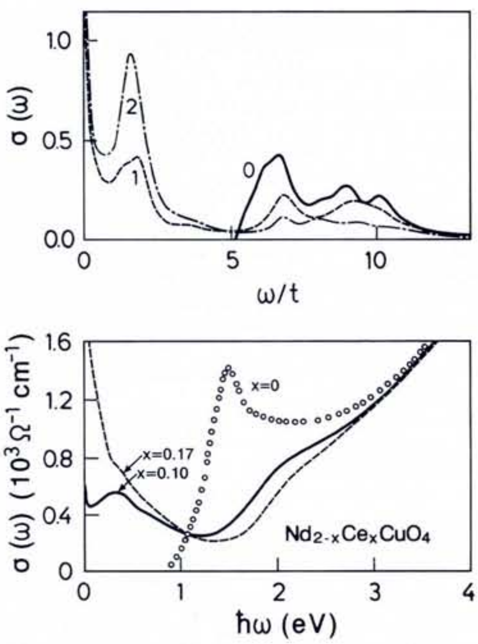

Fig. 5 — (a, upper) Doping dependence of optical conductivity of $Nd_{2-x}Ce_xCuO_4$ redrawn after [4]. (b, lower) $\sigma(\omega)$ calculated for the Hubbard model using a 3x3 site cluster. The absorption across the Hubbard gap $\sigma(\omega > 5)$ decreases rapidly upon doping with 10% and 20% holes. Intensity is shifted to lower energy and appears as broad absorption in the gap. In addition, the model shows Drude absorption at $\omega = 0$ which also grows in proportion to the number of holes. The remaining gap at low energy is probably due to finite-size effects. (From [5])

der is suppressed by the holes above a critical concentration δ_c of a few percent. A few calculations have estimated the spin-wave velocity as function of δ, which vanishes at δ_c. As the hole concentration increases further, a "large" Fermi surface is established with an enclosed volume given by the electron concentration per spin $n_\sigma = (1-\delta)/2$. The transition from the small to the large Fermi surface is presently not understood.

Response Functions

Drude peak and mid-infrared absorption

The considerations above strongly suggest that the high-T_c superconducting materials are close to a Mott-Hubbard type insulator-to-metal transition [1]. Much data on the electronic response to various external perturbations support this picture. One interesting example is the frequency-dependent conductivity $\sigma(\omega)$. When La_2CuO_4, for example, is doped with holes one finds for the real part of the conductivity a Drude-like peak and a broad continuum filling the gap (Fig. 5), the integral over which is proportional to the doping concentration (remember that a metal is characterised by a Drude absorption of the form $\sigma(\omega) = \sigma(0)/[1-i\omega\tau]$ with $\sigma(0) = ne^2\tau/m$ and we identify n with δ). The scattering rate $1/\tau$, however, behaves in a very unusual manner in that $1/\tau \sim \omega$ over a wide range of frequencies.

The Drude peak and the absorption in the gap (called mid-infrared absorption) can be explained by studying numerically the *t-J* model (so-called finite cluster calculations). While Eq. 3 shows that the Drude peak follows from the quasiparticle peak of the one-electron Green's function, the mid-infrared absorption stems from $G_{inc}(\omega)$: it is the result of a strong coupling of charge carriers to spin degrees of freedom.

A "marginal" Fermi liquid

An interesting phenomenological approach to the electronic response has been suggested under the name "marginal" Fermi liquid theory [6]. This approach is based on the observation that a number of the anomalous effects observed in the resistivity, Raman scattering intensity, nuclear relaxation rate, and other quantities can be qualitatively explained by simply assuming a high density of low-energy excitations, with the property that only the temperature appears as an energy scale. One immediate consequence is that the scattering rate, equivalent to the inverse lifetime of the electronic excitations, is given by

$$1/\tau \sim \max.\{|\omega|, T\} \qquad (4).$$

This is in contrast to ordinary Fermi liquid theory where $1/\tau \sim \max.\{\omega^2, T^2\}$.

No microscopic justification for Eq. (4) has been provided up to now. But the qualitative agreement with some of the experiments is startling. Note the agreement of Eq. 4 with the findings for $\sigma(\omega)$ discussed above. One consequence of Eq. 4 is that there are no well-defined quasiparticles near the Fermi energy owing to the short lifetimes. There is also no longer a jump in the momentum distribution $n(\underline{p})$ at $|\underline{p}| = p_F$, the Fermi momentum, a hallmark of Fermi liquid theory.

Magnetic response

Of particular interest is the magnetic response when the systems are perturbed by neutron scattering or time varying magnetic fields (nuclear magnetic resonance or NMR). Neutron-scattering experiments established early on that spin correlations incommensurate with the lattice structure persist at hole concentrations at which magnetic long-range order no longer exists. For example, in $La_{1.85}Sr_{0.15}CuO_4$ the antiferromagnetic Bragg peak at $q_0 = (\pi,\pi)$ in the undoped system splits into four quasi-elastic peaks at $\underline{q} = [\pi \pm \delta_q, \pi]$ and $[\pi, \pi \pm \delta_q]$ on moderate doping. The relative sharpness of these peaks in the neutron scattering cross-section (*i.e.*, in Im $\chi(\underline{q},\omega)$ where χ is the magnetic susceptibility) implies a magnetic coherence length $\xi_m \approx 4.5\ a_o$ at low temperatures (10 K), where a_o is the Cu-Cu distance in the plane.

There still exist well-defined magnetic excitations in these strongly doped superconducting systems. Just such a splitting of the peak at $[\pi,\pi]$ was expected from earlier quantum Monte Carlo calculations on small clusters for the Hubbard model [7]. Conventional band-structure computations of the magnetic susceptibility seem to give a splitting of the Bragg peak along the (π,π) direction instead. At higher temperatures, the magnetic coherence length decreases, reaching $\approx 2a_o$ at 100 K.

NMR provides another important experimental tool. A temperature-dependent coherence length of the form $\xi(T) \sim 1/(T + \theta)$ is required for $T \geq T_c$ to account for the Cu relaxation rates. Since this technique is a *local* probe it measures a momentum space average of Im $\chi(\underline{q},\omega)$ for $\omega \to 0$. A consensus concerning the results of these two techniques has not yet been reached, one reason being the lack of precise calculations of the magnetic susceptibility.

Superconductivity

The ultimate aim of all theoretical efforts with regard to the high-T_c materials is to explain why the superconducting transition temperatures are so high. Two burning questions are: which interactions lead to the formation of the superconducting state, and what is the form of that state? Owing to the complexity of the materials, progress towards answering both questions has been rather limited. There is unambiguous evidence, however, that the superconducting state is a pair state, *i.e.*, one in which electrons form pairs [8]. Furthermore, experiments show clearly that in the superconducting state there is a weak Josephson-type coupling between the Cu-O planes.

Although most theoretical studies focus on the properties of a single Cu-O plane, the coupling between planes is also important. A key parameter in superconductivity is the coherence length ξ which is found to be as small as $\xi_\parallel \approx 20$ Å within the Cu-O planes and $\xi_\perp \approx 5$ Å perpendicular to them. The length $\xi_\parallel$ is of the same order as the distance between the doped holes, in contrast to conventional superconductors where ξ is 2 to 4 orders of magnitude larger than the distance between charge carriers. The high-T_c materials therefore lie between two limiting cases. A large number of pairs overlap in the case of large ξ, as in the Bardeen-Cooper-Schrieffer (BCS) theory for superconductivity, and it is of limited use to speak of a single pair. The other limit is that of well-separated pairs (bipolarons) which form well above the superconducting transition temperature. Below T_c, these bosonic electron-pairs become superfluid [9].

Numerous experiments suggest that one is much closer to the BCS case than to a conventional Bose condensate. Experiments also show that unlike in ordinary metals, the ratio between T_c and the Fermi temperature T_F (related by Boltzmann's constant to the Fermi energy, *i.e.*, T_c/T_F) is no longer small. The ratio is therefore no longer a good expansion parameter, and corrections for strong coupling are expected to be large. But strong coupling effects cannot be directly calculated or easily estimated. This shows why it is unrealistic to expect a quantitative microscopic theory of T_c in the near future.

Table 1 — Experiments on high-T_c superconductors that indicate a strong coupling betwen phonons and those electrons which become superconducting.

– Raman (infrared active) phonons show frequency shifts at T_c or at the upper critical field which are much larger than in ordinary superconductors [13].
– Heat conduction, to which phonons contribute predominantly, shows a strong increase as drops below T_c. This results from an increase in the phonon mean-free path due to a freezing out of electronic excitations; it allows the electron-phonon coupling constant to be estimated.
– There are pronounced isotope shifts of T_c in many high-T_c superconducting compounds below and above optimal doping concentrations. Isotope shifts are a classical indicator for identifying lattice vibrations as source of attraction in ordinary superconductors, although one cannot rule out this mechanism from the absence of a shift. However, it appears to be a general trend that at optimal doping (corresponding to the highest T_c in a given class of materials) the isotope shift is practically zero. This suggests that a mechanism not involving phonons is responsible for the high T_c.
– Small changes in lattice structure lead in some systems to a complete disappearance of superconductivity.
– Features related to $\alpha^2F(\omega)$, the phonon density-of-states multiplied by the coupling constant squared, have been observed in tunnelling measurements. This is a clear signature of phonon-mediated contributions to electron attractions.

Symmetry of the order parameter

Still uncertain are the symmetries which are broken at the superconducting transition. Gauge symmetry is broken in the BCS ground state due to the establishment of off-diagonal long-range order at T_c. The order parameter consisting of the Cooper-pair amplitude changes its phase by 2α if a phase α is added to all one-electron states. Symmetries of the lattice may also be broken at T_c, as in p- or d-wave pairing, where the order parameter has a lower symmetry than, for example, the Fermi surface. There is growing experimental evidence that the order parameter vanishes along lines in momentum space. There have even been suggestions that parity and time-reversal symmetries are broken by a radically different mechanism of superconductivity, possibly only in two dimensions (anyon superconductivity). Experiments rule out this form of symmetry breaking so we shall not discuss this interesting superconducting state.

Strong electron correlations

The form of the order parameter is intimately connected with the type of interaction which leads to the formation of electron pairs. Leaving aside for the moment the electron-phonon interactions, one would like to know whether or not the *t-J* model, for example, can have a superconducting ground state.

The question of hole-hole interactions within the model has been studied by numerical diagonalisation as well as analytically by the method of string states [10]. The latter calculations accurately reproduce the former; they also specify when numerical results suffer from the use of clusters that are too small. It is generally found that two holes attract each other with a binding energy depending on the ratio J/t. One contribution to the attraction is the energy gain due to a reduced number of broken bonds $\underline{S}_i\underline{S}_j$ when the holes sit next to each other. In this case this number is 7, while it is 8 when the holes are well separated. Additional contributions arise from changes in the spin correlations in the vicinity of the pair. This energy gain is counter-balanced by a loss of kinetic energy which arises when the two holes are constrained to stay close to each other. It turns out that this type of charge-carrier attraction favours d-wave pairing, in agreement with recent calculations for small clusters which have found a d-wave pair state for a one-quarter full band in the range $2 < J/t < 3$ [11].

For larger values of J, the holes and spins phase separate. One reason for this is, of course, because Coulomb repulsion is neglected in these models. The repulsive Hubbard model, however, is less susceptible to superconductive pairing and phase separation. Although there exist indications for a superconductive instability at low temperatures, long-range superconducting pairing correlations have yet to be established by quantum Monte-Carlo calculations [7].

A study of the one-band Hubbard model with the help of the theory of spin fluctuations [12] leads to sizeable charge carrier attraction resulting in a pair state of B_1 (or possibly A_2) symmetry for a square lattice. Both have a lower symmetry than the square lattice. The results are based on a weak-coupling theory in which the strong correlations enter only through the self-consistency requirement.

Electron-phonon interaction

This leads us to a discussion of the rôle played by the electron-phonon interaction. It appears to be widely accepted that electron-phonon interactions cannot alone explain the high T_c temperatures [1]. Nevertheless, several experiments (some are listed in Table 1) give direct evidence for a strong coupling between phonons and those electrons which become superconducting.

Studies of the phonon frequency-shifts and of the heat conductivity based on the standard Migdal-Eliashberg theory of the interplay of phonons and superconducting electrons lead to an electron-phonon coupling constant $\lambda \approx 2\text{-}3$. This is in rough agreement with values of $\lambda \approx 1.0\text{-}1.5$ derived from conventional band-structure theory and from frozen phonon calculations in which the strong electronic correlations are left out. Values of 2-3 would be sufficient to explain the high T_c values. Using such large values of λ, it was shown that the linear dependence of ρ_{ab} on T can at least be explained in the 100-400 K temperature range; the $T^{3/2}$ behaviour observed in overdoped $La_{2-x}Sr_xCuO_4$ remains unexplained.

What is asked for is a unified description of both, namely of the strong electron-phonon interactions and strong electron correlations. Such a theory is presently not in sight.

Conclusions

It has become obvious that the doped cuprates are not simply conventional, anisotropic metals. They are instead characterised by strong electronic correlations which change substantially excitation spectra and transport properties. We have discussed models for low-energy excitations and presented arguments why they are expected to correctly describe the physics of these systems in the low-energy regime. Yet the models are still complex enough that so far only partial and mainly qualitative explanations of the various experimental results could be provided. This is why the fields of high-T_c superconducting materials and strongly correlated electrons remain a challenge for both experimentalists and theorists.

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