Magnetohydrodynamic Instabilities in Controlled Fusion Experiments

R. Gruber

Centre de Recherches en Physique des Plasmas
Association EURATOM — Confédération Suisse
Ecole Polytechnique Fédérale de Lausanne, Switzerland

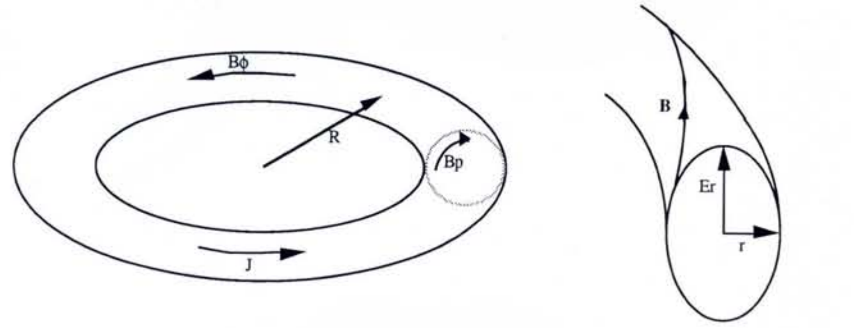

Fig. 1 — Toroidal plasma configurations. Left) Toroidal magnetic field B_θ and the poloidal magnetic field B_p owing to the presence of the toroidal current J. R is the mean large radius.
Right) Combination of two magnetic field components $\boldsymbol{B} = (B_p, B_\theta)$ produces twisted field lines. r is the small radius, A = R/r the aspect ratio and E the aspect ratio or elongation of the plasma cross section.

Several technical and physical problems must be understood and solved on the road towards a viable thermonuclear fusion reactor. Some of the more fundamental involve the many types of instabilities which can enhance energy outflow and even destroy the magnetic confinement of fusion plasmas. Knowledge about these instabilities derived from supercomputer calculations is a prerequisite for a good understanding of the results of existing experiments, and for designing new devices.

Much of the effort at the EPFL has been devoted to the study of magnetohydrodynamic instabilities in axisymmetric toroidal configurations such as tokamaks or reversed field pinches, where the toroidal magnetic field is produced by external magnetic field coils and the poloidal field originates from an induced toroidal current (Fig. 1). We have also investigated the behaviour of helically twisted toroidal devices such as stellarators where the toroidal and poloidal fields are generated by external coils. An example of a three-dimensional (3-d) configuration is the Wendelstein VII-X (W7X) stellarator (see cover illustration) which is to be built at the Max-Planck Institut für Plasmaphysik. One of the final goals of these ideal 3-d MHD stability computations is to find stable configurations in which the plasma energy in a relatively dense gas ($>10^{20}$ particles per m^2) at a temperature of 10^8 K can be magnetically confined for at least 1 second.

Ideal MHD Model

The most violent global instabilities, arising on timescales of microseconds, are those described by the linear, ideal, magnetohydrodynamic (MHD) equations for the linearized motion of a magnetically confined plasma around its equilibrium state. In this model, we neglect kinetic effects as well as non-ideal effects arising from a finite resistivity and viscosity. It is well-suited to the very high temperature (on the order of 10^8 K) plasma arising in model experiments relevant to reactor devices because, for example, including a finite resistivity only alters results for the ideal case over timescales on the order of milliseconds, which are three orders of magnitude longer than the timescale of linear modes.

Before taking into consideration any effect of non-ideality on stability, we have to make sure that the ideal MHD model gives a stable equilibrium configuration. However, although the assumed model is simple and non-dissipative, and the resulting eigenvalue problem symmetric, it is difficult to identify general characteristics of the eigenvalue spectrum.

As a first step we must compute the *static, ideal MHD equilibrium* solution by solving the nonlinear pressure balance equation

$$\nabla \boldsymbol{p} = \boldsymbol{J} \times \boldsymbol{B},\ \boldsymbol{J} = \nabla \times \boldsymbol{B},\ \nabla \cdot \boldsymbol{B} = 0 \qquad (1)$$

where $\boldsymbol{P}$ is the plasma pressure and $\boldsymbol{B} = (B_p, B_\phi)$ is the magnetic field. For our investigations we used the VMEC program [1] developed at the Oak Ridge National Laboratory in which curvilinear coordinates adapted to the geometrical form of the fusion device are chosen to minimize the computational effort.

The stability problem resulting from a perturbation of the equilibrium solution can be expressed in its variational form as

$$\delta (W_p + W_v - \omega^2 K) = 0 \qquad (2)$$

Here W_p, W_v and $\omega^2 K$ are the plasma potential energy, the energy of the vacuum magnetic field surrounding the plasma column and the plasma kinetic energy, respectively. These energies have to be expended in order to displace the plasma from its equilibrium position. The displacement $\xi = \xi(\boldsymbol{r},t) = \xi(\boldsymbol{r}) \exp(i\omega t)$ of the plasma column from its equilibrium position is the unknown eigenfunction of the symmetric eigenvalue problem, Eq. 2, and the quantity ω^2 is the eigenvalue. A configuration is unstable if $-\lambda = \omega^2 < 0$.

Axisymmetric Devices

Extensive stability computations for tokamaks were performed in the 1980's. The goal of these studies was to maximize the efficiency of a fusion experiment. The confinement efficiency is given by β, the ratio between the plasma pressure and the confining magnetic field energy density, and reactor-relevant values should exceed 0.05.

In a tokamak configuration, the homogeneity of the equilibrium solution with respect to the toroidal angle leads

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Ralph Gruber graduated from the ETH Zürich and received a Ph.D. in plasma physics in 1976 from the Ecole Polytechnique Fédérale de Lausanne, Switzerland. He was for five years leader of the EPFL's GASOV Group and also responsible for generating a programming environment for scientific users. As a research scientist at the CRRP, EPFL, CH-1007 Lausanne, he is presently developing algorithms for parallel computing.

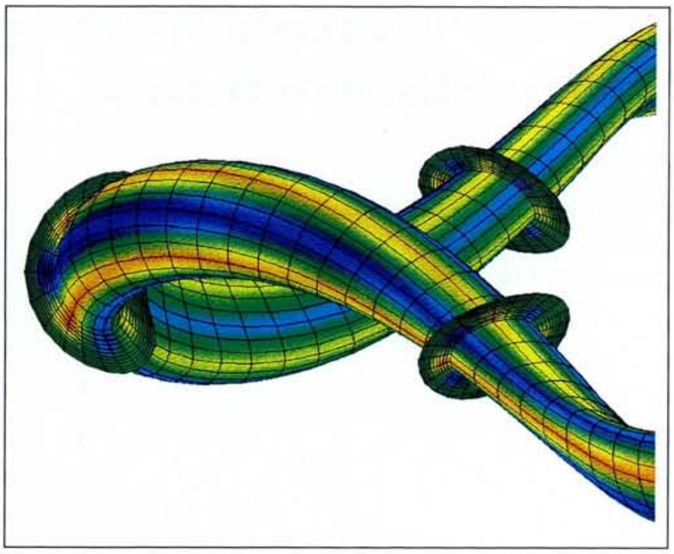
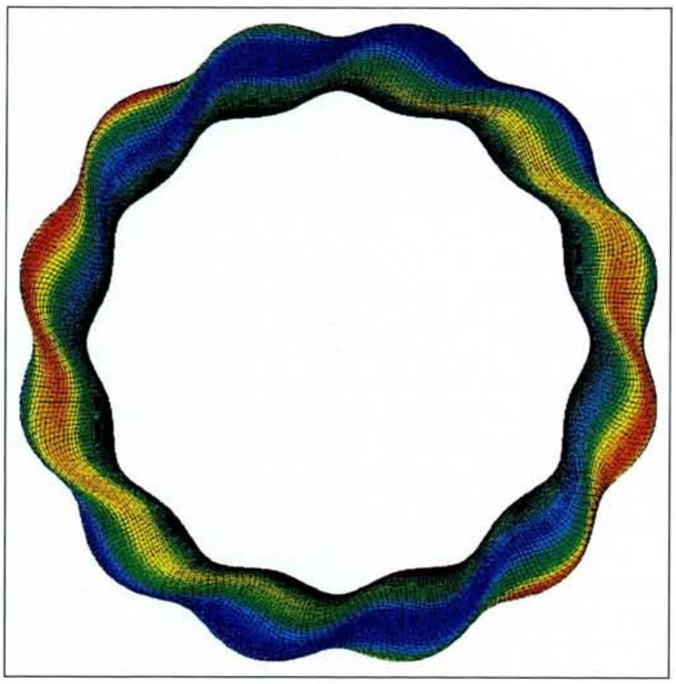


Fig. 2 — Helias configuration. A computer generated image of a portion of about one period long of the toroidal surface of the perturbed pressure distribution for a four field period Helias configuration on an internal flux surface where the instability is concentrated (the perturbed pressure attains maximum values in the yellow/red regions, and minimum values in the blue/violet regions). The instability structure from this internal surface to the plasma - vacuum interface is shown at three toroidal cross-sections. The configuration has $\beta = 0.02$ and the component with a poloidal mode number $m = 4$, and a toroidal mode number $n = 3$ dominates the unstable mode structure. Note that the superimposed coordinate system follows the instability.



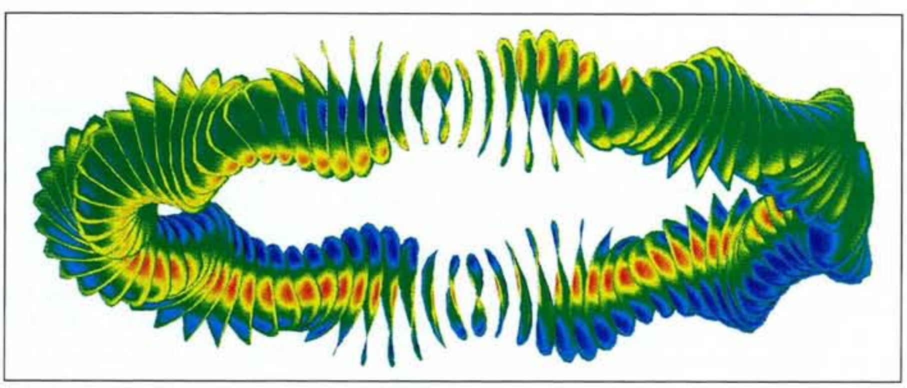


Fig. 3 — The ATF Stellarator.
a, upper) The perturbed pressure distribution at the plasma - vacuum interface for the standard ATF configuration with a finite toroidal plasma current in the same direction as the magnetic field (the pressures are colour coded as in Fig. 2). The configuration is weakly unstable with a dominant $m = 3$, $n = 2$ structure at $\beta = 0.03$. The coordinate system in which the calculations were performed is superimposed on the colour-coded representation of the plasma surface.
b, lower) The perturbed pressure distribution for a $n = 1$ instability on 108 toroidal cuts.

to a complete decoupling of the toroidal eigenmodes. This implies that stability studies of such devices are two-dimensional (2-d) problems for each toroidal mode number n. Several 2-d stability packages [see, *e.g.*, 2] were employed in designing axisymmetric tokamak devices such as the Joint European Torus (JET) and the Tokamak Fusion Test Reactor (TFTR) which have been built and operated successfully. They are also being used to design reactor-like devices such as the International Test Experimental Reactor (ITER) which is an international collaborative effort for the next step in magnetic fusion energy research. Indeed, a substantial fraction of the computational effort for magnetic fusion energy research realized on Cray computers over the last 10 years has been devoted to the 2-d stability problem.

Limits

An important outcome of these parametric studies for axisymmetric configurations is what one now calls the Troyon limit [3] stating that the largest stable value of β which can be achieved is

$$\beta_{max} = \alpha\mu_o JA/RB \qquad (3)$$

subject to the restriction $q_s \geq 2$. Here, J is the total induced toroidal current, A $(= R/r)$ the aspect ratio and RB_θ the toroidal magnetic flux at the plasma surface. The parameter α was reported in [4] to be 2.5 when the quantities are measured in the MKS system. The so-called safety factor at the plasma surface is

$$q_s = \int \frac{rB_\phi d\theta}{2\pi RB_p}$$

and B_θ, Bp, R, r and θ are the toroidal and poloidal fields, the major and minor radii and the poloidal angle, respectively (see Fig. 1b).

The scaling law (3) has been confirmed by experimental measurements [5]. It predicts that the maximum achievable β is proportional to the total indu-

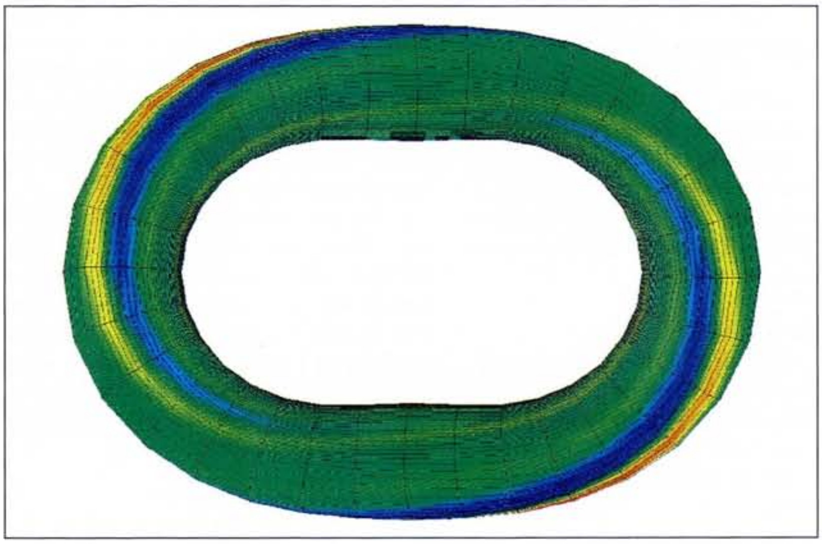


Fig. 4 — Race-track configuration. The perturbed pressure distribution at the plasma - vacuum interface for a race-track plasma configuration with an effective aspect ratio of four (colour coded as in Fig. 2).

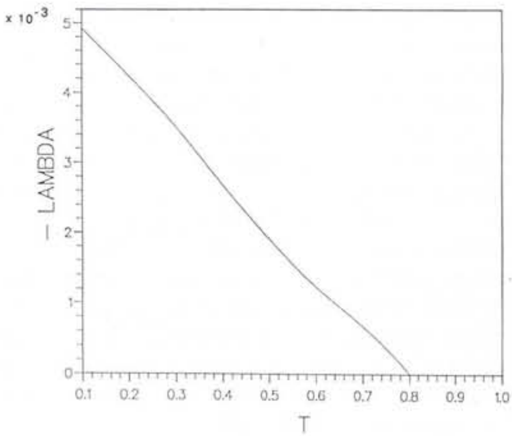
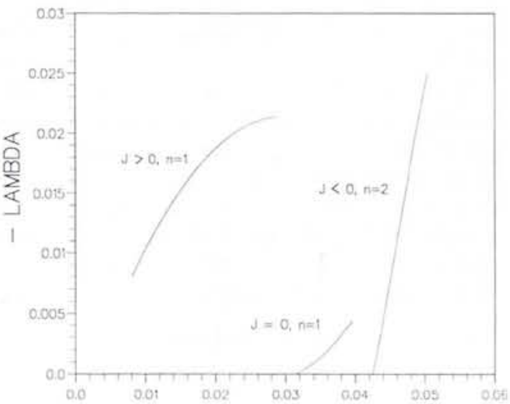


Fig. 5 — The calculated eigenvalues $-\lambda = \omega^2$ of the most unstable eigensolution as a function of the parameter T which governs the transition from a stellarator with a planar axial elongation E of 2 and five equilibrium field periods ($T = 0$) to the W7X device ($T = 1$). The configuration is stable ($\lambda > 0$) for $0.8 < T < 1.0$.

ced current J and not to J^2 as expected. It also tells us that for a fixed total current J, altering shape parameters such as the elongation or the triangularity (the so-called "D-shape") of the cross-section of the reactor vessel does not increase the confinement efficiency. If we fix q_s to its optimum value of 2, we can increase β by increasing the elongation E (see Fig. 1b) or by accentuating the bean-like shape of the plasma cross-section, with the additional requirement that the total current must be increased for both cases.

Fig. 6 — Converged eigenvalues λ for the ATF stellarator as a function of the confinement efficiency β for different values of the net toroidal field current J driven near the plasma surface, imposed by the $n = 1$ and $n = 2$ families of the ideal MHD instability modes. The addition of a small current in the direction of the magnetic field to enhance the twist of the field lines improves stability (λ > 0 achieved at larger values of β).



3-D Configurations

After having succeeded in computing the stability behaviour of 2-d configurations, we decided to develop TERPSICHORE, a computer program for computing stability in three dimensions [6]. For an efficient tool, it is important to carefully choose the numerical methods and their implementation on parallel vector computers. In order to minimize the matrix size, we perform a Fourier expansion in the two angular directions (in the poloidal and toroidal coordinates) and apply a non-conforming finite hybrid element approach [4] for the radial direction.

The coordinate system was chosen on the basis of analytical considerations of unstable modes which are of interest. Specifically, the curvilinear coordinates follow the instability structure along the magnetic field lines to reduce the coupling between Fourier modes. It is then necessary to consider only a few mode pairs in describing instability with sufficient precision, thus reducing the overall size of the computation matrix. This attractive feature of the coordinate system can be seen in Fig. 2, showing, for the four-period Helias stellarator concept, the perturbed pressure of an unstable mode on the flux surface with maximal displacement.

Effect of twisting

Our stability code has been applied to study existing and prospective 3-d configurations shown in Figs. 2-4 and in the cover illustration. Stability studies with TERPSICHORE and the CAS 3-d code [7] for the proposed *W7X stellarator* (cover illustration) have confirmed the prediction that the machine will be stable up to $\beta = 0.045$, a value close to that needed for a reactor-relevant plasma. In Fig. 5 we show a major result of the study performed to arrive at this prediction, where the eigenvalue λ of the most unstable eigensolution is plotted as a function of T which is a parametric measure of the helical twist of the magnetic axis to the device (a planar circular axis is represented by $T = 0$ the W7X configuration, for which the axis is highly twisted, by $T = 1$). One sees that the eigenvalue decreases with an increasing helical twist of the magnetic axis so that the configuration becomes stable ($\lambda > 0$) at $T = 0.8$ and remains stable up to $T = 1.0$.

Stability studies for existing devices, notably the *ATF stellarator* (Fig. 3) at the Oak Ridge National Laboratory, USA, have revealed a new type of a stable regime that also depends upon twists in the magnetic field (Fig. 6). Stellarators have always been designed and built in such a way that no externally induced plasma currents flow in the device except for those driven internally by the plasma pressure gradient. However, for the ATF, we were able to show that by adding a small net toroidal current in the direction of the magnetic field (in order to enhance the twist of the field lines),

β could be increased from 0.032 (with no net toroidal current) to 0.042 (with a toroidal current). This phenomenon can be seen in Fig. 6 in which is plotted the converged eigenvalues as a function of β for three different values of J, the net toroidal current:

a) $J = +0.5$: the externally applied imposed twist of the magnetic field is reduced and the plasma configuration as a result remains unstable to global, external modes with a toroidal mode number $n = 1$ at β values well below 0.01;

b) $J = 0$: corresponds to the standard configuration ($\beta < 0.032$);

c) $J = -0.5$: the twist of the magnetic field is increased which causes the resonant surface for the $m = 1$, $n = 1$ component (where m is the poloidal mode number) of the mode structure to move from the vacuum region into a region of high magnetic shear within the plasma. This causes the $n = 1$ family of modes to become stabilized up to $\beta = 0.05$. The $n = 2$ family of modes imposes the limiting value at $\beta = 0.042$.

The latter is promising result because the effect can be realized by driving a small current near the plasma surface using well-known radio frequency current drive techniques. It also demonstrates that by taking into account new parameters one can adjust the stability of magnetic fusion devices.

In the future, new types of devices such as the Helias type of stellarators (Fig. 2) and racetrack tokamaks (Fig. 4) will be studied in more detail using supercomputers running the TERPSICHORE program in order to identify and understand regimes of stable operation which could lead to improved designs for the next generation of fusion experiments and reactors.

Conclusions

Using computer simulation techniques to study the stability behaviour of reactor-relevant controlled fusion experiments we have been able to obtain new insights into the underlying physics, notably:

— For tokamaks, a scaling law has been found that states that the maximum achievable plasma pressure, as given by β, the ratio of the plasma pressure to the confining magnetic field energy density, is proportional to the total current J, and not to J^2 as expected. Furthermore, altering shaping parameters such as the elongation or the triangularity of the cross-section can only lead to an increase in β if the total current is increased at the same time.

— For the proposed W7X stellarator, helically twisting the magnetic axis stabilizes the plasma configuration.

— For Oak Ridge's ATF stellarator, the addition of a small net toroidal current in the direction of the magnetic field (in order to enhance the twist of the field lines) increases β from 0.032 to 0.042. This result clearly contradicts the established theory that the stability properties of stellarators are deteriorated by induced net toroidal currents.

Supercomputer simulations therefore allow novel plasma configurations to be studied, an important feature since reactor-relevant fusion experiments are very expensive, and it will no longer be possible to build a fundamentally new type of device without having performed intensive stability computations beforehand.

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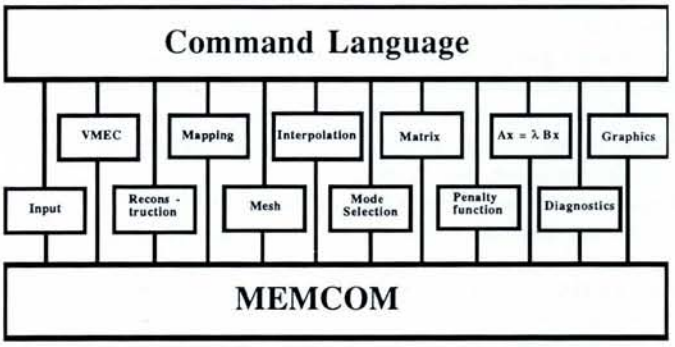
The TERPSICHORE Project

The modular structure of the TERPSICHORE computer program. Using the ASTRID programming platform, a common memory and a command language is shared between the the various modules that perform specific tasks such as mapping the MHD equilibrium, constructing the stability matrix elements, interpolating, etc.



TERPSICHORE is a high-speed, efficient computer package that was developed to study the three-dimensional (3-d) stability of plasma in reactor-relevant devices. It presently comprises six basic modules involving: the interface to the magnetohydrodynamic (MHD) equilibrium, reconstruction of the MHD equilibrium, mapping of the MHD equilibrium from the equilibrium coordinates to the stability coordinates, construction of the stability matrix elements, solution of the eigenvalues, and analysis and diagnostics of the results. It is intended to incorporate additional modules that will: generate an adaptive radial mesh by interpolating the equilibrium quantities; minimize the number of selected Fourier mode pairs by using an algorithm based on an expert system approach (this would replace the tedious manual selection of repetitive choices); add a penalty function integral contribution to the potential energy of the variational form (see Eq. 2, page xx) in order to eliminate destabilized modes belonging to the stable continuous spectrum.

The overall modular structure of TERPSICHORE program, which is embedded in the ASTRID programming platform [Bonomi E. *et al.*, *GASOV Report 26* (EPFL, Lausanne) 1990], is shown in the figure above. The choice of very advanced numerical methods and a design which runs efficiently on vector-parallel computers with either shared or distributed memories (an operating speed of 2 Gflop/s has been achieved on the EPFL's Cray YMP8 machine — a feature that led to the team being awarded the Cray Gigaflop Performance Prize in 1989 and 1990 — have made TERPSICHORE an effective tool for plasma physicists. It enables the generation of significant results in a relatively short time — crucial when studying 3-d devices where the parameter space is much wider than for 2-d geometries and many more cases need to be treated. For if we cannot simulate the