

The Chiral Anomaly

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How a discrepancy in the current algebra formulation of the decay of the neutral pion into two photons was resolved in terms of a quantum mechanical mechanism for symmetry breaking that has become an essential ingredient of models in particle physics.

John Bell and I met and became acquainted in 1967, when I went to CERN for a year-long research visit, soon after finishing my doctoral studies at Cornell. At that time, particle physics theory was dominated, as it happens from time-to-time, by a single idea: there was broad agreement among theorists what the important problems were and how they should be solved, although one can hardly remember the details of the programme. But attaching my scientific activity to a consensus was not my ambition; I had much admired the independent attitude of one of my research supervisors at Cornell, Ken Wilson. So I looked among the staff at CERN for someone who pursued interesting issues that were neither "central" nor "important", and I was delighted to find such a scientist in John Bell. Moreover, he was generous in giving his time: he tolerated my coming to his office and appeared willing to discuss without limit. I appreciated the magnitude of his generosity only years later when I too became installed in an office and people began coming in and taking my time to talk about things.

The Pion Decay Puzzle

There began for us a period of wide-ranging conversations — and not only about physics — which acquainted me with the many issues that concerned John. But nothing was then said about his work on quantum mechanics — he did not at that time describe it to me and I did not know of it. Current algebra interested John very much. Within its framework one can understand the low energy behaviour of elementary particles, without making a commitment to a dynamical model, unknown in the 1960's, while today's "standard model" resists solution in the low energy domain. The approach seemed successful, complete and exhausted by the late 1960's, yet there remained discre-

pancies between theoretical predictions and experimental verification.

John was particularly impressed with an analysis by his good friend Martinus Veltman, and also D. Sutherland, to the effect that the neutral pion could not decay into two photons if the charge-neutral and gauge invariant axial vector (chiral) current is conserved, as it was then taken to be in current algebra applications. Because the decay does, in fact, occur in Nature, while the Sutherland/Veltman argument appeared incontrovertible, John stressed that the subject of current algebra must not be closed until this puzzle is resolved and urged a study of the chiral current.

This was the second time I received such advice: in my final student days Wilson suggested a critical examination of the apparent conservation of the axial vector current in the Baker-Johnson-Willey theory of massless electrodynamics, with which he had his own disagreements.

I was therefore willing to research this topic, but since the existing discussions were straightforward and the conclusions immediate, it was hard to see how a useful probe could be launched. I asked fellow theorists for suggestions but the subject did not spark interest. I do recall two mathematically oriented colleagues, Henri Epstein and Raymond Stora, offering a diagnosis that in retrospect proved prescient: in their opinion one could not rely on current algebra analyses because physicists treat cavalierly singular products of distributions. But their prognosis that a cure will be found if one uses rigorous rather than heuristic mathematics did not appeal. In fact, the decisive suggestion did not come from a theorist but from an experimentalist.

Reformulating the σ -model

One of CERN's civilized activities, to which John frequently invited me, consists of taking an afternoon drink in the cafeteria, where we would continue our conversations together with people who joined us. On one occasion, Jack Steinberger — John's friend and colla-

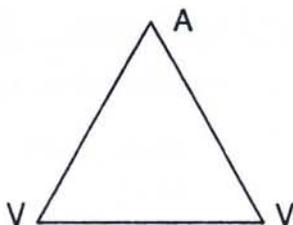
borator on charge particle formalism — was at the table and asked about our current interests. When we described to him the $\pi^0 \rightarrow 2\gamma$ puzzle, he expressed amazement that theorists should still be pursuing a process that he, an experimentalist, calculated almost twenty years earlier. He had found excellent agreement with experiment, while also noting a discrepancy between results obtained when the pion coupled to nucleons by pseudovector and pseudoscalar interactions (pions, nucleons and photons were the only particles in Steinberger's model, and it was believed that equivalent results emerge for pseudovector and pseudoscalar pion-nucleon coupling).

There, at that table, came to us the realization that Steinberger's calculation would be identical to the one performed in the dynamical framework of the σ -model, which was constructed to realize current algebra explicitly. We reasoned that within the σ -model we could satisfy the current algebraic assumptions of Sutherland/Veltman and also obtain good experimental agreement in view of Steinberger's result, thereby resolving the $\pi^0 \rightarrow 2\gamma$ puzzle.

Guided by Steinberger's paper (at that time we were not familiar with the work of his contemporaries, H. Fukuda and Y. Miyamoto, and only dimly aware of subsequent contributions by J. Schwinger), we quickly established that the amplitude describing correlations between the three currents appearing in the problem — two vector currents to which the two photons couple and one axial vector current to which the pion couples — is given in lowest order (one loop) perturbation theory by the now famous triangle graph depicted in Fig. 1. The amplitude is determined by Feynman rules only up to an overall ambiguity, owing to ultraviolet divergences, even though the amplitude is finite. Moreover, while the ambiguity may be resolved by enforcing current conservation, it is impossible to maintain conservation of all *three* currents, as was assumed in the current algebra calculation.

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Fig. 1 — Triangle Feynman diagram for the three-current correlation amplitude that leads to quantum mechanical symmetry breaking. Two vector currents V and one axial vector current A form the three corners, at which virtual fermions are created/annihilated. Propagation of the virtual fermions, indicated by solid lines, prevents the currents from being conserved. A similar effect arises with three axial vector currents.



Thus, we found that the σ -model's symmetries, which underlie current algebra and which should guarantee the conservation of the respective currents, cannot be maintained when the model is quantized. In the absence of these symmetries, pion decay is no longer considered to be forbidden.

Anomalous Symmetry Breaking

Our work resolved the $\pi^0 \rightarrow 2\gamma$ problem, by exposing a purely quantum mechanical mechanism for symmetry breaking, which is the modern interpretation of Steinberger's discrepancy, and these days is described as "anomalous breaking of a symmetry", although once the surprise has worn off, it is better named "quantum mechanical symmetry breaking" [page 72, 6].

The "anomaly" was substantiated by S. Adler, who working independently came to a similar conclusion about the (absence of) symmetries in massless electrodynamics, and building on our work established, with W. Bardeen, the important fact that higher perturbative orders do not modify the one-loop calculation of pion decay. Further confirmation came from Wilson, who used our theory as a case study for his non-Lagrangian models of current algebra, based on his operator product expansion. The early period of research on this subject culminated in a phenomenological description of quantum mechanical symmetry breaking in terms of an effective Lagrangian, constructed by J. Wess and B. Zumino [1] who apparently were not aware of our result.

In time, this work, which arose from clearing up a corner of current algebra, grew to affect much of particle physics. It became an important ingredient of model building, both for speculative string theories and for the conventional "standard model", where, among other things, it enforces colour triality and explains the numerical equality of quark and lepton degrees of freedom, thus predicting the existence of the elusive top quark [1].

John maintained an amused interest as our calculation became transformed in various contexts, and was shown to be a consequence of diverse physical and mathematical considerations [1]: symmetry breaking aspects of the

Dirac sea (R. Feynman), anomalous transformation properties of the functional integral measure (K. Fujikawa), the necessary effect of high-energy modes on low-energy physics (V. Gribov), quantum field theoretic manifestation of Berry's phase, local version of the Atiyah-Singer index, and cosmological properties of gauge groups (L. Faddeev). The last two mathematical connections seeded a remarkable collaboration between mathematics and physics, which is still flourishing. On the other hand, the physical world itself became threatened by the anomaly because, as Gerard 't Hooft showed [2], it catalyzes baryon decay, but fortunately at a sufficiently slow rate to cause no immediate concern.

In spite of these wide-ranging generalizations, John preferred the simple triangle graph calculation [page 72, 6]. He always stressed the element of choice that exists in resolving the calculational ambiguity, thus putting different faces on the nature of the anomaly — a freedom that is obscured in the more abstract and high-powered approach.

Indeed John was rather diffident about the entire matter. In this he showed one of his striking qualities: modesty about his own work, praise for the work of others, but skepticism in the face of inflated claims, even if they were extolling his own contributions. When he eventually described to me his famous analysis of the foundations of quantum mechanics [page 72, 1], he called that research "a hobby".

Fractional Charge Quantum Numbers

After I left CERN in 1968, we had many occasions to meet and talk about interesting topics, but our discussions never again resulted in a joint publication. The closest we came to this happened when I described to John the phenomenon of fractional charge quantum number [3], which had become physically relevant [4]. He found this interesting but characteristically was at first skeptical that a fractional value could be an eigenvalue. Upon elaborating the precise circumstances in which a sharp observable arises, he published with R. Rajaraman [page 72, 21] an analysis that contributed to the understanding and acceptance of this fascinating

idea that today has also gained wide currency among physicists.

In all my contacts with John I was always made aware of his overwhelming intellectual precision and honesty. These are the qualities that made him such an incisive critic and therefore a wonderful colleague. Precisely this attitude lay behind his scientific achievements, which are informed by clarity of observation about previously murky subjects.

The same attitude characterized his approaches outside science, for example to social and political questions. Many physicists profess humane and liberal values, but often these become obscured by personal emotion and prejudice. In the last quarter century, issues of Vietnam, Ireland and Palestine offer a dramatic opportunity for displaying social conscience in search of justice. John recognized and spoke on these matters clearly. Already in 1967/68, I heard him analyze America's role in Vietnam in terms that did not gain acceptance until years later; his opinions on the two other tragedies remain in the minority even today, but one hopes that here too his ideas are merely ahead of their time.

I liked John very much and together with many colleagues I shall miss him. He was an outstanding scientist and helped us do good science, which is one reason why we become physicists. Moreover, many enter our field not only for the opportunity of exploring Nature in its most fundamental workings, but also for what we perceive as the purity and honesty of the profession. These qualities sometimes get submerged by pressure of personal ambition, struggling for achievement and recognition, but John Bell never lost them, and in this way he reminded us of the ethical motivation for becoming a physicist.

ACKNOWLEDGEMENT

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ADDITIONAL REFERENCES

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