

Superfluid ^3He

Theory and Recent Experiments

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Superfluid liquid ^3He is a unique system within condensed matter physics because it shows very sophisticated phenomena which are at the same time quite manageable theoretically.

In this article we shall try to demonstrate that in spite of its limited availability and relatively recent discovery 19 years ago, superfluid ^3He is worthy of continued research.

We shall briefly review theoretical developments and describe some related recent experimental results. Several open questions are raised but many interesting topics are either left out or mentioned only in passing.

The Many-Body Problem

One of the great unsolved problems of theoretical physics is that of many interacting particles. Problems involving a small number of units such as the solar system or light atoms can be handled with modern computers by solving the equations of motion for each particle. This approach is not feasible when the number of particles grows; it fails totally in condensed matter systems of macroscopic objects. The many-body problem becomes especially difficult if one is interested in phenomena and properties that do not exist in a few-body problem. For example, concepts like phase transition and electrical conductivity are well defined only in the limit of a system of infinite size.

Quasiparticles

Many approximate methods have been developed for the many-body problem, but with few exceptions an exact

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solution is only found under the following condition: that a coordinate system can be identified where the interaction between the new variables, or *quasi-particles*, is weak. Familiar examples are phonons in a crystal and spin waves in a ferromagnet. The theory of gases falls into the same category because collisions between molecules (when the interaction is strong) are rare.

A further example, which will be our main topic here, is provided by the Fermi liquid theory of liquid ^3He and of metals: the atoms of the liquid (electrons of the metal) form a strongly interacting system. At temperatures that are low compared to the Fermi temperature T_F (1 K for ^3He ; ≈ 10000 K for metals), the degrees of freedom in the system are dramatically reduced because of the Pauli exclusion principle. The result is a dilute gas of quasiparticles. In this approach the structure of a quasiparticle in terms of the constituent particles remains undetermined, but its properties can be parametrized by a reasonable number of measurable quantities.

Theory of Fermi liquids

The residual interaction between quasiparticles is usually taken into account by perturbation theory. A more interesting situation occurs in ^3He and in several metals: the residual interaction is attractive and in spite of being weak, it brings about a phase transition to a qualitatively different state at low tem-

peratures (≈ 2 mK in ^3He ; a few K in metals). The theory which also incorporates this superfluid (or superconducting) state is known as the *quasi-classical theory of Fermi liquids*.

One may perhaps argue that further studies are uninteresting because the many-body problem has already been solved; all that remains is straightforward calculation of the measurable quantities. We would like to stress, however, that superfluid ^3He verges on being the most sophisticated condensed matter system that does not seriously suffer from the uncertainties implicit in the many-body problem.

Bulk Superfluid Phases

Superfluidity in ^3He was discovered at Cornell University in 1972 in the course of an investigation of the effect of compressional cooling on the melting curve of ^3He . Despite early conjectures based on the properties in *solid* ^3He , it very soon became apparent that two new features at $T \approx 2$ mK, named A and B, originated from superfluid phases of *liquid* ^3He (the phase diagram of ^3He is shown in Fig. 1).

In superfluid ^3He and metallic superconductors, the quasiparticles of the normal Fermi liquid form pairs owing to the weak attractive interaction. All these so-called Cooper pairs are restricted to be in one single state of a pair owing to the Pauli principle. The superfluid properties are a consequence of the macroscopic occupation of the one-pair wave function.

In most superconductors, the spin part of the pair wave function is a singlet and the orbital part is an s-wave

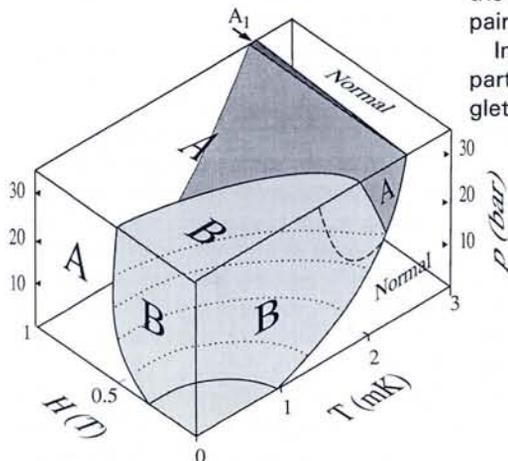


Fig. 1 — Phase diagram of ^3He showing the superfluid phases (A, A₁ and B) in the space of temperature (T), pressure (P) and magnetic field (H). The normal Fermi liquid exists above 3 mK and the solid phase at pressures above 34 bar. The vortex-core transition observed in the rotating B phase is shown by a dashed line at $H = 0$.

state. There is a wealth of evidence showing that the Cooper pairs in superfluid ^3He are spin triplets and are in a p-wave state.

Critical temperature

The pairing channel for ^3He is roughly understood using simple arguments, but there is no reliable calculation of the superfluid transition temperature T_c . This is because T_c depends on many-body processes that can neither, for practical reasons, be measured in the normal state nor calculated. However, once T_c is known, several properties of the superfluid phases can be calculated using normal state parameters, such as the quasiparticle effective mass and their scattering amplitudes.

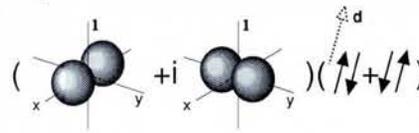
Order parameter

The center-of-mass part of the Cooper-pair wave function is called the *order parameter*. In superconducting metals it is a scalar function because there is only a single spin-singlet s-wave state: the energy of the superconducting state is independent of the phase of the complex-valued order parameter. Being a macroscopic variable, the phase must have a definite value symmetry with respect to the phase so (gauge) must be broken spontaneously.

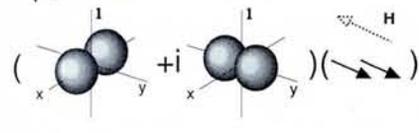
The basic reason for the complexity of superfluid ^3He is that there exist three p-wave states as well as three spin triplet states. This means that the order parameter is a complex-valued 3×3 matrix. Evidently, the energy is independent of both the phase and the orientation of the pair. It is also approximately independent of the orientation of the spin and orbital parts separately because the "spin-orbit coupling" is very weak. Symmetries must be broken, and depending on external parameters, this happens in a variety of ways, giving rise to several different phases and to transitions between them.

In the absence of perturbations, symmetry is spontaneously broken in two ways resulting in two bulk phases in the pressure-temperature plane (see Fig. 1). Their Cooper pair wave functions are illustrated in Fig. 2. In the A phase the orbital part of the Cooper pair wave function has unit angular momentum component on a quantization axis I (i.e. $L_I = +1$). The spin part has a vanishing component along another quantization axis d (i.e. $S_d = 0$). The symmetry is clearly broken in spin-space rotations that turn d , and in orbital-space rotations that turn I . An important feature is that an orbital rotation around I can be undone by a phase shift. Thus the symmetry is broken only in the relative transformation of the two.

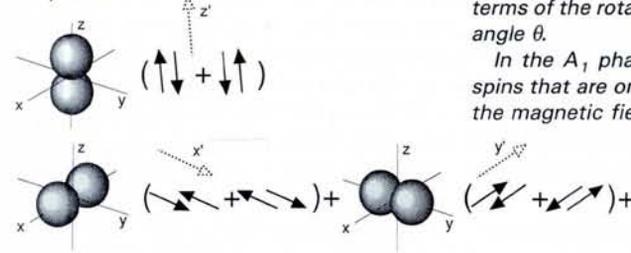
A phase



A₁ phase



B phase



In the B phase, the orbital and spin parts do not separate but corresponding to each orbital wave function p_x , p_y and p_z , there is a spin part with a vanishing component along the x' , y' and z' axes. Here x , y and z and x' , y' and z' are two independent orthogonal coordinate systems whose relative orientation is often defined in terms of a rotation axis n and an angle θ . The relative rotational symmetry of spin and orbit spaces is spontaneously broken in addition to the gauge symmetry.

Theoretical predictions

Both the A and B phases are predicted by calculations, i.e. the *tricritical pressure*, where the two phases are in equilibrium with the normal state, is calculated to lie below the solidification pressure. The thermodynamic properties of the B phase are estimated fairly accurately at low pressures, but at higher pressures they depend sensitively on the quasiparticle scattering amplitude which is poorly known. The predicted tricritical pressure is therefore rather inaccurate (off by 7 bar) and there exists no calculation of the A-B transition line beyond the tricritical point.

The most direct evidence of the order parameter structure probably comes from *collective modes*. There exist a total of 18 collective modes corresponding to the 18 real degrees of freedom in the order parameter. They have an especially clear structure in the B phase, where almost all of them have also been detected experimentally. The broken symmetries lead to Goldstone modes: three spin-wave modes and a

Fig. 2 — The Cooper pair wave functions of the superfluid phases A, A₁ and B. The orbital wave functions p_x , p_y and p_z are illustrated as shaded spheres, and the spins are shown as arrows. The radial parts of the wave functions are not shown.

In the A phase, the orbital part has a unit angular momentum component on the quantization axis I ; the spin part has a vanishing component along another quantization axis d .

In the B phase, the orbital and spin parts do not separate. For each orbital wave function along axes x , y and z there is a spin part with a vanishing component along axes x' , y' and z' of another orthogonal coordinate system with a relative orientation defined in terms of the rotation axis n and the rotation angle θ .

In the A₁ phase, the Cooper pairs have spins that are only able to lie antiparallel to the magnetic field H .

zero-sound mode correspond to spin-orbit rotations and gauge transformations, respectively. Another set of three-fold degenerate and non-degenerate modes lies at the pair-breaking energy 2Δ , and at intermediate energies there are two five-fold degenerate modes. The splitting of all the degeneracies that is observed in a magnetic field is well accounted for by theory.

Strong Magnetic Fields

A third bulk phase occurs in a strong magnetic field H . This A₁ phase is a variation of the A phase in which the Cooper pairs have spins that are only antiparallel to H (Fig. 2). Superfluidity in strong magnetic fields has recently become a very active area of ^3He research (experiments in fields up to 16 T will probably be conducted shortly).

Ultrasonic experiments at Cornell University have established the phase diagram up to 9 T, the strongest magnetic field where superfluidity has been observed in ^3He . They have confirmed an anomaly in the sound attenuation in the vicinity of the A₁-A transition [1]. Experiments at the University of Leiden, also at 9 T, indicated an anomaly in the same regime in a measurement of the viscosity [2]. It was concluded that this phenomenon results from a rearrangement of the spatial distribution of the order parameter near the A₁-A transition, or possibly from a new superfluid phase. Another intriguing result arising from the viscosity measurements is that the A₁ phase, unlike other superfluid phases, shows a minimum in the viscosity as a function of temperature.

Perhaps the most intricate observation is the increase of the superfluid transition temperature from 2.8 mK up to 3.3 mK when a highly polarized solid ^3He sample is quickly melted by decompression [2]. Model calculations suggest an increase of up to ≈ 20 mK under suitable conditions.

Inhomogeneous B Phase

Complicated forms of the order parameter can be generated by external forces. The perturbation can be a magnetic field, the surfaces of a container, rotation of the system, or a boundary condition causing for example, superflow or the formation of an interface between two phases. The spin-orbit coupling due to the interaction between the dipole moments of the ^3He nuclei also acts as a perturbation which is on the order of 10^{-5} times the superfluid condensation energy. Qualitatively different behaviour can be obtained depending on the strength of the perturbation, and a corresponding hierarchy of length scales can appear.

Magnetic field

Consider for example the B phase in a container: at atomic distances from the wall of the container, many-body processes beyond the quasiclassical theory dominate. For instance, a magnetic solid layer can build up and some model has to be used to describe the scattering of quasiparticles from this layer.

The quasiclassical theory is applicable on approaching a distance ξ_0 (≈ 50 nm) from the wall called the coherence length. The order parameter is strongly distorted on this scale but at larger distances it approaches the bulk form.

A calculation on the scale ξ_0 gives the boundary condition for the order parameter in the bulk. It is found that the rotation axis \mathbf{n} has two degenerate orientations in magnetic fields below 3 mT and four at higher fields.

At distances ξ_H (typically from 50 μm up to several mm's depending on the field), the B phase approaches the orientation in which \mathbf{n} is parallel (or antiparallel) to the field. Experimentally, a first order transition can be induced by adding superflow [3]. Theoretically, this effect is understood as arising from a transition between two different surface orientations of \mathbf{n} (see Fig. 3) [4].

Experimental studies of the inhomogeneous B phase are primarily made using nuclear magnetic resonance (NMR), ultrasonic and ion transmission measurements. The shift of the NMR frequency from the Larmor value is proportional to $(nH)^2$. One can thus deduce the distribution of $(nH)^2$ and

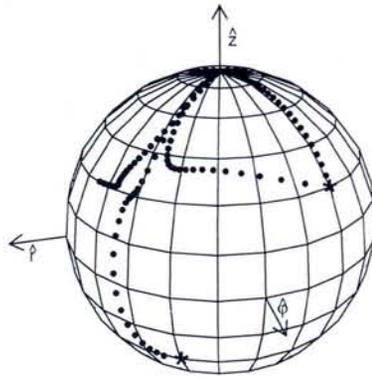


Fig. 3 — A representation of the B phase texture in a cylinder. The unit sphere represents the end points of the vector \mathbf{n} , and the "northpole" is the direction of the cylinder axis. The vector \mathbf{n} traverses the dotted paths as a function of the radial coordinate r of the cylinder. The four paths correspond to four different counterflow velocities increasing from right to left. In all cases \mathbf{n} is parallel to the cylinder axis at $r = 0$, but on the surface of the cylinder the paths of \mathbf{n} tend toward two different minima of the surface energy, indicated by crosses [4].

possibly also some eigenfrequencies of trapped spin-wave modes. Attenuation of ultrasound in the superfluid phases originates from the dissociation of Cooper pairs and from the excitation of collective modes. Information about the orientation of the order parameter can be obtained because both processes depend on this parameter. Unfolding the data may be complicated but, unlike NMR, ultrasonic probing works at all magnetic fields. Exploiting ions is based on their anisotropic mobility. It should be noted however, that several other methods, e.g. the mechanical response of an oscillating object (see below), also yield information about the order parameter.

Rotation

Besides a magnetic field, a particularly suitable "field" for superfluid ^3He involves rotation of the system. Uncharged superfluids cannot rotate as a solid body. Instead, the vorticity of the rotation is concentrated in quantized vortex lines. A slightly different hierarchy of lengths than those above is encountered around a single vortex line. On approaching a vortex from outside, the \mathbf{n} vector starts to deviate from the field direction at distances ξ_H . At distances ξ_D (≈ 10 μm), the angle θ starts to deviate from its bulk value (104°) fixed by the weak spin-orbit coupling. Finally, at distances ξ_0 , the order parameter deviates strongly from the B-phase form.

The first successful experiments on rotating superfluid ^3He were carried out in Helsinki in 1981 [3]. The whole cryostat (see cover) is mounted on air bearings and circulation of cryogenic liquids is maintained by cryoadsorption pumps; adiabatic nuclear demagnetization cooling is used as in most milli-Kelvin cryostats. Measurements were initially made by NMR and later using ions. Ultrasound has recently been used with a new rotating cryostat in Finland to study ^3He under rotation up to angular velocities of 5.5 rad/s [5].

Vortex cores

A first order transition-line was unexpectedly observed within the rotating B phase (see Fig. 1) [3]. This phase change can be understood as a rearrangement of the order parameter in the vortex core (on a length scale ξ_0) [6]. In the simplest case the order parameter would vanish at the vortex line, as it does in metallic superconductors or in the Bose superfluid ^4He . In ^3He -B this state is unstable towards two different ways of forming non-zero order parameter. In the *A phase core vortex*, reflection symmetry in the plane perpendicular to the vortex line is spontaneously broken. In the *double core vortex*, rotational symmetry around the vortex axis is also broken forming two separated minima of the superfluid density.

A calculation of the free energies near the superfluid T_c shows that the two types of vortices correspond to those observed at high and low pressures. The transition between the two is obtained at a pressure that is roughly the same as the measured one. Calculations for vortex cores at temperatures far below T_c have not been reported.

Inhomogeneous A Phase

The A phase has very special flow properties because of the coupling of orbital rotations and phase. The angular-momentum quantization axis \mathbf{l} is uniform in the direction of the flow at low velocities. With increasing velocity this state turns out to be unstable towards helical deformation of \mathbf{l} . Moreover, time-dependent states not involving vortices are possible: the flow energy can be dissipated to the quasiparticles through turning \mathbf{l} only. Theory predicts uniform states, stable helices and time-dependent textures depending on the flow rate and magnetic field; the experimental phase diagram vaguely resembles the theoretical one [7].

Non-singular vortices

The coupling of orbital rotations and phase also allows *non-singular* vortices

that have a finite A-phase order parameter everywhere. Different structures for both singular and non-singular vortices have been predicted depending on the magnetic field and on the density of vortices [6].

NMR experiments show that in moderate magnetic fields (≈ 30 mT) the vortices are non-singular and doubly quantized [3]. Theoretically, however, singular, singly quantized vortices were found to have the lowest free energy, indicating that nucleation processes rather than small free energy differences determine the equilibrium states of vortices. Indeed, later measurements using ion focusing found two different vortex types depending on the acceleration rate of the cryostat. These are identified as singular (slow acceleration) and non-singular vortices. Ultrasonic techniques have revealed very recently a first order transition in the vortex state at low magnetic fields; and the first experiments on rotating $^3\text{He-A}_1$ are presently being performed.

One of the long-term goals of experiments on superfluid ^3He in rotation is the **direct** imaging of the individual vortices using either optical or ion focusing methods.

Small Geometries

The bulk phases become deformed in geometries approaching the size of the coherence length ξ_0 and components of the order parameter that have their orbital parts perpendicular to the wall inevitably have to vanish at the wall. Parallel components are also suppressed to the extent that the scattering of quasiparticles from the surface is non-specular (*i.e.* non mirror-like). The suppression results in reductions in the transition temperature and the superfluid density.

Scattering

Relatively diffuse rather than specular scattering has been observed in several experiments using narrow pores and films. Moreover, when a small amount of ^4He is added, the atoms tend to coat the surfaces of the pores (owing to its larger mass and thus smaller zero-point motion in the surface potential). It has then been observed that the scattering is shifted near the specular limit by coatings as thin as a monolayer. The shift is perhaps surprising because the surface irregularities are probably much larger than the thickness of a monolayer.

Josephson effects

Josephson effects demonstrate the existence of phase coherence of the

order parameter. In Josephson phenomena in superconductors, two bulk metals typically have a weak link which provides the partial overlap of the two macroscopic wave functions.

A practical form of a weak link for liquid helium is an orifice connecting the two reservoirs. Despite several attempts starting in the 1960's, unambiguous proof of the existence of Josephson behaviour was not observed, even in superfluid ^4He , until 1985 when a group at Orsay succeeded in observing Josephson steps in helium, at first in ^4He and subsequently in $^3\text{He-B}$ [8].

The experimental set-up is shown schematically in Fig. 4. The orifice has the shape of a rectangular duct, $0.3 \times 5 \mu\text{m}^2$ in cross section, bored in a $0.2 \mu\text{m}$ thick nickel plate. The chamber acts as a Helmholtz resonator, pumping electrostatically liquid helium in and out at the resonance frequency of a few Hz; the amplitude of the resonating diaphragm is measured.

The lower part of Fig. 4 shows a typical staircase pattern in $^3\text{He-B}$ with the steplike structure that is analogous to the one in a SQUID (superconducting quantum interference device). The observations can be fitted assuming that the supercurrent J_s through the orifice as a function of the phase difference $\Delta\theta$ has a slanted-sine form: $J_s = J_c \sin(\Delta\theta - cJ_s)$, where J_c and c are adjustable parameters. Smaller, unexplained features appear at high pressures.

Preliminary calculations of the current-phase relation in $^3\text{He-B}$ with identical boundary conditions on both sides of the weak link give results that are qualitatively the same as those for a metallic superconductor. However, this is a special case because there are many degenerate boundary conditions which need not be the same on the two sides. A qualitatively new current-phase relation has indeed been found with antiparallel \mathbf{n} vectors. Many other cases have not been studied, including the

high field case or having a different phase on each side. Symmetry breaking solutions such as those found in vortices may also arise.

Spin Hydrodynamics of the B Phase

Early NMR experiments on the B phase detected surprisingly slow decay of the free precession signal in a non-uniform magnetic field. This is now understood as an initial tipping of magnetization from the field direction by say 90° and a splitting into two domains [9]. In one (high field) domain the magnetization returns to a direction parallel to the field \mathbf{H} ; in the other domain the tipping angle increases to $> 104^\circ$ and the magnetization and the \mathbf{n} vector rotate homogeneously about \mathbf{H} at an angular velocity that is equal to the Larmor frequency at the domain interface.

Soft Josephson effect

Spin supercurrents analogous to mass supercurrents are allowed in superfluid ^3He as a result of the spin structure of the Cooper pairs. These currents do not exhibit the "hard" Josephson effect because the spin current is not conserved (by the spin-orbit interaction). However, a "soft" Josephson effect has been identified by a group in Moscow [9] using the homogeneously precessing domain (HPD): two containers are connected by a channel whose dimensions can be rather large ($> \xi_H$). HPD's are generated in both containers and a difference in the rotation angle of \mathbf{n} around \mathbf{H} arises if the frequencies of the HPD's are not equal. Slip in the rotation angle takes place when \mathbf{n} can turn in the direction of the field. This happens either uniformly over the cross section of the channel or *via* the motion of a "spin vortex".

Persistent Currents

Phase slips in the Josephson effect can be largely understood on the basis of the theory for static superfluids. This

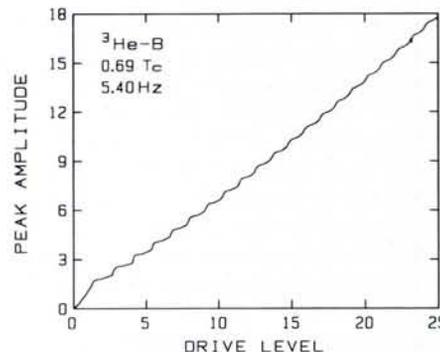
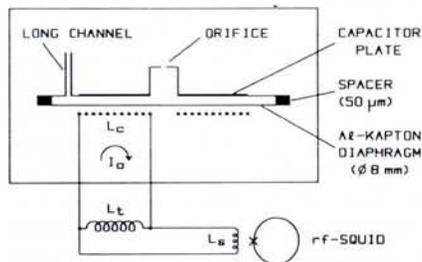


Fig. 4 — The Helmholtz resonator of the Orsay group, and a typical staircase pattern of the Josephson effect in $^3\text{He-B}$ [8]. A "weak link" is formed by separating two reservoirs by a small orifice through which liquid helium is pumped electrostatically. The peak amplitude of the resonating diaphragm is measured as a function of the drive voltage.

changes when the flow channel becomes wider and there is a cross-over to vortex nucleation which determines the magnitude of non-decaying currents.

The existence of persistent currents in ^3He was verified in 1984 by determining the angular momentum L of the superfluid circulating in a torus-shaped container using a gyroscopic technique [10]. The torus was packed with fine powder because, experimentally, the critical velocity is higher in small geometries. The decay time (Fig. 5) of the current in the B phase was found to have an immeasurably high value (> 48 hours). The magnitude of the critical current can be roughly understood using dimensional arguments but no microscopic calculations exist.

A transition line in the critical velocity was observed. It is not the same as the line in the pressure-temperature plane but it resembles that for the vortex core transition (Fig. 1). The origin of this transition remains unclear [6].

The experiments did not at first show persistent currents in the A phase. However, more recent measurements at Cornell indicate a small "nearly" persistent current in $^3\text{He-A}$ with a measurable rate of decay (1%/day).

Transformation Kinetics

There exists a serious difficulty in understanding nucleation of the transition from the A to the B phase: the free energy difference between the phases is very small and the interface energy is rather large (\approx superfluid condensation energy in a layer of thickness ξ_0). These features give a predicted nucleation rate for thermal activation that is 100 orders of magnitude smaller than is seen. The discrepancy is not so severe for the reverse transition because small remnants of the A phase may persist within the B phase in corners, etc. Cosmic rays were suggested to be the cause, but experiments did not establish a correlation between nucleation and the cosmic ray flux.

Once nucleated, the interface will travel across the liquid. In most first-order transitions, the speed of the interface is determined by thermal transport. However, the A \rightarrow B transition can supercool so much that this mechanism does not give a finite velocity. Instead, the speed seems to be determined by the momentum transfer of quasiparticles scattering from the interface.

Dynamics of Quasiparticles

An inhomogeneous order parameter is a very weak perturbation on a quasiparticle whose structure is determined by energies on the order of the Fermi

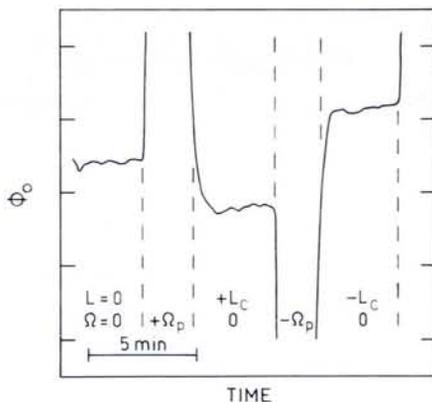


Fig. 5 — Primary data of a gyroscopic persistent current measurement in $^3\text{He-B}$. This experiment was carried out in a rotating cryostat which was alternately spun to angular velocities of $+\Omega_p$ and $-\Omega_p$. The torus experienced a torque during rotation owing to the Coriolis effect; this is seen in the figure as a huge "off-scale" shift in the amplitude of vibration ϕ_0 of the gyroscope. When the cryostat is again brought back to rest, ϕ_0 is determined by the gyroscopic tilting of the torus owing to the persistent angular momentum, L_c . The vertical difference of ϕ_0 after $+\Omega_p$ and $-\Omega_p$ rotations is proportional to L_c [10].

energy $k_B T_F$ where k_B is Boltzman's constant. The momentum p_F of a quasiparticle therefore cannot be changed drastically by the order parameter, in contrast to scattering from a solid wall. Instead, so called Andreev reflection takes place: an incoming particle-type excitation combines with another particle forming a Cooper pair, and a hole-type excitation is left travelling in the opposite direction [11]. The momentum change in Andreev reflection is only on the order of $(T_c/T_F)p_F$.

The number of excitations in the B phase decreases exponentially at low temperatures because of the finite energy gap Δ . Collisions become infrequent, and below $0.3 T_c$ the mean free path of the quasiparticles exceeds the dimensions of a typical experimental chamber. In this limit, hydrodynamic concepts such as viscosity and thermal conductivity need to be re-examined.

Experiments at low temperature also allow studies with ballistic quasiparticle beams. For example, Andreev reflection might be demonstrated directly. Secondly, the A phase has two nodes in the energy gap, a feature that is predicted to lead to unusual hydrodynamic behaviour at low temperatures.

The ballistic regime has been studied at Lancaster University using vibrating resonators as experimental probes. Substantially lower temperatures ($\approx 100 \mu\text{K}$) than before have been reached with a specially designed nuclear refri-

gerator [12]. The resonators are typically superconducting wires bent into semicircles with the ends anchored solidly. Quasiparticles can be generated by vibrating the wires at high enough velocities; they can be detected by similar wires that sense the incoming quasiparticle "wind". Several unexpected findings were made, many of which are still unexplained. For example, emission takes place when the velocity of the wire is one quarter of the theoretical (Landau) velocity. A factor of one-half can perhaps be accounted for by back-flow around the object, but the problem of the other factor of one-half remains.

Conclusions

The greatest impact of superfluid ^3He research arises because this exotic substance provides a concrete example of a very complicated system that can still be rather well understood theoretically using a reasonable amount of computation. There remains, however, an important challenge for theory to resolve the remaining unexplained experimental results. Nevertheless, the various superfluid phases of helium allow measurements of several parameters that are difficult to extract from normal state measurements. These can be used to test theories aiming at a solution of the many-body problem.

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