Superformed Bands in Atomic Nuclei

A Fascinating Recent Discovery

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The recent discovery of a new type of exotic state, the so-called superformed (SD) state in rapidly rotating, extremely distorted atomic nuclei has ushered in a new area of physics called high-spin spectroscopy. The measurement of discrete SD states at angular momenta lying in the 40 to 60 $\hbar$ range is fast becoming a standard technique in nuclear γ-ray spectroscopy thanks to improved experimental skills and the introduction of multi-detector arrays.

The term "superdeformation" was first used by Cohen, Plasil and Swiatecki [1] in their study of nuclear shapes at equilibrium in the presence of a fast rotation. The analysis was based on a model of a rotating liquid drop and the predicted SD states were related to the Jacobi instability known from the behaviour of astronomical objects such as stars, planets and asteroids.

The analogy between astronomical objects and atomic nuclei should not, however, be pushed too far. First of all, the gravitational forces active in the former are of a long range (as compared to the dimensions of a system) whereas the nuclear forces are of a short range. Second, astronomical objects are governed to a very good approximation by the laws of classical mechanics while the atomic nucleus is a domain of quantum phenomena. Nevertheless, the fascinating discovery of SD states in nuclei seemed to prove the existence of analogous phenomena in systems differing greatly in their sizes and masses.

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The authors' research involves nuclear theory, primarily structure under conditions of fast rotation.

Origin of Superdeformed States

In order to understand superdeformation it is useful to recall some of the characteristic features of nuclear behaviour under extreme conditions such as rapid rotation or considerable distortion of nuclear shape. To a first approximation, the rapid rotation of an atomic nucleus may be considered as the purely classical (*i.e.* nonquantized) motion of a portion of nuclear matter that disregards the individual nuclear degrees of freedom. It is well known that a portion of a classical liquid which was originally spherical in shape changes under rotation into an oblate distribution of matter around the axis of rotation. Such a shape can be approximated by an axially symmetric spheroid. However, at very high angular momenta the oblate shape may become unstable, favouring a triaxial (Jacobi) ellipsoid and eventually a strongly prolate shape. It turns out that a similar transition may take place in rotating nuclei. The Jacobi instability in nuclei is thought to occur in light, medium, and heavy nuclei (say, roughly up to $A \approx 250$) and at angular momenta ranging from $I = 0$ up to $I \approx 100\,\hbar$ (in the region of $A \approx 150$).

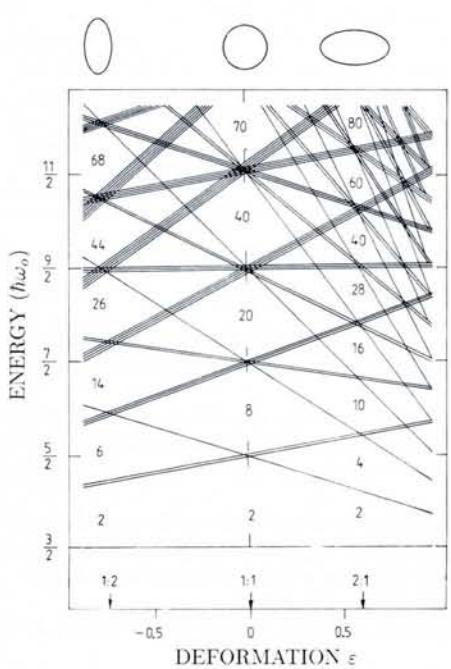

Fig. 1 — Single particle level spectrum of an axially symmetric harmonic oscillator as a function of the quadrupole deformation ε. Orbital degeneracies (N-$n_z + 1$) are indicated by the parallel lines and the numbers 2, 4, 6 etc. inside the box are the magic numbers. The arrows mark the characteristic deformations corresponding to a $\omega_\perp/\omega_z$ ratio of 1/2, 1/1 and 2/1.

Shell Structure

Another and perhaps more important reason for the formation of nuclear SD states stems from the essentially quantum nature of the nucleus which manifests itself as a shell structure. This nature is best understood in terms of an irregular distribution of the quantized energy levels in the nucleus exhibiting bunches of close-lying states.

The nuclear "shell" structure successfully explains most nuclear features such as the occurrence of especially stable systems ("magic" nuclei), the abundance of isotopes in nature, *etc.* Crucial in the search for the origin of superdeformation is the observation that a pronounced shell structure may also appear in the extreme condition of very large distortion. It transpires that some idealized many-body quantum systems may favour very distorted shapes in addition to the shell structured spherical shape. Consider, for example, the axially deformed harmonic oscillator with frequencies $\hbar\omega_z$ and $\hbar\omega_\perp$, the eigenstates of which (shown in Fig. 1) depend on the two quantum numbers N and n_z:

$$e(N, n_z) = \hbar\omega_0[N + 3/2 - \varepsilon(3n_z - N)/3]$$

where $\varepsilon = (\omega_\perp - \omega_z)/\omega_0$ is the deformation parameter and $\omega_0 = \frac{1}{3}(2\omega_\perp + \omega_z)$. The relation implies that for any

shape with $\omega_\perp/\omega_z = p/q$ the single particle levels show degeneracies if p and q are integer numbers (note that the ratio of the harmonic oscillator frequencies is the inverse of the ratio of axes defining the equipotential surfaces). This situation occurs for the "magic" numbers of particles corresponding to configurations with filled "shells" *e.g.* when the spherical magic numbers ($p/q = 1$) are 2, 8, 20, 40, 70... One can see in Fig. 1 that especially large degeneracies arise for $p/q = 2$ corresponding to a very elongated shape with an axis ratio of 1:2 ($\varepsilon = 0.6$) where the SD magic numbers are 2, 4, 10, 16, 28...

Analogues

The occurrence of similar values (1:2) for the ratios of axes was postulated several years ago for some nuclear configurations, notably certain actinide nuclei, in connection with the discovery of spontaneously fissioning nuclear isomers. It is believed that the SD states discovered up to now in the rare earth region have the same origin. However, while the 1:2 shape in the actinide nuclei is essentially related to the close vicinity of the fission channel, in the SD rare earth nuclides the large distortion is partly due to fast rotation superimposed on a favourable shell structure.

Analogues of the SD states can also be found in very light nuclei. The 7.66 MeV resonant state in the ^{12}C nucleus which is well-known in astrophysics may demonstrate the existence of a very elongated intrinsic shape (roughly speaking built up from three α-particles in a row). This should be contrasted with the ground-state of ^{12}C that corresponds to an oblate distribution of matter (three α-particles in a plane).

The single-particle nuclear spectrum calculated using realistic average potentials does not, of course, have a simple axially symmetric harmonic oscillator structure. Fig. 2 illustrates the independent-nucleon energy levels for the Woods-Saxon average potential plotted as a function of the nuclear quadrupole deformation β_2 (roughly proportional to ε) [2]. Regions having a low density of levels exist at specific particle numbers, *e.g.* at neutron number $N = 86$ and proton numbers $Z = 64,66$ at $\beta_2 \approx 0.6$. Thus gadolinium and dysprosium nuclei with $N \approx 86$ have been proposed since 1976 [3] as being good candidates for observing low-lying SD states.

Discovery of Superdeformed States

The first reliable indication of the presence of high-spin nuclear states at very large deformations was obtained

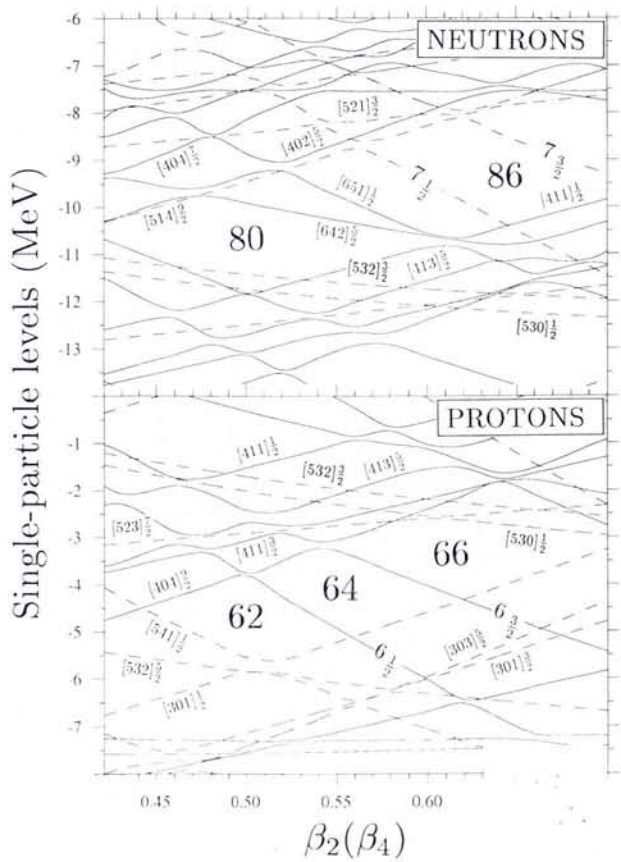

Fig. 2 — Single particle Woods-Saxon levels for neutrons (top) and protons (bottom) plotted versus the quadrupole deformation β_2 [2]. The normal parity orbitals are labelled by means of the asymptotic oscillator quantum numbers $[Nn_z\Delta]\Omega$ while the intruder states are labelled using the principal oscillator quantum number N and the Ω quantum number.

from experiments using the γ-γ energy correlation method in quasicontinuum spectroscopy. A compound nucleus is formed at very high angular momentum as the product of a reaction with heavy ions. A few particles (usually neutrons) then evaporate to give the low-temperature, high angular momentum state that de-excites by radiating several γ-rays: energy correlations between two γ-lines are observed.

The energy of a nucleus is given by the well-known rotational formula

$$E_I = \hbar^2 I(I+1)/2J$$

where I denotes the nuclear angular momentum in units of $\hbar$ and J the moment of inertia of a nucleus. The energy difference between two γ-rays coming from adjacent transitions in the rotational band is related to the nuclear moment of inertia as follows

$$\Delta R_\gamma = (E_{I+2}-E_I)-(E_I-E_{I-2}) = 4\hbar^2/J$$

A two-dimensional plot of γ-γ correlations ($E_{\gamma 1}$, $E_{\gamma 2}$) consequently shows two ridges lying parallel to the main diagonal and separated by a distance $4\hbar^2/J$. Moreover, since J is a sensitive function of the nuclear deformation one can also determine β_2 in the states that emit the radiation.

The γ-γ correlation experiments performed in 1983 to 1984 for the nucleus ^{152}Dy ($Z = 66$, $N = 86$!) yielded two ridges separated by a very small distance indicating a very small moment of inertia for certain nuclear states. The estimated value of β_2 was about 0.6, *i.e.* in agreement with the predicted value for superdeformation.

Escape-suppressed detectors

The discovery of superdeformed states was made possible by the introduction of escape-suppressed spectrometers that revolutionised the field of γ-ray spectroscopy. These multi-detector systems consist of germanium detectors (having a very high energy resolution for γ-rays) surrounded by scintillator shields (made of NaI, BGO or BaF_2 scintillators) operating in anti-coincidence to reduce the Compton background. One of the first of these new-generation spectrometers, the Daresbury detector TESSA2, was used for the quasicontinuum experiments [4].

A real turning-point was, however, the direct observation of a discrete SD rotational band in ^{152}Dy by the Daresbury-Liverpool-Copenhagen group in 1986 [5] using TESSA3. The γ-ray spectrum of the band is shown in Fig. 3: the energy difference ΔE_γ between two

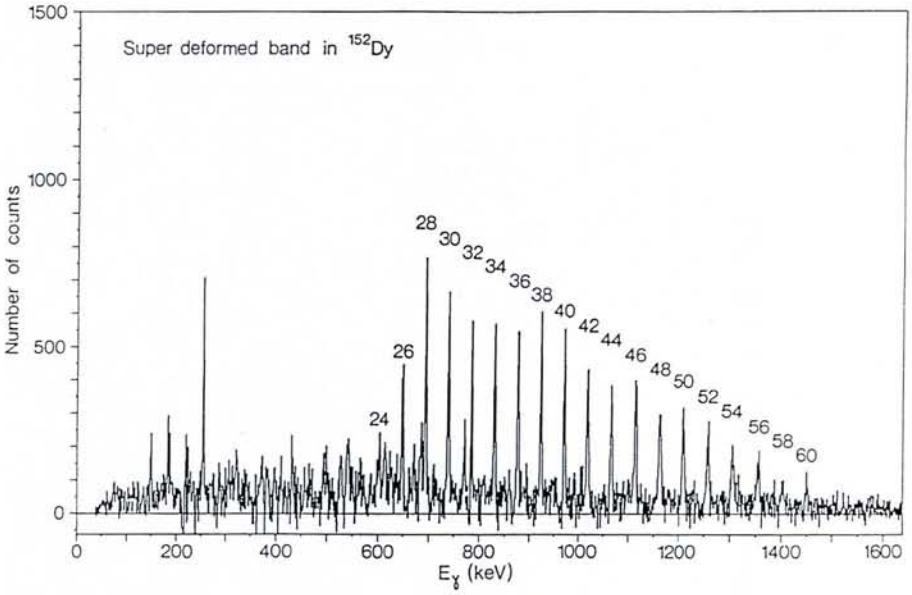

Fig. 3 — A spectrum of the SD band in ^{152}Dy: the numbers indicate the angular momentum as a multiple of $\hbar$ (from Ref. 5).

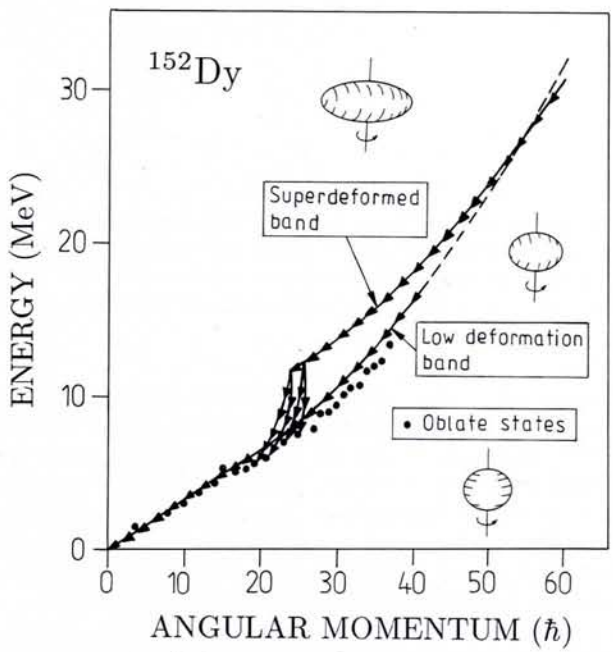

Fig. 4 — Near-yrast states in ^{152}Dy demonstrating nucleus shape coexistence.

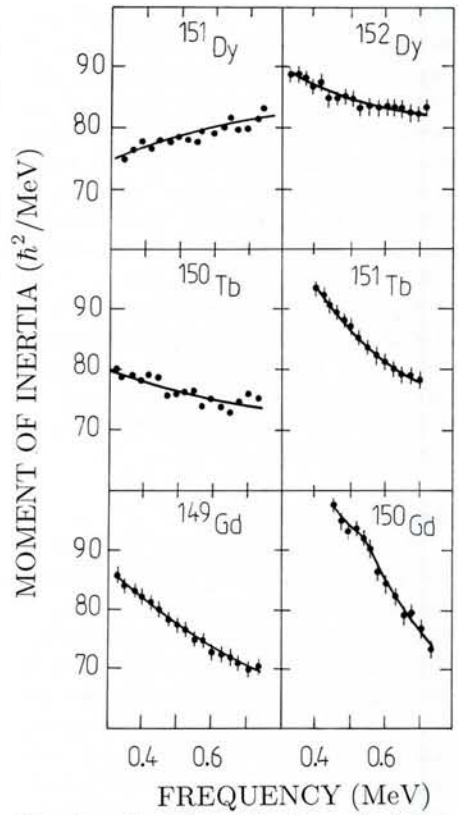

Fig. 5 — Experimentally determined moments of inertia in several SD bands.

adjacent γ-rays is almost constant and equal to 47 keV ($J \approx 85\ \hbar^2$/MeV, $\beta_2 \approx$ 0.6). This result is in perfect agreement with the estimate derived from the γ-γ correlations.

A nucleus with an angular momentum of 60 $\hbar$ makes about 10^{20} rotations per second. It emits the very fast electric quadrupole (E2) γ-radiation on slowing down. In 1987 it became possible to determine the reduced transition probability B(E2) within a SD band in ^{152}Dy [6]. The quadrupole transitions connecting SD states were found to be super-collective, of the order of 2500 Weisskopf units. The value of β_2 deduced from the transition quadrupole moment Q_t ($\approx$ 19 eV) *via* the proportionality $Q_t^2 \sim B$(E2) was in agreement with previous estimates. It should be noted that typical values of Q_t in rotational bands of *normal* deformation for rare earth nuclei are only 5-7 eV.

SD rotational bands have now been observed in nine rare earth nuclei ($^{146, 148, 149, 150}$Gd, $^{150, 151}$Tb, $^{151, 152, 153}$Dy). In addition, the transition quadrupole moments of SD bands have been measured for $^{149, 150}$Gd and ^{152}Dy.

Nuclei in Superdeformed States

The high-spin spectrum of ^{152}Dy is one of the most spectacular examples of nuclear shape coexistence. The energies of known near-yrast states (*i.e.* of those states that appear at the lowest energy at a given angular momentum) in ^{152}Dy are plotted in Fig. 4 as a function of the angular momentum. At low rates of spin the yrast states are very weakly deformed. In the region $20 < I < 40\ \hbar$ the yrast line has a non-collective character where the lowest states can be interpreted as many-particle, many-hole excitations of a weakly deformed oblate shape ($-0.2 < \beta_2 < -0.1$). The angular momentum in these structures is almost perfectly aligned along the symmetry axis of the nucleus.

For $20 < I < 50\ \hbar$ the collective band appearing close to yrast can be interpreted as an elongated triaxial configuration with $\beta_2 \approx 0.3$. The SD band with $\beta_2 \approx 0.6$ has a lower energy at $I \geq 54\ \hbar$. In the spin region $22 < I < 40\ \hbar$, three different shapes of the nucleus (oblate, near-prolate or triaxial and SD) coexist.

The SD bands found in several nuclei close to ^{152}Dy have revealed many unexpected features. One anticipated rather similar, rigid rotor like, rotational behaviour for such strongly deformed systems, but it transpires that the observed rotational patterns show pronounced variations with both particle number and angular momentum. The experimentally determined moments of inertia for SD bands in several rare-earth nuclei are plotted in Fig. 5 as a function of the rotational frequency $\hbar\omega$ defined using the standard canonical relation $\hbar\omega = dE/dI \approx E_\gamma/2$. While in $^{151, 152}$Dy and ^{150}Tb the moments of inertia are fairly constant, in the remaining isotopes J shows a significant dependence

on ω. In particular, the observed variations of the moment of inertia for ^{150}Gd are much larger than those seen in the neighbouring nuclei.

Intruder orbitals

In the presence of rotation, nuclear configurations can be labeled using the parity (π) and signature (r) quantum numbers. The latter symmetry is related to the invariance of the intrinsic nuclear Hamiltonian with respect to rotation by 180° about the axis of collective rotation. To distinguish between configurations belonging to the same (π,r) family, approximate quantum numbers must be introduced. Calculations based on the deformed shell model theory (see *e.g.* [2, 7] explain many of the observed properties of SD bands in terms of characteristic *intruder* orbitals originating from the high-N oscillator shells as they approach the Fermi surface at large deformation. For the nuclei of interest the intruder orbitals are the $N = 6$ proton states (originating from the $1i_{13/2}$ subshell) and the $N = 7$ neutron states (originating from the $1j_{15/2}$ subshell) — see Fig. 2. Because of their large intrinsic angular momenta these states are strongly influenced by the Coriolis force. The Lund group [7] has demonstrated that the observed variations in the moments of inertia in the SD bands shown in Fig. 5 can be attributed to the number of occupied high-N states. The different high-N occupation levels also account for the variation in deformation with particle number.

The intruder orbitals that become occupied in the SD bands lie at energies which are much above ($\approx$ 8-10 MeV) the Fermi surface for typical ground-state deformations. SD band spectroscopy therefore offers the unique possibility of probing relatively well-defined shell model configurations at extreme conditions. By extracting the positions of high-N levels at large deformations and extrapolating them back to normally deformed or spherical shapes we may in the future learn about the energy of high-j subshells that cannot be accessed otherwise.

Superfluidity

It is well known that the nuclear moments of inertia at low spins are much smaller than the corresponding rigid-body values. The reason is the short-range, two-body residual interaction that couples pairs of nucleons in time-reversed orbits. This interaction, known as *pairing*, leads to nuclear superfluidity characterized by the order parameter Δ, the pairing gap. At high spins, the Coriolis force tries to align the single particle angular momenta along the axis of nuclear rotation. It is therefore expected that nuclear superfluidity should break down at very high rotational frequencies. This phenomenon, known as the nuclear Meissner effect, was predicted in 1960 by Mottelson and Valatin. The near-rigid rotational pattern of SD bands suggests that the examination of SD states can give information about nuclear superfluidity under extreme conditions.

Owing to the very large $Z = 66$ and $N = 86$ single-particle gaps occurring at similar deformations (see Fig. 2) the SD configuration in the ^{152}Dy band can be understood in terms of a *doubly magic* structure. Superfluid-type correlations in this band are strongly quenched and are mainly dynamical (*i.e.* the pairing interaction cannot form a boson condensate) because the density of single particle states is very low. Conquently, the moment of inertia in ^{152}Dy is very close to the rigid-body limit and is almost independent of ω. This contrasts with the dramatic variation in the observed moment of inertia of ^{150}Gd (see Fig. 5) associated with the presence of quasiparticle excitations characteristic of the superfluid phase [2].

Perspectives

Without deprecating the unquestionable experimental success of γ-ray spectrometry, one has to stress that prediction of the presence and properties of high-spin SD states preceded the discovery of these states by almost a decade. The large amount of high-spin discrete spectroscopy data on SD states that has now been accumulated makes it possible, for the first time, to examine fine details of the underlying shell structure.

Recent experimental discoveries have greatly stimulated theoretical work on the nature of very elongated nuclear shapes. Although some questions concerning the structure of SD states have already been answered, many problems remain. One of the most puzzling is the mechanism by which the SD bands are populated. Why are special states so favoured among the many possible configurations that appear at excitation energies around 8 MeV (a typical neutron separation energy)? On the other hand, we know that as soon as the SD states are formed the relative in-band intensity remains almost constant. This result suggests that they hardly mix with the many close-lying levels because of their very distinct intrinsic structures.

Another interesting problem is the rapid depopulation of the SD bands (in none of the observed cases has the decay path from the SD band been established experimentally!). Calculations have given relatively detailed predictions of features of *excited* 1:2 states only, such as the higher lying rotational bands and the low and high frequency collective vibrations, all of which are expected to have very different properties as compared to the case for "normal" deformation.

Theory shows that very elongated structures should also exist in other regions of the periodic table, *e.g.* in $A \approx 80$ and $A \approx 190$ nuclei. It has also been suggested that even more elongated systems with 1:3 shapes ($\beta_2 \cong 1.1$!) arise. It will be interesting to see of these predictions prove to be as successful as those made in the 1970's.

Proposals for a new generation of spectrometer arrays are being discussed in the USA (GAMMASPHERE) and Europe (EUROBALL). These magnificent instruments will certainly be able to shed light on many basic aspects of the physics of superdeformation, and reveal new phenomena that will invite further study.

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Reactor Restarted

The ILL reactor was brought back into normal operation just before Easter after a 2½ month shut-down ordered by the French nuclear safety authorities following ILL's chance discovery that the reactor had been working since its commissioning in 1970 at about 10% above the authorised power limit.

The backlog of experiments will be whittled away by the year's end. Meanwhile, proposals for reassigning responsibility for reactor operation are being considered by the authorities, together with reports of safety studies. The reactor is modular in design and with component replacement, theoretically has a infinite life. Few components need early replacement in view of the higher than expected dose levels.