Cell Mapping

The above discussion emphasizes the importance of investigating the global behaviour of the system equation (for example Eq. (2)), *i.e.* with initial conditions everywhere in the phase plane. With each initial condition the system may — depending on the damping parameter — require the integration of typically 500-1000 RF cycles before the steady state is obtained with certainty; furthermore, since a reasonable coverage of initial conditions typically requires a 100×100 element grid, the computer time for such an investigation becomes very expensive indeed.

A solution to this problem recently proposed is a method called cell to cell mapping [5]. In that method the phase plane is divided into, for example, a two dimensional 100×100 grid of cells. With given initial conditions in the centre of one cell, say *(i, j)*, the system is integrated for one RF cycle and the final coordinates are noted. These coordinates belong to another cell which we denote *(m, n)*. Thus we have defined a mapping of cell *(i, j)* on to cell *(m, n)*. The mapping of all cells in the grid is investigated that way, each cell requiring the integration of one RF cycle. The rest of the analysis is sorting, a process that is relatively inexpensive in computer time. For example fixpoints (periodic solutions) are found by asking if some cells map onto themselves (possibly after several iterations). The corresponding basin of attraction for each fixpoint is found by tracing backwards the iterations leading to the fixpoint. Specially interesting regions in phase space can be investigated by doing a local cell mapping over a smaller region.

Fig. 3 shows as an example a cell mapping computed with a 75×75 grid of cells. The parameters are chosen such that the system is chaotic with a strange attractor in the phase plane. As demonstrated in the figure, the (period 7) attractor found by cell mapping provides an approximation to the strange attractor which has been obtained by direct numerical calculation. The basin found by cell mapping is almost identical to the basin of attraction for the strange attractor. Thus the cell mapping method gives a quick overview of the global behaviour of the system, with a minimum of computer time.

Other phenomena that require knowledge about the different attractors and their basins of attraction are the so-called crises, of which an interior crisis and a boundary crisis have been clearly identified in Josephson junction systems.

A crisis results when increasing a system parameter to a critical value causes a strange attractor to collide with an unstable periodic orbit in phase space. It is called a boundary crisis if the chaotic attractor disappears as a result of the collision. Instead it may undergo a sudden expansion, in which case it is referred to as an interior crisis. A crisis may also cause intermittency and thus give rise to approximately $1/f$ noise behaviour. Crises can be observed directly by a simple measurement of the current-voltage curve of a Josephson junction irradiated with microwaves. For example, the interior crisis typically manifests itself as a gradual loss of phaselock on the so-called RF-induced current steps at harmonics of the voltage $\hbar\omega/2e$. Since the voltage of these steps are used as a voltage standard with a precision of about 10^{-7}, any small loss of phaselock will have dire consequences. Thus any voltage standard based on Josephson junctions must be carefully dimensioned so as to avoid chaos. For the new types of voltage standards with many (hundreds of) series connected junctions, this is a non-trivial problem.

Conclusion

As I hope to have demonstrated, the Josephson junction is a marvellous device with many practical applications. It is at the same time a model system for all types of chaos. Many practical applications typically involve operating close to the onset of chaos, and the detailed understanding of chaos in Josephson junctions is thus important. Since "chaotic noise" and thermal noise have approximately the same amplitude, and since their interaction is highly non-linear, such an understanding is by no means trivial to obtain.

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Anniversary Seminar

In honour of the Past Presidents and the Honorary Members of EPS, the 20th anniversary of the founding of the Society and the 90th anniversary of the founding of the Italian Physical Society are being celebrated in a Seminar on "Past and Future of Physics in Europe", appropriately in Pisa on 18/19 June 1988. The meeting is open to EPS Members; no registration fee will be charged.

The Seminar which follows a meeting of the EPS Executive Committee is being organized by the new EPS President, Professor R.A. Ricci (see page 56). Further information may be obtained from him through the EPS Secretariat in Geneva.