Non-classical Light

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Antibunching and squeezing are characteristics of light beams that cannot be explained by classical theory. There is considerable current interest in developing sources of non-classical light and understanding its basic properties.

Direct manifestations of the quantum nature of light are difficult to find, despite the leading role played by Planck's quantization hypothesis in initiating the development of quantum mechanics. It might be expected that the photon aspects of light would be revealed through particle-like behaviour in straightforward optical detection experiments. However, light can only be detected via the intermediary of some electronic process, whose own particle-like nature obscures the character of the optical field. It turns out that only certain kinds of light are capable of displaying their non-classical properties in optical detection experiments, and manifestations of the quantum effects are relatively subtle.

Photodetection Noise

The photoelectric detection of light occurs by the excitation of electrons from bound states to free states. The randomness in the times of occurrence of the electronic transitions produces corresponding statistical fluctuations in the photocurrent, which exhibits a form of shot noise. These fluctuations are correctly predicted by the semi-classical model in which the active material of the photodetector is treated quantum mechanically, but the light is treated as a coherent classical electromagnetic cosine wave. There is no need to take account of quantization of the light.

If this quantization *is* taken into account, then the same photodetection process can be treated by a fully quantum-mechanical theory. The classical cosine wave has a quantum analogue known as the coherent state, which is closely approximated by the light of a single-mode laser operated well above threshold. The quantum theory for photodetection of coherent-state light predicts exactly the same shot-noise photocurrent fluctuations as does the semi-classical theory.

The basic origin of the shot noise is however different in the two models. In the semi-classical model, it is a consequence of the discreteness of the photoelectric generation of free electrons. In the quantum model, the shot noise is a property of the light itself that is transferred to the electrons in the photodetection process. This distinction may seem to be of purely academic interest since the two models predict no differences in experimental results. However, differences do arise when other kinds of light are considered. According to the semi-classical model, the shot noise is the *minimum* current fluctuation that can occur for any kind of light, since the noise is an intrinsic property of the photoelectric effect. The minimum is realized for a classical coherent cosine wave, while light that has additional classical fluctuations produces photocurrent noise additional to the shot noise.

Such fluctuations can also be treated in the quantum model, and they likewise give rise to excess noise in the photocurrent. They are sometimes referred to as *photon bunching*, since the effect can be pictured as a tendency of the photons, or more accurately the photodetections, to occur in temporal clusters. However, the quantum model can also accommodate optical fluctuations that are below the level that characterises coherent state light. A very simple example, although difficult to produce experimentally, is an optical excitation that contains exactly n photons. This clearly has a zero uncertainty or fluctuation in photon number, and it produces a photocurrent whose noise is less than the classical shot-noise limit. Other, more easily realisable kinds of light exhibit the same property, referred to as *photon antibunching* since the photodetections occur with greater regularity than they do for the purely random shot noise.

The field of quantum optics is mainly concerned with phenomena that are embraced by the quantum theory of light, but are not allowed in semi-classical theory. The challenge of the field is sharpened by the tendency for optical detection processes to cause both models to predict the same observations. However, clear-cut observations of quantum phenomena have recently been made, and there is considerable current interest in developing sources of non-classical light with sufficient strength for more precise measurements and for various applications. In order to understand these observations, we need to look at optical detection in a little more detail.

Direct Detection

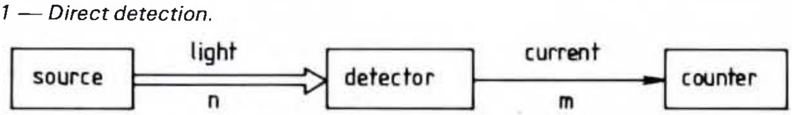
Fig. 1 shows the simplest kind of photon-statistical experiment in which light of variable photon-number n falls directly on a photodetector. Suitable processing of the photocurrent can provide the statistical distribution of the number m of photocounts recorded during repeated time intervals of duration T. We assume that T is much shorter than any characteristic fluctuation time of the incoming light, and the photocount fluctuations then provide a faithful record of the optical fluctuations. In particular, it can be shown that the normalised second factorial moments of the two distributions are equal,

$$\langle n(n-1)\rangle / \langle n\rangle^2 = \langle m(m-1)\rangle / \langle m\rangle^2. \qquad (1)$$

The angle brackets on the left denote averages over the photon number distribution of the light, while those on the right denote averages over the photocount distribution. Normalisation by the square of the mean removes the quantum efficiency of the detector, which is often considerably less than unity in practice.

Fig. 2 is a pictorial representation of the quantum-mechanical coherent state. The large arrow represents the complex electric field; its length in suitable field units is equal to the square root of the average photon number. Its phase angle increases linearly with the time to produce a real field whose average value oscillates with a cosine time-dependence. A classical cosine wave can be represented by the same complex-field arrow. However, the quantum light necessarily has some uncertainty, and for the coherent state, this is represented by the circle attached to the tip of the arrow. The uncertainty can be resolved as shown into equal phase and amplitude contributions, each of magnitude ½, independent of the average photon number. The classical wave is approached for every large $\langle n\rangle$, when the uncertainties become relatively insignificant.

Fig. 1 — Direct detection.



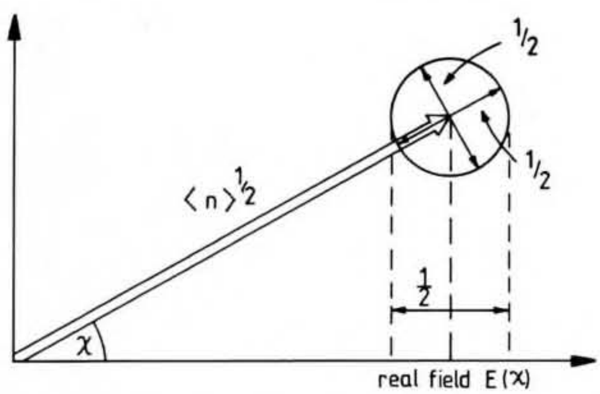


Fig. 2 — Coherent state complex field and uncertainty circle.

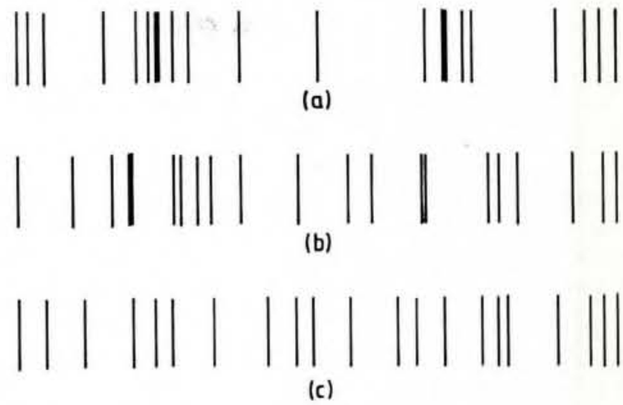


Fig. 3 — Photocount sequences for light that is (a) bunched, (b) coherent, and (c) antibunched.

The mean and fluctuation of the photon number are the important quantities for direct detection. The detected quantity is essentially the square of the field amplitude, and for the geometry of Fig. 2, this is

$$(<n>^{½} \pm ¼)^2$$
$$= <n> \pm ½<n>^{½} + 1/16. \quad (2)$$

Thus for a photon number sufficiently large that the final term on the right can be neglected, the photon-number uncertainty is

$$\Delta n = <n>^{½}. \quad (3)$$

The photon-number variance is correspondingly

$$<n^2> - <n>^2 \equiv (\Delta n)^2 = <n>, \quad (4)$$

and substitution of this result produces unity on the left-hand side of (1), so that the photocount variance is

$$(\Delta m)^2 = <m>. \quad (5)$$

Variances equal to the mean values of distribution are characteristic of Poisson random processes, and these expressions demonstrate the transfer of the shot-noise fluctuation (4) in the light to the photocount shot noise (5) according to the quantum theory. It is the same shot noise (5) that arises in semi-classical theory from the statistics of the production of photoelectrons by light that has no fluctuations at all.

Now consider a light beam that has exactly n photons, so that the photon-number variance vanishes,

$$(\Delta n)^2 = 0. \quad (6)$$

Rearrangement of (1) gives an expression for the photocount variance

$$(\Delta m)^2 = <m>[1- <m>/n] \quad (7)$$

These are smaller than the shot-noise variances, (4) and (5) respectively, corresponding to probability distributions that are narrower than Poisson distributions with the same mean values. The light is said to have *sub-Poisson* statistics or alternatively is described as being antibunched. Light with these characteristics falls outside the scope of classical theory. Fig. 3 represents the sequences of photocounts for light that is bunched, coherent, and antibunched.

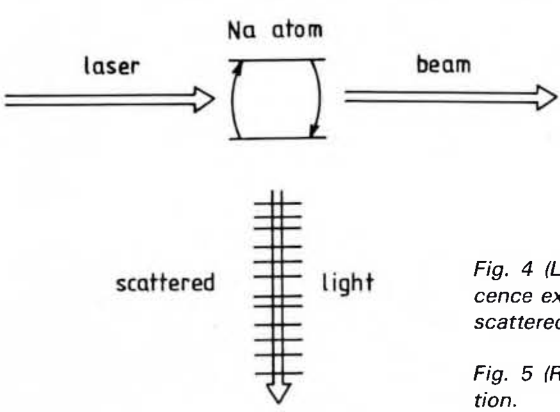
The observation of antibunched light is clearly of great interest in view of the direct evidence that it provides for the presence of optical quantization, but experiments in this area are very difficult. Thus, although many processes, particularly in non-linear optics, are predicted to produce antibunched light, the departures of the detection statistics from the Poisson distribution characteristic of coherent light are usually very small. The most extensive measurements to date use resonance fluorescence scattering, the theoretical and experimental work having been carried out by Cohen-Tannoudji, Walls and Mandel and their collaborators.

The bare bones of the experiment are represented in Fig. 4. An excited state of a single sodium atom is resonantly excited by a beam of coherent laser light. Spontaneous emission produces scattered light, which is observed at right-angles to the laser beam. With only one scattering atom, successive spontaneous emissions are separated by an average time determined by the radiative lifetime of the excited state. Thus photocounts of the scattered light are very unlikely to occur with time separations much smaller than this, and the photocount sequence has the form shown in Fig. 3 (c), corresponding to antibunched light.

Antibunching typically occurs in very weak light beams, making experimental detection difficult. In the case of resonance fluorescence, it is clear from Fig. 4 that the random superposition of scattered beams from several atoms will remove the desired effect. The experiments accordingly used a weak atomic beam so that the measurements were mainly made with a single atom in the field of view of the photodetector. The experimental results are in very good agreement with the detailed quantum theory of resonance scattering.

Homodyne Detection

Direct detection measures the photon-number, or intensity characteristics of a light beam, but provides no information on its phase properties. Phase-sensitive detection is exemplified by the



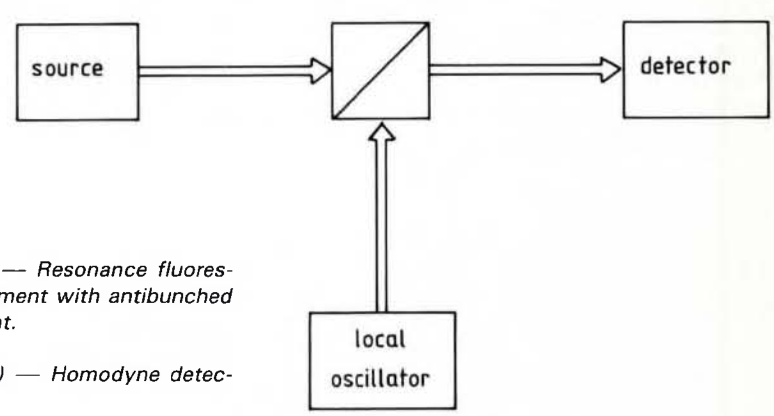


Fig. 4 (Left) — Resonance fluorescence experiment with antibunched scattered light.

Fig. 5 (Right) — Homodyne detection.

homodyne detector represented in Fig. 5. Here the electric field of coherent light of known properties supplied by a local-oscillator laser is superposed on the field of the signal light beam by a beam splitter. The superposed light falls on a photodetector, which essentially measures the square of the sum of the two fields. With equal frequencies for the signal and local oscillator, the homodyne detector has a DC output. The known local oscillator mean field is usually chosen to be much larger than that of the signal. The measurement then provides a value $E(\chi)$ for the signal field *via* the cross term in the square of the superposition. The phase angle χ of the measured field is determined by the known phase of the local oscillator field, which can be varied to study the phase dependence of $E(\chi)$.

The description of the previous paragraph applies equally to the classical and quantum-mechanical models of light, but differences again emerge when fluctuations are considered. From the quantum viewpoint, both the signal and local oscillator inputs contribute fluctuations to the measured superposition. However, the beam splitter can be so designed that almost 100% of the signal field is superposed on only a small fraction of the local oscillator field. Yet, this can be made so large that it nevertheless dominates the superposition (as assumed above) and its noise contribution is suppressed, since this is independent of amplitude, as shown by the uncertainty circle in Fig. 2. The homodyne measurement can thus be performed in such a way as to detect only the signal-field fluctuations $\Delta E(\chi)$.

Suppose first that the signal is itself in a coherent state of the kind represented in Fig. 2. The measured field and its uncertainty are represented by projections on to the horizontal axis. It is clear from the geometry of the figure that as the local oscillator phase is varied, the measured uncertainty is ½, independent of the angle χ. The signal is said to have phase-independent noise. The

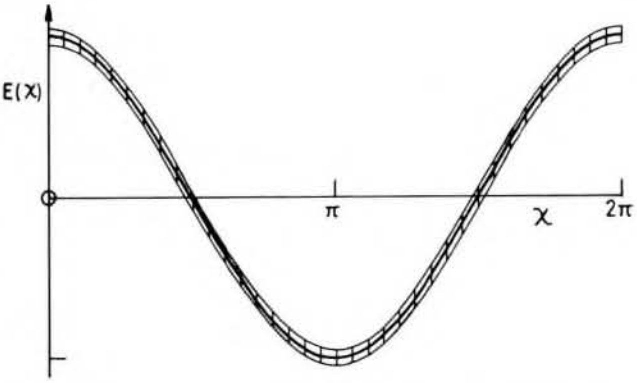


Fig. 6 — Phase dependence of the detected mean field and its uncertainty for a coherent signal.

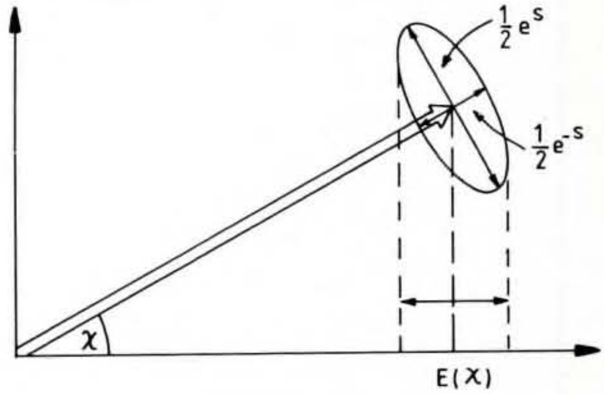


Fig. 7 — Squeezed state complex field and uncertainty ellipse.

phase variation of the mean signal field and its constant uncertainty are illustrated in Fig. 6. In the semi-classical theory of homodyne detection the same level of field fluctuation occurs and is ascribed to the shot noise from the photoelectric process.

The coherent state of Fig. 2 shows a special case of the distribution of uncertainty allowed more generally by quantum mechanics. It is possible to show that field measurements for different local oscillator phase angles must conform to an uncertainty relation,

$$\Delta E(\chi_1)\Delta E(\chi_2) \geq \tfrac{1}{4}\,|\sin(\chi_1-\chi_2)|. \quad (8)$$

The uncertainty of ½ for the coherent state is the minimum value allowed by quantum mechanics for phase-independent noise. More exotic states are possible when the noise is allowed to vary with the phase angle. It follows from (8) that the noise can be made arbitrarily small for a selected angle χ, but compensating increases in the noise then occur at other phase angles, particularly those at right-angles to χ. Light whose field uncertainty falls below ½ for some phase angles is said to be *squeezed*. Such uncertainties lie below the shot-noise limit of the semi-classical theory, and squeezed light is another example of an optical excitation that can only be described quantum mechanically.

Fig. 7 is a pictorial representation of a squeezed state, analogous to the coherent state picture in Fig. 2. The complex field amplitude and phase are the same but the uncertainty circle is now replaced by an ellipse, with axis lengths as shown. The positive quantity s is the squeezing parameter, and the coherent-state circle is recovered for $s = 0$. The change from the coherent state can be interpreted as a decrease in the amplitude uncertainty, compensated by an increase in the phase uncertainty. It is obvious from Fig. 7 that the uncertainty in a homodyne measurement of the real signal field, given by the projection of the ellipse on to the real axis, varies with phase angle χ. The form of this phase-dependent noise is illustrated in Fig. 8. It is seen that minimum noise occurs at the points of maximum amplitude in the mean signal field, and this is a consequence of the particular orientation of the uncertainty ellipse chosen in Fig. 7. With the field vector left unchanged, other orientations of the ellipse axes correspond to different combinations of phase and amplitude noise, and minimum field noise occurs at the phase angles χ for which the ellipse minor axis is parallel to the horizontal axis.

Resonance fluorescence scattering and many non-linear optical processes are capable in principle of generating squeezed light, but experiments are again difficult owing to the need for careful elimination of extraneous noise sources and the often small size of the predicted effect. The single successful observation to date used non-linear four-wave mixing in sodium, the theoretical and experimental work having been carried out by Shapiro, Walls, Yuen, and Slusher and their collaborators. This non-linear process employs the transfer of energy from two counter propagating laser beams to two other light beams generated with their frequencies symmetrically disposed on either side of the laser frequency. For appropriate phase angles, the generated light showed squeezing corresponding to a fluctuation strength 7% smaller than the classical shot-noise limit.

Conclusions

The study of non-classical light is clearly important for understanding the characteristic properties of quantum fields and their relation to the properties of the corresponding classical fields. There are also possible practical uses for the low noise figures achievable with non-classical light. Thus antibunched and squeezed light could offer advantages in direct-detection and coherent-detection optical communication systems, the latter having been subjected to detailed study by Yuen and Shapiro. The narrow phase or amplitude definition available with squeezed light has been evaluated by Caves as a means of achieving the extremely sensitive optical interferometry required for gravitational wave detection. The present need is to develop non-classical light sources of sufficient strength and variety to pursue these objectives.

FURTHER READING

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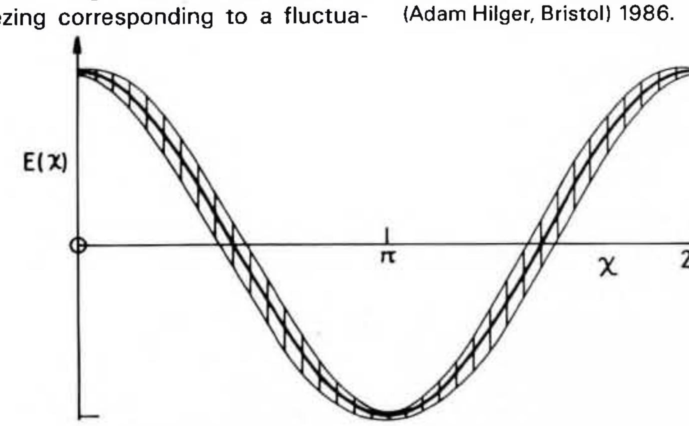

Fig. 8 — Phase dependence of the detected mean field and its uncertainty for a squeezed signal.