Macroscopic Quantum Phenomena in Josephson Elements

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Josephson's original paper on the effect named after him has long been a classic. Until recently, the experimental manifestations of Josephson tunnelling have also been classical in the sense that the phase difference across the junction could be regarded as a classical variable. In the past few years, however, progress in microfabrication and the understanding of the importance of external noise have enabled us to build Josephson circuits that no longer behave classically but instead, although macroscopic in size, display various quantum mechanical effects. For example, macroscopic quantum tunnelling and quantization of energy have been demonstrated experimentally. Other effects such as the existence of the superposition of wave functions on a macroscopic level and of Bloch oscillations have been predicted. The study of these

quantum effects was initiated by Leggett¹⁾ who questioned the validity of quantum mechanics on a macroscopic level. A central issue is the fact that as one begins to couple a macroscopic system to external or unspecified internal degrees of freedom, an unavoidable dissipation sets in. The major challenge to the theory has been the need to account for this dissipation within a rigorous quantum mechanical framework and to determine the influence of dissipation on various quantum phenomena²⁾.

In Fig. 1 we summarize the essential features of the classical Josephson effect. The single degree of freedom that characterizes the effect is the phase difference φ across the junction. By equating the currents flowing through the three components of the resistively shunted junction (Fig. 1b) to the sum of

the applied current and the Johnson noise current $\hat{I}(t)$ generated by the resistance, one finds:

$$C\hbar\dot{\varphi}/2e + (1/R)(\hbar\dot{\varphi}/2e) + \partial U(\varphi)/\partial(\hbar\varphi/2e) = \hat{I}(t) \quad (1)$$

The first term on the left-hand side arises from charging of the capacitance, the second from the dissipative current that flows through the resistor, and the third from the supercurrent $I_s = I_c \sin \varphi$ and the applied current I_x . For the moment, we have ignored the normal current due to tunnelling of quasiparticles. One gains considerable insight into the predictions of Eq. (1) by realizing that it is completely analogous to the equation for a particle moving in a tilted washboard potential. There is a one-to-one correspondence between the capacitance, phase difference, and charge for the junction and the mass, position, and momentum of the particle. The potential describing a current biased junction with $I_x < I_c$ is illustrated in Fig. 1c. In the stationary state $V = \dot{\varphi} = 0$, the junction (or particle in the local minimum) may also undergo oscillations with a frequency $\omega_p = \omega_{po} [1 - (I_x/I_c)^2]^{1/4}$ where the plasma frequency is $\omega_{po}^2 = 2eI_c/\hbar C$. The classical description is valid provided $k_B T \gg \hbar\omega_{po}$, where k_B is Boltzmann's constant and T is the absolute temperature.

It is obvious that Eq. (1) contains Planck's constant, $\hbar$, the presence of which underscores the quantum mechanical origin of the Josephson effect. However, each factor of $\hbar$ is divided by the charge of a Cooper pair, $2e$, and the quantity $\hbar/2e = \Phi_0/2\pi$ is not small; Eq. (1) is a classical equation of motion.

Provided one can ignore the dissipative term, the general quantum mechanical description is a straightforward extension of Eq. (1). The charge Q on the capacitor and (periodic functions of) the phase difference $\hbar\varphi/2e$ are treated as non-commuting variables, so that

$$Q = (\hbar/i) \partial/\partial(\hbar\varphi/2e) \quad (2)$$

The junction is then described by the Hamiltonian

$$H_0 = Q^2/2C + U(\varphi) \quad (3)$$

This description can be completely justified on the basis of microscopic theory

Fig. 1 — (a) A Josephson tunnel junction consists of two superconducting electrodes separated by an insulating barrier. The Cooper pairs in each electrode are described by a macroscopic wave function of the form $\Delta \exp(i\Phi)$. A supercurrent I_s , involving the tunnelling of Cooper pairs through the barrier, flows if there exists a phase difference $\varphi \equiv \varphi_R - \varphi_L$ across the barrier according to the relation $I_s = I_c \sin \varphi$; I_c is the critical current of the junction, the maximum supercurrent it can sustain. In the presence of a voltage V across the barrier, the phase difference evolves as $\dot{\varphi} = 2eV/\hbar$. The self-capacitance C of the junction allows a charge Q to accumulate.

In (b), an external shunt resistor R has been added to the junction. If we impose a current I_x , current conservation leads to Eq. (1), where the potential is $U(\varphi) = -(\hbar I_c/2e)\cos \varphi - I_x \hbar\varphi/2e$. This potential is plotted in (c) for $I_x < I_c$. The equation of motion, Eq. (1), is analogous to that for the motion of a particle in this tilted washboard potential. In (d), we show a superconducting loop interrupted by a Josephson tunnel junction. The potential is $U(\varphi) = -(\hbar I_c/2e)\cos \varphi - (\Phi - \Phi_x)^2/2L$, where Φ is the total magnetic flux threading the loop and Φ_x is the externally applied flux. Flux quantization leads to the constraint $\varphi/2\pi = \Phi/\Phi_0$, where $\Phi_0 \equiv \hbar/2e$ is the flux quantum.

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Fig. 2 — Observation of MQT. To observe MQT, the Josephson tunnel junction is cooled to millikelvin temperatures in a dilution refrigerator. Extensive filtering against room temperature noise is essential. The junction is current-biased in the zero voltage state with an appropriate tilt of the washboard in Fig. 1c so that the lifetime against tunnelling into the free running state has a reasonable value, say $1\mu\text{s}$ to 10ms . The elapsed time between the application of the current bias and the appearance of a voltage across the junction is measured. This process is repeated a large number of times (10^4 to 10^5) to obtain a distribution of lifetimes from which a mean value for the tunnelling rate is calculated. In one of the experiments ⁵⁾, the tunnelling rate was expressed as an escape temperature, T_{esc} , defined as $\Gamma = (\omega_p/2\pi) \exp(-\Delta U/k_B T_{\text{esc}})$ in both the classical and quantum regimes. In the classical limit, T_{esc} is equal to the bath temperature T , while in the quantum limit T_{esc} is a constant, independent of temperature (see the expressions for Γ_q and Γ_t). The measured values of T_{esc} are plotted vs. T ; the solid line is $T_{\text{esc}} = T$. As the temperature is lowered from 1K , the measured values of T_{esc} are initially equal to T , eventually flattening off to a constant value when MQT takes over from thermal activation. Using junction parameters determined in situ in the classical regime, the authors found excellent agreement between the measured low temperature value of T_{esc} and the $T = 0$ prediction indicated to the left of the ordinate axis.

³⁾ From this treatment, $\hbar\phi/2e$ and Q emerge naturally as quantum mechanical variables, provided one is justified in ignoring the dynamics of all the other degrees of freedom, for example, the magnitude of the order parameter.

Given the Hamiltonian of Eq. (3), we expect to observe all the quantum effects predicted by elementary quantum mechanics. For example, the junction prepared in a metastable minimum of the potential (Fig. 1c) can escape by a process that has become known as macroscopic quantum tunnelling (MQT) into the non-zero voltage state: In the parlance of the mechanical analogue, the particle has tunneled through the local potential maximum and runs freely

down the tilted washboard. In the WKB approximation, the tunnelling rate at $T=0$ is

$$\Gamma_q = a_q (\omega_p/2\pi) \exp(-7.2 \Delta U/\hbar\omega_p).$$

Here, a_q is a known prefactor and ΔU is the height of the barrier, which like ω_p is controlled by the bias current I_x . At a non-zero temperature, the junction may also escape by thermal activation over the barrier, a process which, in the classical limit and for moderate damping, occurs at a rate

$$\Gamma_t = (\omega_p/2\pi) \exp(-\Delta U/k_B T).$$

Thus to observe quantum tunnelling we require $\hbar\omega_p \gtrsim 7.2 k_B T$. For the junction parameters $C = 5\text{pF}$ and $I_c = 10\mu\text{A}$, this constraint implies $T \lesssim 60\text{mK}$; this temperature range is readily accessible in a dilution refrigerator.

It should be emphasized that in the process of macroscopic quantum tunnelling the state of the entire junction — characterized by ϕ — changes spontaneously as it makes a transition through a region of ϕ -space that is classically inaccessible. This effect has been observed experimentally by several groups ^{4,5)}. A description of one such experiment in which the effects of damping were negligible appears in Fig. 2. The results are in quantitative agreement with theoretical predictions.

Another quantum mechanical effect, namely the quantization of the energy in a bound state, has also been observed in a Josephson junction. Given the Hamiltonian of Eq. (3), we naturally expect the eigenstates of a particle confined to a well to have discrete energies. Strictly speaking, the potential $U(\phi)$ shown in Fig. 1c is not bounded below, and the eigenstates are not well defined. However, provided the period of oscillation in one of the minima, $2\pi/\omega_p$, is short compared with the lifetime $1/\Gamma_p$ against macroscopic quantum tunnelling out of the metastable state, one would expect to observe quasi-eigenstates. In other words, the wave packet centred at the minimum has only a tiny penetration under the barrier, and one neglects this leakage in the calculation of the energy levels. If the junction, prepared in a metastable state, is exposed to microwave radiation, one observes an enhancement in the escape rate from the zero voltage whenever the energy difference between two eigenstates coincides with the microwave frequency. Fig. 3 describes the experiment ⁶⁾ and shows one of the results; the agreement between the observed and predicted resonance is very good. It should be emphasized that the anharmonicity of the potential well causes the energy level splittings to decrease with increasing ener-

gy, producing distinct resonances, thereby enabling one to evade the correspondence limit. If the potential were harmonic, all the energy level splittings would be identical; one would observe only a single resonance, and one would not be able to distinguish between classical and quantum behaviour.

Several other macroscopic quantum effects have been suggested. One is the existence of the superposition principle on a macroscopic level in a system consisting of a superconducting loop interrupted by a single Josephson junction ¹⁾. For an applied flux of $\Phi_0/2$, the two absolute minima occur at the same potential energy, as indicated in Fig. 4. The degeneracy of their ground states is lifted by the tunnelling between the wells. The two lowest eigenstates, one symmetric the other antisymmetric, are separated in energy by an amount δ proportional to the tunnelling rate. If we prepare the system in the left-hand minimum, that is, in a superposition of two eigenstates, it should perform coherent oscillations between the two wells at a frequency $\delta/\hbar$. Unfortunately, the range of parameters for which the effect is observable is more restricted

Fig.3 — Observation of quantized energy levels ⁶⁾. The energy levels in a particular well of the tilted washboard potential are shown in the inset. Note that the spacings of the levels decrease with increasing energy. At an appropriate temperature there is a significant thermal population of the lower-lying excited states. The escape rate $\Gamma(0)$ from the zero voltage state is first measured as described in Fig. 2. One then applies 2GHz microwaves to the junction and remeasures the escape rate $\Gamma(P)$, where P is the microwave power, as a function of the bias current. As the current is decreased, the spacings of the energy levels increase. Whenever a particular spacing corresponds to the microwave frequency, the escape rate is resonantly enhanced. For example, the peak in the escape rate at the right-hand side of the figure results from the microwave excitation of the particle from the ground state to the first excited state, in which its lifetime is substantially reduced. The second and third peaks correspond to transitions from the first excited state to the second and from the second to the third, respectively. The positions of the resonances are in excellent agreement with theoretical predictions.

than for the examples described above ⁷). In particular, the effect of dissipation (see below) is much stronger and may even change the picture qualitatively ⁸). Furthermore, one must take care to minimize the perturbation due to the measurement process ⁷).

A second predicted macroscopic quantum effect is concerned with Bloch oscillations ⁹) in junctions where the charging energy of a Cooper pair $(2e)^2/2C$ is larger than or comparable to the Josephson coupling energy, which, in turn, must exceed $k_B T$. Thus for $T = 1$ K, one requires $C < 2e^2/k_B T \approx 3$ fF. For zero bias current, the Josephson potential is periodic in the phase difference, and the wave function is thus also periodic, apart from a phase factor, $\Psi_{n,Q_x}(\phi + 2\pi) = \exp(2\pi i Q_x / 2e) \Psi_{n,Q_x}(\phi)$. Physically, Q_x represents an external charge introduced onto the electrodes through (weakly coupled) external leads ¹⁰). From our experience with periodic potentials and Bloch states in solid state physics, we are not surprised to learn that coupling of the potential wells turns each discrete energy level into an energy band. By increasing Q_x by means of an external current we can sweep through the energy bands, causing the energy and hence the voltage across the junction to oscillate at a frequency $f = I/2e$. This result is potentially of practical importance because of the link it provides between the Josephson voltage-frequency relation $2eV = hf$ and the quantum Hall relation $V = (h/ne^2)I$.

Finally, we briefly discuss the effects of dissipation on the quantum phenomena described above. The pioneering paper on this subject is that of Caldeira and Leggett ²), who considered a model in which the single degree of freedom ϕ describing the junction is coupled linearly to a bath of harmonic oscillators. If the distribution of amplitudes and frequencies of these oscillators and the coupling strengths to them are chosen suitably, in the classical limit the model describes a system in which the damping is proportional to the velocity $\dot{\phi}$. Thus, the dissipation is linear in the voltage across the junction, that is, it is ohmic, and the current noise generated by the resistance has a Gaussian distribution.

An alternative approach to the problem of dissipation starts with the microscopic tunnelling Hamiltonian for a Josephson junction ³). In the absence of dissipation, this model reduces to the Hamiltonian of Eq. (3). In general, however, the model contains a dissipative element due to the tunnelling of single electrons. This element is non-linear rather than ohmic, and the discrete

charge transfer process leads to shot noise rather than to Gaussian noise.

In many respects, the effect of dissipation is to make the system behave in a more classical way, even though it is fully quantum mechanical. For example, dissipation reduces the spread of the wave function describing a particle in a well. As a result, the MQT rate is lowered by a factor that depends exponentially on the strength of the dissipation. For weak dissipation, the coherent oscillations in the double well potential of Fig. 4 decay exponentially with time, so that the system ends up in one or other of the minima with equal probability ¹²). At the same time, the splitting of the energy levels in Fig. 4 is reduced, as are the widths of the energy bands in the periodic potential. For strong dissipation, a phase transition ^{8, 10, 13}) is predicted to occur in a double well or periodic potential. Above a critical strength of the dissipation, the system becomes localized: if prepared in one of the potential wells, the particle will remain there, and there will be no coherent oscillations. In the case of very strong dissipation, it is often sufficient to use a quantum Langevin equation in which the particle is described classically while the noise driving term has a quantum spectral density. Both Gaussian ¹⁴) and shot noise ³) have been analyzed in this approach.

Of the various predicted effects of dissipation, the reduction in the rate of MQT due to an ohmic resistor or single electron tunnelling has been observed experimentally ¹⁵). The results are in qualitative agreement with theory. For example, the factor representing dissipation in the exponent of the MQT rate has been shown to scale as T^2 , in accordance with predictions ¹¹). Measurements have also been made of the noise generated in an overdamped Josephson junction current-biased at a voltage V so that $\hbar\omega_J \gg k_B T$, where $\omega_J = 2eV/\hbar$ is the Josephson frequency ¹⁵). The results were in good agreement with predictions of the quantum Langevin equation.

At this stage, most of the theoretical groundwork has been laid, and the techniques for describing dissipation are well understood. Experimentally, the existence of MQT and of quantized energy levels has been demonstrated unequivocally, while the effects of dissipation on the MQT rate have been shown to be qualitatively in accord with theory. The other effects described above await experimental demonstration. The experiments are difficult, usually demanding very low temperatures, very small devices, and extraordinary screening against

Fig. 4 — Potential of a superconducting loop containing a Josephson tunnel junction with an external flux $\Phi_0/2$. The two lowest eigenstates are split in energy by δ .

external perturbations. Nonetheless, it is to be hoped that some of these effects, at least, will be observed in the course of the next few years.

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