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The Spheromak

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Intuitively, the most attractive form for a confined thermonuclear fusion plasma is a simple sphere which avoids all the problems associated with an inner vessel wall.

Among the most promising, although least mature, of the magnetic confinement geometries proposed for containing high temperature deuterium-tritium fusion plasmas is the spheromak configuration. The simplicity and compactness of this device has been the impetus for innovative and novel theoretical and experimental investigations during the past decade. Although still considered an "alternative" or "supporting" concept, the spheromak is now beginning to demonstrate an ability to heat and confine plasmas approaching those of thermonuclear interest.

Confinement Geometry

The spheromak is related to but distinct from several other magnetic confinement concepts. Like the tokamak and reversed field pinch (RFP), the spheromak magnetic fields are axisymmetric, or independent of the cylindrical angle ϕ, Fig. 1. Also, like these other devices, large currents flow in the plasma itself which produce magnetic fields which in turn confine the plasma. The spheromak differs, however, in that only one set of external coils are necessary to provide the required externally generated magnetic fields. This leads to a non-linked coil geometry, similar to a simple magnetic mirror machine, and offers considerable simplification.

If we follow an individual magnetic field line in Fig. 1 as the toroidal angle ϕ increases around the device, we see that it remains in a two-dimensional surface $\psi(R,Z)$ = constant. These constant ψ values form nested toroidal surfaces encompassing the magnetic axis. Since both electron and ion orbits make tight spirals around individual magnetic field lines, these nested magnetic surfaces form the lowest order basis for the spheromak confinement scheme, as they do for tokamaks and RFP's. We characterize these as closed field line geometries, as opposed to open field line systems like magnetic mirror machines where individual magnetic field lines pass out of the confinement region and may intersect the surrounding walls. The open field line confinement physics relies on anisotropy in the velocity space distribution function and on the establishment of confining electrostatic fields, whereas closed field line confinement does not.

It is useful to think of the spheromak axisymmetric magnetic field as being composed of two parts, a poloidal part

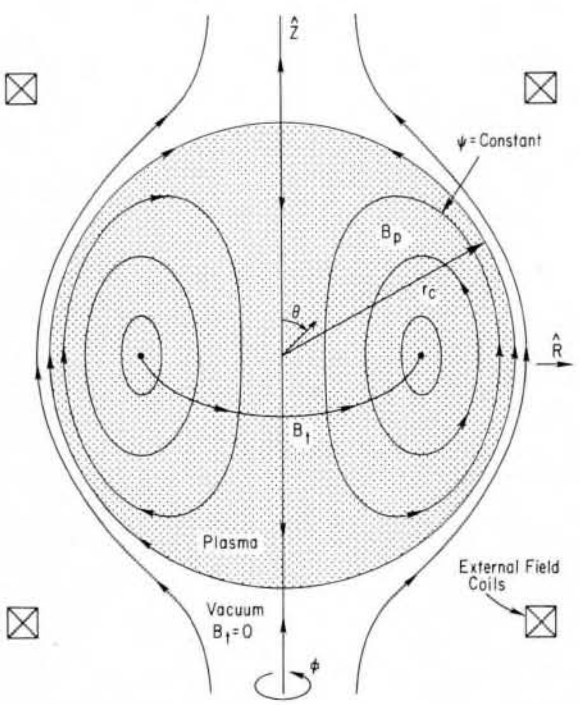

Fig. 1 — The spheromak equilibrium is independent of the cylindrical angle ϕ. The magnetic field lines form closed magnetic surfaces ψ = constant.

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constrained by axisymmetry and the condition $\nabla \cdot \mathbf{B} = 0$ to be of the form

$$\mathbf{B}_p = (2\pi)^{-1}\nabla\phi \times \nabla\psi(R,Z),$$

and a toroidal part

$$\mathbf{B}_T = g(R,Z)\nabla\phi.$$

The total magnetic field $\mathbf{B} = \mathbf{B}_p + \mathbf{B}_T$ is thus completely described by these two two-dimensional scalar functions $\psi(R,Z)$ and $g(R,Z)$. Under certain mild assumptions, namely that the plasma is a non-rotating equilibrium fluid adequately characterized by a scalar pressure, it can easily be shown that both the toroidal field function $g(R,Z)$ and the fluid pressure $p(R,Z)$ must be single valued functions of the poloidal flux function $\psi(R,Z)$, *i.e.*, $g = g(\psi)$ and $p = p(\psi)$. These functions are related to each other by the force-balance, or equilibrium equation:

$$\mathbf{J} \times \mathbf{B} = \nabla p \qquad (1)$$

or, using Ampere's law $\nabla \times \mathbf{B} = \mu_0\mathbf{J}$,

$$(2\pi)^{-2}R^2\nabla \cdot R^{-2}\nabla\psi + \mu_0 R^2\, \mathrm{d}p/\mathrm{d}\psi + g\, \mathrm{d}g/\mathrm{d}\psi = 0. \qquad (2)$$

The distinguishing feature of the spheromak configuration is that the toroidal field, and hence $g(\psi)$, goes to zero at the last closed surface separating the confined plasma region from the surrounding vacuum region. This implies, through Ampere's law, that no toroidal field magnets are needed to produce the spheromak toroidal fields, as is the case in tokamaks and RFP's. The toroidal field is non-zero only in the actual plasma region and is produced entirely by poloidal currents flowing in the plasma itself. This feature means that it is not necessary for any external structure to pass through the centre hole of the doughnut-like toroidal surfaces, and thus the confinement region becomes topologically spherical as opposed to toroidal. The inherent simplification this affords is substantial. The aspect ratio of the plasma-vacuum interface can be made to approach unity. The toroidal field magnets, normally the most expensive component in a toroidal confinement device, are eliminated. Shielding requirements are greatly eased by the change in topology. In addition, there is the possibility that the spheromak structure can be translated in the direction of the symmetry axis, raising the possibility of separate formation, thermonuclear burn, and disposal regions.

Magnetic Helicity

A concept central to the discussion of the stability, formation, and decay of the spheromak is that of magnetic helicity. If we temporarily restrict consideration to a volume with no magnetic field lines penetrating the bounding surfaces, then we can define the global magnetic helicity and the magnetic energy in that volume as

$$K = \int \mathbf{A} \cdot \mathbf{B}\, \mathrm{d}V, \qquad (3)$$

and

$$W = \int \tfrac{1}{2}\, \mathbf{B}^2\, \mathrm{d}V, \qquad (4)$$

where **A** is the vector potential associated with the magnetic field **B**, *i.e.*, $\mathbf{B} = \nabla \times \mathbf{A}$. Because of the gauge freedom in **A**, the transformation $\mathbf{A} \rightarrow \mathbf{A} + \nabla\chi$ leaves **B** unchanged for any scalar field χ, even one that is multi-valued. To be definite we must also stipulate that **A** is single valued in the sense that the line integral of $\oint \mathbf{A} \cdot \mathrm{d}\ell$ around any closed curve lying within the volume is equal to the magnetic flux $\int \mathbf{B} \cdot \mathrm{d}\mathbf{S}$ through the area enclosed by the curve.

A simple example serves to illustrate that the magnetic helicity K measures the topological linkage of the magnetic field. Consider the situation in Fig. 2 where we have two intertwined magnetic flux tubes, one containing flux ϕ_1 and the other containing flux ϕ_2. By definition, the magnetic field is everywhere tangential to the boundaries of the flux tubes so that the magnetic flux, $\phi = \int \mathbf{B} \cdot \mathrm{d}\mathbf{S}$, through any cross-section of either of the tubes is a constant, equal to ϕ_1 or ϕ_2. Assuming **B** to vanish in the volume outside the two flux tubes, we can easily calculate the helicity by integrating over the volumes inside,

$$K = \sum_{1,2} \int \mathbf{A} \cdot \mathbf{B}\, \mathrm{d}V = \sum_{1,2} \oint \mathbf{A} \cdot \mathrm{d}\ell \int \mathbf{B} \cdot \mathrm{d}\mathbf{S} = 2\phi_1\phi_2 \qquad (5)$$

where the integration over each of the two flux tube volumes gives the same result. We see that if the tubes were not linked, the helicity of the configuration would be zero since the integral $\oint \mathbf{A} \cdot \mathrm{d}\ell$ would be zero in each flux tube integral.

If the spheromak plasma is modelled as a perfectly conducting fluid, without electrical resistance, and surrounded by a perfectly conducting wall, it can easily be shown that the global magnetic helicity K is an exact invariant. In fact, for such a plasma, the individual helicity associated with each of the infinite number of flux tubes in the plasma is an invariant, remaining constant in time as the plasma deforms or moves. This statement is equivalent to saying that in a plasma fluid with infinite conductivity, magnetic field lines are frozen into the fluid, and since the fluid velocity is continuous, magnetic field lines cannot break or coalesce to change their topological properties.

The special significance of the global magnetic helicity K comes from consideration of a more realistic plasma model in which we allow small departures from the perfect-conductivity approximation. In such a plasma, topological properties of the magnetic field are no longer strictly preserved, and lines of force may break and coalesce. However, as first noted by J.B. Taylor [1]), in such a plasma the changes in field topology are accompanied by only very small changes in the field itself and the integral $K = \int \mathbf{A} \cdot \mathbf{B}\, \mathrm{d}\tau$ over the entire volume will be almost unchanged. The effect of the topological changes is merely to redistribute the integrand among the field lines involved. If surrounded by perfectly conducting walls, the global helicity K will still be a good invariant even though the individual helicity on each flux tube is not.

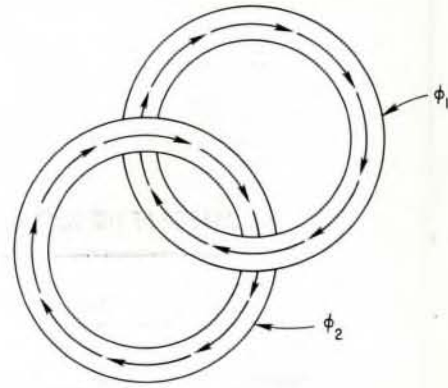

Fig. 2 — Helicity K is a measure of the topological linkage of the magnetic field. If two magnetic flux tubes with flux ϕ_1 and ϕ_2 interlink, helicity is $K = 2\phi_1\phi_2$.

Taylor's hypothesis, then, is that a non-perfect plasma will undergo some turbulent relaxation, but it will be such as to take the plasma into its lowest possible energy state consistent with the constraint that the global helicity K does not change. One is able to show that this state is such that the electrical current is everywhere proportional to the magnetic field.

$$\nabla \times \mathbf{B} = \lambda\mathbf{B} \qquad (6)$$

where λ is a single proportionality constant determined by the global helicity. Such an equilibrium configuration with constant λ has become known as the "Taylor State".

To the extent that the Taylor principle applies, the magnetic helicity K is the fundamental property of the spheromak. Global helicity is created or injected during the formation scheme, and eventually decays to zero due to resistive dissipation. At any intermediate time the fields and currents are uniquely determined by Eq. (6) with λ being determined by the instantaneous value of the global helicity.

The Taylor state equilibrium described by Eq. (6) is a particular solution of the more general equilibrium Eqs. (1) and (2) that has g proportional to ψ and $p = 0$ everywhere. This state is force free, having $\nabla p = 0$ everywhere, and is thus unsatisfactory as a fusion equilibrium magnetic confinement configuration. Although Eq. (6) is useful as a first approximation, detailed stability analysis must be performed to determine what other stable equilibria are present in the neighbourhood of the Taylor state that allow a finite pressure gradient and thus provide confinement. Also, in a geometry without conducting walls in contact with the plasma, the Taylor principle certainly does not apply and mode analysis must be performed to locate instabilities and stable configurations.

Stability

The central question one asks of any proposed magnetic confinement scheme is its stability. The spheromak configuration is assumed to be symmetric with respect to rotations in the cylindrical angle ϕ (axisymmetric), and to possess symmetry with respect to reflection about the midplane. We must ask whether any perturbation allowed by the dynamics which breaks this symmetry is energetically favourable, that is, will take the configuration to a lower energy state. If so, the system is unstable.

There are many levels of mathematical description for a magnetically confined plasma. These differ in their complexity and sophistication, and in their realism in describing the true physical situation. It is useful to classify instabilities with respect to what level of description is necessary for the instability to appear. This classification also clarifies the free energy driving the instability and the magnitude and scaling of the growth rates for unstable perturbations.

Perhaps the most troublesome of the spheromak instabilities are the tilting, the shifting, and the vertical mode [2]. These are global, rigid displacements of the plasma, present in its simplest description, that of a rigid current carrying ring. A spheromak carrying a toroidal current I_p in the positive ϕ direction requires an externally supplied magnetic field with a component in the negative z-direction, to produce a radially inward force which balances its intrinsic radially outward directed expansion force. This external field can be adequately characterized as having a strength B_0^z and a magnetic field index $n = -(r/B_0^z)(\partial B_0^z/\partial r)$. It is easily shown that the rigid spheromak ring will be unstable to at least one of the modes of Fig. 3 for any value of the field index n. If $n < 0$ the vertical mode is unstable, if $n > 0$ the shifting mode is unstable, and if $n < 1$ the tilting mode is unstable. The growth rates for these instabilities are large, scaling like $\gamma \sim (I_p B_0^z/M)^{1/2}$, where M is the total mass of plasma. These correspond to times of the order of 1-10 µs for present spheromak experiments.

Each of these fundamental modes of instability has a simple physical interpretation. The tilting mode arises since the magnetic moment of the spheromak is aligned antiparallel to the external vertical field. A lower energy state can be obtained by the plasma tilting 180°, although it will no longer be in radial force balance when it does so. The shifting and vertical modes correspond to the plasma ring displacing itself into a region of weaker external field strength, and thus lowering its interaction energy.

Although several innovative methods for controlling these instabilities with energetic particles or plasma rotation have been proposed, the only method which has been experimentally demonstrated relies on the placement of external conductors in close proximity to the shell. Induced currents appear in these conductors when the plasma is displaced, producing a restoring force which tends to push the spheromak back to its symmetric state. Although effective, the necessity for having solid conductors in close proximity to the plasma compromises its flexibility and desirability. Also, active feedback systems are necessary to stabilize these motions for times long compared to the resistive decay times of the conductors.

The next useful level of description is the "ideal MHD" model [3]) where we allow non-rigid deformations of the plasma, but treat it as a perfectly conducting fluid. Extensive analysis of the stability of the spheromak using the ideal MHD model shows new instabilities if the plasma pressure is too high (pressure driven) or if the toroidal current channel is too peaked (current driven). These instabilities, while also having large growth rates comparable to the rigid mode growth rates, are avoidable if we limit the maximum pressure and current density. These ideal MHD stability limits are given approximately [5,6]) by: $\mu_0 p_0 < B_0^2/10$, and $\mu_0 I_p < 10\ aB_0$ (mks units) where p_0 and B_0 are the central values of the plasma pressure and toroidal field, I_p is the total plasma toroidal current, and a is the plasma minor radius. These limits are not unduly restrictive.

If one next adds finite electrical resistivity to the fluid model of the plasma, the stability picture changes considerably. Normal mode analysis says that a new class of resistive instabilities are present. These resistive instabilities may tap predominantly the free energy source associated with the plasma pressure, in which case we call them "resistive interchange" or "resistive ballooning" modes, or they may tap the free energy associated with the electrical current flowing parallel to the magnetic field lines, in which case we call them "tearing" modes.

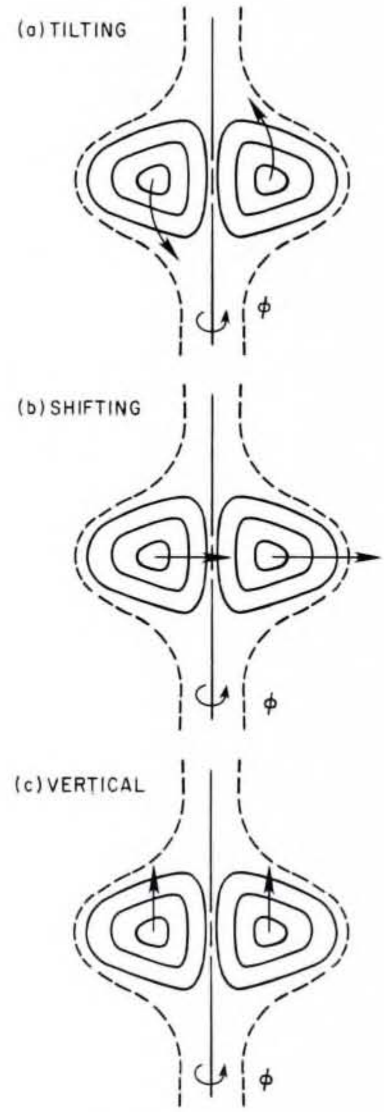

Fig. 3 — Rigid instabilities include (a) tilting, (b) shifting, and (c) vertical motion.

The resistive modes are predominantly localized in small bands around "rational" magnetic surfaces in which the magnetic field lines close upon themselves after traversing m times the short way around the torus and n times the long way around. An unstable resistive mode will develop large perturbed currents in the resistive inner region surrounding one of these rational surfaces causing increased magnetic reconnection and transport of plasma across the magnetic field lines. Rational magnetic surfaces with $m = 1$ and $n =$ (small integers) have the most stringent stability criteria.

The stability criteria with respect to these resistive instabilities also limit the allowable current and pressure distributions in the plasma, but the limits are much more restrictive. Only a small range of current distributions are found

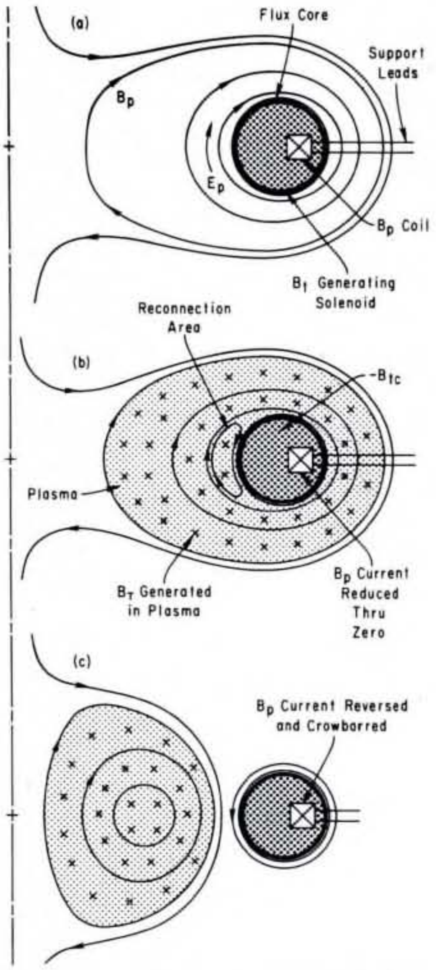

Fig. 4 — The S-1 inductive formation method begins with (a) the plasma linked to the flux core. As currents are induced into the plasma (b) reconnexion occurs and (c) the spheromak separates from the structure.

to be stable to tearing instabilities, and these are ones close to the Taylor state, Eq. (6). Also the resistive interchange or ballooning modes set rather low limits on the allowable pressure. The precise pressure limits set by these instabilities depends on many factors, including details of the geometry, the collisionality regime, and the Larmor radius size of the ions. However, it is clear that they set limits as much as an order of magnitude less than the pressure limits set by ideal modes.

The growth rates associated with the resistive modes are several orders of magnitude smaller than those for ideal modes. This, coupled with the highly localized nature of these instabilities makes it plausible that their effects are rather benign, serving only as a mechanism for allowing the plasma to "relax" back to a configuration near the Taylor state once it has deviated too far. The efficiency and desirability of the spheromak as a fusion plasma confinement configuration depends on how far on the average it can deviate from the force-free Taylor state. Ultimately, this question will have to be answered experimentally.

Formation Methods

There are presently at least four distinct methods of forming spheromaks that have been demonstrated. These are:

1) by a magnetized coaxial gun using currents through electrons [4])
2) by a combination of θ and z pinch discharges utilizing both electrode and inductive techniques [5])
3) by a conical theta pinch technique utilizing fast inductive discharges and
4) by an electrodeless inductive scheme utilizing a flux core [6]).

These schemes differ in detail, but all have the common feature of producing both toroidal and poloidal field components, and in affecting a change in topology to break off field lines that are open or linked with external structures to produce a free spheromak with closed magnetic surfaces. We illustrate in Fig. 4 the inductive flux core method used in the S-1 Spheromak at Princeton University.

The S-1 flux core is a thin walled stainless steel toroidal tube that contains current-carrying windings that go around both the short way (TF windings) and the long way (PF windings). Each of these sets of windings are connected through leads to separate external voltage sources, the TF source and the PF source. As these circuits are pulsed, large currents appear in the windings and, through inductive coupling, in the surrounding plasma. Initially, the plasma surrounds, or is linked to, the flux core (4a). As the vertical field and currents are increased, the plasma separates from the flux core with each magnetic surface reconnecting on the small major radius side (4b). When the reconnection is complete, a free non-linked spheromak plasma has been created (4c).

Experimental Results and Prospects

Over half a dozen spheromak devices are now operating in the USA, Japan, and Europe. Experiments exist in the USA at the University of Maryland, LANL, PPPL, and the University of Washington; in Europe at the Universities of Essen and of Heidelberg, and in Japan at the Universities of Osaka and of Tokyo. Temperatures of the order of 100 ev, sufficiently high to overcome the low-z impurity radiation barrier, have been achieved on several of these devices. Densities are typically 0.3 to 10 $\times$ 10^{14} cm^{-3}, with toroidal currents in the range 0.1 to 1.0 MA, producing peak magnetic fields of 0.5 T. Discharge duration times up to 2 ms have been obtained on several of these devices, which have major radii in the range of 9-60 cm.

Magnetic probe measurements confirm that the plasma in these devices indeed tends to relax into a state that lies near the Taylor minimum energy state. Specific resistive modes have been identified as effecting this relaxation, in good agreement with the theoretical onset conditions. This implies that the final plasma state is relatively insensitive to the details of the formation process, but depends primarily on the amount of magnetic helicity injected during formation. A noteworthy experiment on the CTX device has demonstrated near steady-state maintenance for over 5 ms by continuous injection of helicity from an external electrode source [4]).

It has been demonstrated that the global tilt and shift modes can be stabilized by passive conductors. This has been accomplished by solid conductor walls, by "bird cage" wire mesh enclosures, and by a figure-8 coil, which is a specially designed wire loop that is twisted to give the current pattern needed to stabilize these instabilities, but to have near zero inductance with the axisymmetric coil systems.

Although the temperature and confinement parameters obtained to date in the spheromak fall considerably short of those obtained in tokamaks, the intrinsic attractiveness of this configuration beckons us to continue its investigation. Steady improvement in these parameters over the last few years encourages us to believe that continuing success in the development and optimization of this concept will be forthcoming.

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