Ultrasonic Spectroscopy of Superfluid Helium 3

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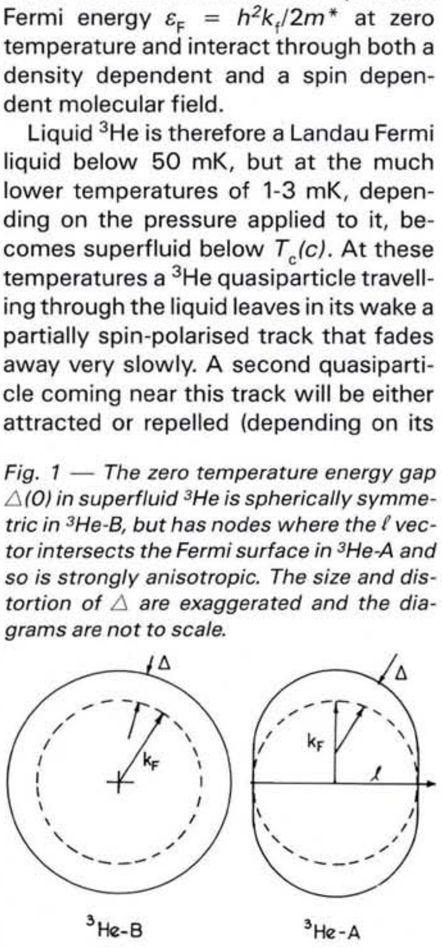
Many of the collective modes of the complex order parameter of superfluid helium 3 couple to the fluctuations of density produced by ultrasonics. Studies of the resonances of these modes and their splitting by magnetic fields constitute an ultrasonic spectroscopy of the superfluid phases of ^{3}He.

Helium 3 atoms condense into a colourless, transparent liquid at 3.19 K, in which their zero point motion is so strong that they only crystallize when a pressure of about 3.4 MPa is applied. This quantum liquid of uncharged fermions is subject to the same Fermi statistics that are so familiar for conduction electrons in metals, but with a Fermi temperature, $T_F \cong 1$ K. At lower temperatures, $T << T_F$, the interactions between the ^{3}He atoms can be represented by Landau's theory of a Fermi liquid. Each ^{3}He atom of mass m collects a screening cloud of other atoms to become a quasiparticle of effective mass m^*, where m^*/m increases from about 2.1 at zero pressure to about 4.6 at the melting pressure. These quasiparticles fill all the momentum states up to the Fermi energy $\varepsilon_F = h^2k_f/2m^*$ at zero temperature and interact through both a density dependent and a spin dependent molecular field.

Liquid ^{3}He is therefore a Landau Fermi liquid below 50 mK, but at the much lower temperatures of 1-3 mK, depending on the pressure applied to it, becomes superfluid below $T_c(c)$. At these temperatures a ^{3}He quasiparticle travelling through the liquid leaves in its wake a partially spin-polarised track that fades away very slowly. A second quasiparticle coming near this track will be either attracted or repelled (depending on its spin) and so effectively interacts with a spin-dependent force with the first quasiparticle. Such a mechanism can lead to correlated pairs of parallel-spin particles and so favours spin triplet pairing ($S = 1$) rather than the spin singlet pairing ($S = 0$) of the Cooper pairs in superconductors. A similar argument can be applied to the density fluctuations leading to an attractive, spin-independent, interaction.

Fig. 1 — The zero temperature energy gap $\triangle(0)$ in superfluid ^{3}He is spherically symmetric in ^{3}He-B, but has nodes where the ℓ vector intersects the Fermi surface in ^{3}He-A and so is strongly anisotropic. The size and distortion of $\triangle$ are exaggerated and the diagrams are not to scale.



The Bose condensate of correlated pairs of ^{3}He quasiparticles is separated by an energy gap $\triangle$ from the excited states (Fig. 1), so that $2\triangle(T)$ is the minimum energy required to excite two quasiparticles from a correlated pair at temperature T. The theoretical value for $\triangle(0)$ is 1.76 k_BT_c, so that the range of energy gaps, conveniently expressed as frequencies $\triangle(0)/h$, for superfluid ^{3}He is from 28 MHz for the zero pressure fluid ($T_c \cong 1$ mK) to 102 MHz for the melting pressure fluid ($T_c \cong 2.8$ mK). Since $\triangle(T)$ rises from zero at T_c to within 1% of $\triangle(0)$ at about 0.30 T_c, ultrasonic phonons in the frequency range 5-100 MHz have proved to be excellent probes of the phases of superfluid ^{3}He.

Three phases have been studied: The *A phase* exists at pressures above the polycritical point (2.15 MPa) in zero magnetic field and at all pressures from 0 to 3.4 MPa in moderate magnetic fields ($B > 0.4$ - 0.6 T, depending on the pressure); the A_1 *phase* in finite magnetic fields in a small temperature interval between the normal fluid and the A phase; the *B phase* at low temperatures, low pressures and in weak magnetic fields ($B < 0.4$ - 0.6 T). In each of these phases the correlated pairs form large coherent regions having a coherence length $\xi_0 = \hbar v_F/\pi\triangle$, where $v_F = \hbar k_F/m^*$ is the Fermi velocity of the quasiparticles.

The coherence length in ^{3}He ranges from 111 nm at 0 bar to 19 nm at the solidification pressure, that is, from 227 r_0 to 44 r_0, where r_0 is the atomic spacing in the solid. For comparison, the coherence length for the Cooper pairs in

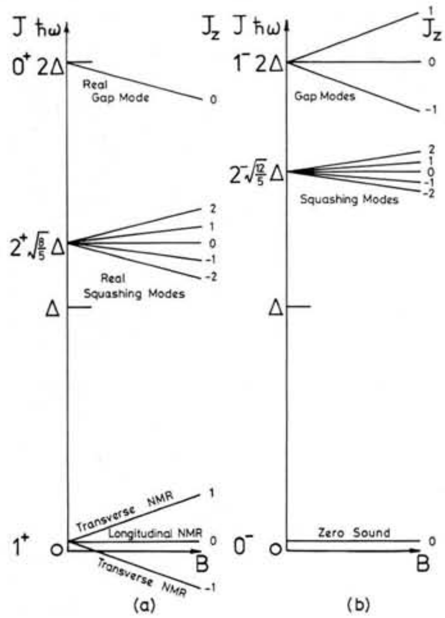


Fig. 2 — Energy levels of the collective modes in ³He-B in the presence of a magnetic field: (a) real, (b) imaginary modes of the complex order parameter. (Diagram courtesy of L. Tewordt, private communication.)

superconducting niobium is 149 r_0 and in aluminium 5600 r_0.

Order Parameter

Since the helium atoms in the superfluid phases of ^{3}He form correlated pairs extending over such large distances compared with the interatomic spacing, they form a Bose condensate that can be described by a *macroscopic* wave function or an order parameter. However, unlike superconductors, where the Cooper pairs have compensated angular momenta ($L = 0$) and spins ($S = 0$), there is now a great deal of evidence to show that the correlated pairs in superfluid ^{3}He have finite angular momenta ($L = 1$) and spin ($S = 1$). In some respects they behave like giant molecules in p-wave, spin triplet states with substates having the possible values $S_z = 0, \pm 1$. In fact, if we neglect some small interactions, the total angular momentum J is a good quantum for the *B phase*, in which all three spin components $S_z = 0, \pm 1$ exist, with a ground state $J = 0$. Since $L = 1$ there are four components of this order parameter, which can be written as a 2×2 matrix with first row $S_z = 1$, $L_z = -1$ and $S_z = 0, L_z = 0$ and second row $S_z = 0$, $L_z = 0$, $S_z = -1$, $L_z = 1$, each component having the common amplitude $\triangle$ (Fig. 1).

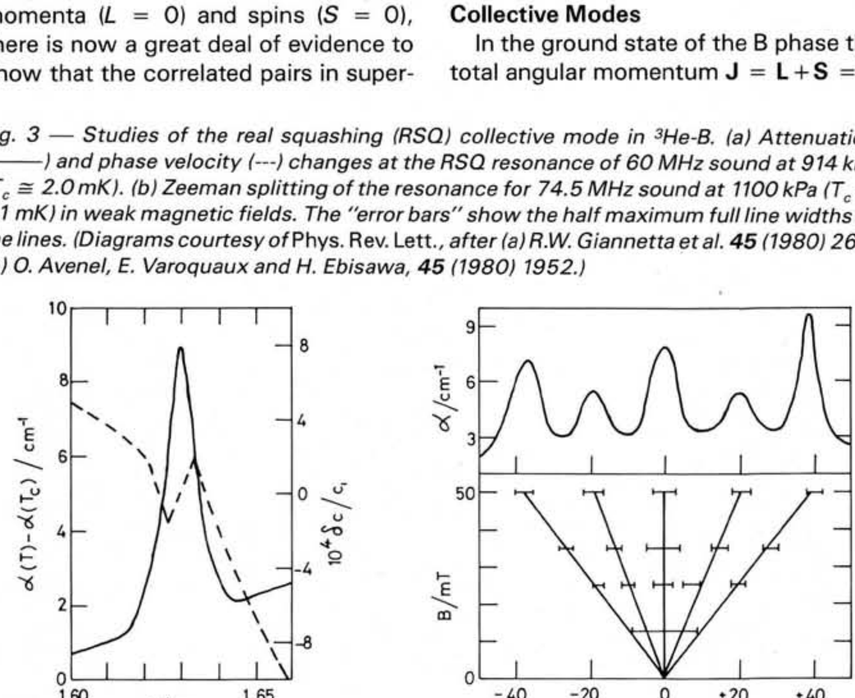
In general, to describe all three phases of the superfluids, an order parameter $d_{j\mu}$ is required in the form of a complex 3×3 matrix, covering the $j = 1,2,3$ dimensions of spin space and the $\mu = x,y,z$ dimensions of momentum (or **k**) space. In the *A phase* only the $S_z = \pm 1$ pairs exist and this leads to an anisotropic superfluid (Fig. 1) in which the orbital angular momentum vector ℓ for a correlated pair has components $\hat{\mathbf{n}} \times \hat{\mathbf{m}} = \ell$ and the energy gap $\triangle(\mathbf{k})$ has nodes along $\mathbf{k} = \ell$, but is axially symmetric about ℓ. In the *A_1 phase*, which is only formed in a magnetic field, the superfluid is completely spin polarized with $S_z = -1$ pairs only. In a high magnetic field (10 T) it has recently been shown to exist over the comparatively wide temperature range 2.5 to 3.1 mK at high pressures near the solidification pressure, but has not been extensively studied.

Collective Modes

In the ground state of the B phase the total angular momentum $\mathbf{J} = \mathbf{L} + \mathbf{S} = 0$ minimises the free energy and the p-wave pairs have an energy gap $2\triangle(T)$ separating their ground state from the quasiparticle continuum. Within this gap lie a number of excited states, corresponding to the collective modes of the complex order parameter, classified in the absence of a magnetic field by *real and imaginary modes* for each value of $J = 0,1,2$ possible from the p-wave spin triplets. In the presence of a magnetic field the degeneracy of the multiplets is removed and the $2(2J+1) = 18$ collective modes are separated. The Landé *g*-factors have been calculated by N. Schopohl and L. Tewordt with the result that an energy level diagram (Fig. 2) can be drawn for all 18 collective modes. The real modes are denoted by J^+J_z and the imaginary ones by J^-J_z.

Each collective mode is a particular combination of oscillations of the components of the order parameter $\delta d_{j\mu}$ about their equilibrium values. The simplest is that for 0^-, $(\delta d_{1x} + \delta d_{2y} + \delta d_{3z})/3^{1/2}$, which is that for *zero sound*. It can be excited by ultrasonics provided the frequency $\nu = \omega/2\pi$ is high enough to satisfy $\omega\tau >> 1$, where τ is the relaxation time between quasiparticle collisions. This zero sound mode then propagates and couples to some of the collective modes; for example, it couples strongly to the imaginary $J = 2^-$, but weakly to the real $J = 2^+$ modes. When the zero sound frequency resonates with the frequency of a collective mode there is a peak in the sound absorption spectrum, which can be extremely large for the imaginary $J = 2^-$ mode.

Fig. 3 — Studies of the real squashing (RSQ) collective mode in ³He-B. (a) Attenuation (——) and phase velocity (---) changes at the RSQ resonance of 60 MHz sound at 914 kPa ($T_c \cong 2.0$ mK). (b) Zeeman splitting of the resonance for 74.5 MHz sound at 1100 kPa ($T_c \cong 2.1$ mK) in weak magnetic fields. The "error bars" show the half maximum full line widths of the lines. (Diagrams courtesy of Phys. Rev. Lett.*, after (a) R.W. Giannetta et al.* **45** *(1980) 262, (b) O. Avenel, E. Varoquaux and H. Ebisawa,* **45** *(1980) 1952.)*



Experimental

The most common method in use today for reaching the temperature range (0.5 - 3.0 mK) required for studies of the ^{3}He superfluids is that of nuclear cooling. The principle of nuclear cooling is similar to that of adiabatic demagnetisation of the electron spins in paramagnetic salts, but uses the much smaller nuclear spins in metals such as copper or indium. The nuclei are partially polarized in high magnetic fields at ^{3}He-^{4}He dilution refrigerator temperatures (below 20 mK) and then demagnetised adiabatically to a low final field.

Most of the ultrasonic measurements have been made with sound cells consisting of two X-cut quartz crystals spaced a few millimetres apart and having fundamental frequencies of 5-15 MHz. Measurements of the absorption, phase velocity and group velocity of short (1-5 µs) ultrasonic pulses generated at odd harmonics of the quartz crystals are made when the attenuation is small (0.1 - 10 cm^{-1}), but in the presence of high attenuation (> 10 cm^{-1}) either propagation through a very thin layer of ^{3}He (0.25 mm) and a delay line, or measurements of the acoustic impedance of a single quartz crystal, have been made.

Three types of thermometer are in use, based on: the measurement by nuclear magnetic relaxation of the magnetic susceptibility of fine platinum powder; the direct measurement of the magnetic susceptibility of cerium magnesium nitrate powder diluted with lanthanum magnesium nitrate (LCMN); the pressure of melting ^{3}He filling a capacitor. Temperature is measured relative to the superfluid transition temperature T_c, since the absolute temperature for these transitions is only known to about 4%, while changes in temperature of a few microkelvin (< 1%) can be recorded.

B Phase Spectroscopy

Spectroscopic measurements are normally made at a fixed ultrasonic frequency $\omega/2\pi$ by sweeping through a collective mode resonance $\omega_i = a_i\triangle(T)$ in a slow cooling or warming cycle. The real $J = 2^+$ mode has been studied in detail by groups led by R.C. Richardson and D.M. Lee at Cornell University, by E. Varoquaux at Université Paris-Sud and by W.P. Halperin and J.B. Ketterson at North Western University. A typical result for this narrow resonance in zero field is shown in Fig. 3(a), where the attenuation peaks at 9 cm^{-1} and the phase velocity shows a characteristic change.

When a magnetic field **B** is applied the energy gap $\triangle$ is no longer uniform for all the spin triplet pairs, but is split into a transverse gap $\triangle_1 \sin\theta$ for $S_z = \pm 1$ pairs and a longitudinal gap $\triangle_2 \cos\theta$ for the $S_z = 0$ pairs, where θ is the polar angle from **B**. For small fields $\triangle_1 \cong \triangle_2$ and the excitation energies of the quasiparticles for $\mathbf{q} \perp \mathbf{B}$ are:

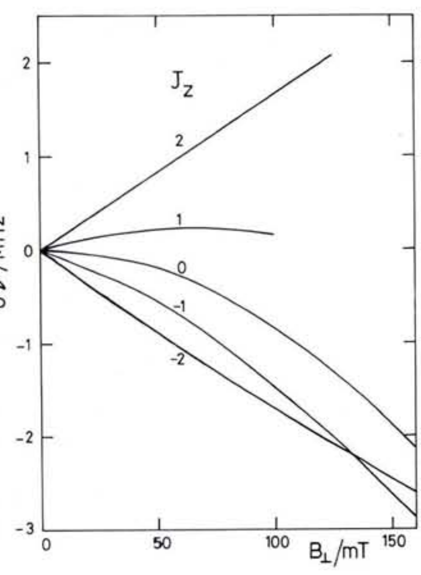
$$E_{\pm}^{2}(k) = \varepsilon_k^2 + \triangle^2 \pm \Omega^2/4 \pm \Omega(\varepsilon_k^2 + \triangle^2\cos^2\theta)^{1/2}$$

where $\Omega = \gamma B_{\mathrm{eff}}$ is a renormalised Larmor frequency. This linear, or Zeeman, splitting of a collective mode resonance was beautifully demonstrated at Orsay by application of 10 - 50 mT fields to the $J = 2^+$ resonance, Fig. 3(b). The splitting of 360 μK/T, or 10 MHz/T, was in fair agreement with the Landé g-factor calculated by Tewordt and Schopohl.

For larger fields the distortion of the energy gap becomes significant, $\triangle_1$ differs significantly from $\triangle_2$ and the quasiparticle energies are no longer given by the simple equation for small fields. When the gap distortion dominates, spin splitting effects similar to the non-linear (or Paschen-Back) splitting of atomic levels are found. By measuring the effects of standing waves in an acoustic cell and using magnetic fields up to 150 mT on low pressure ^{3}He-B, the North Western group demonstrated the type of non-linear splitting that was expected by Tewordt and Schopohl (Fig. 4) and also showed how far the $J_z = 0$ line was shifted from its zero field position.

The attenuation peak due to the imaginary $J = 2^-$ mode is so large that it has not yet been seen in ultrasonic propagation, but the complex phase velocity changes associated with it have been studied over a wide range of pressure by the acoustic impedance technique. Although it is predicted to split (Fig. 2), its splitting is smaller than that of the real $J = 2^+$ mode and so far the large line

Fig. 4 — Paschen-Back splitting of the RSQ mode resonance in moderate magnetic fields observed in ^{3}He-B at 135 kPa and 0.767 T_c ($T_c \cong$ 1.3 mK). In zero field the resonance is at 38.24 MHz. (Diagram courtesy of Phys. Rev. Lett. *after B.S. Shivaram et al.* ***50*** *(1983) 1070.)*



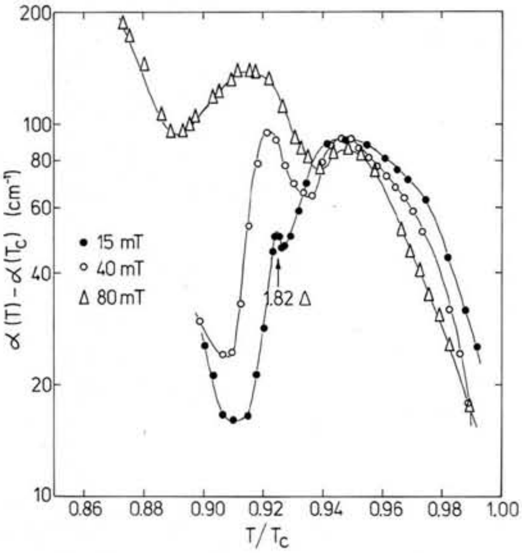


Fig. 5 — Measurements of a gap mode resonance with 44.25 MHz sound in ^{3}He-B at 290 kPa ($T_c \cong$ 1.4 mK) for $B_\perp$ = 15, 40 and 80 mT. This resonance occurs between the squashing mode ($J = 2^-$) peak and the pair breaking maximum (P) and is due to the $J_z = -1$ component of the $J = 1^-$ mode. (Diagram courtesy of American Institute of Physics *after J. Saunders, M.E. Daniels, E.R. Dobbs and P.L. Ward,* AIP Conf. Proc., ***103** (1983) 314.)*

widths that have been studied by propagation in moderate magnetic fields have not revealed its fine structure. In the acoustic impedance technique, the zero sound only couples to the $J_z = 0$ component of a multiplet and so this technique will not show a Zeeman effect.

Recent results in my laboratory, Fig. 5, show at low temperatures the attenuation rising towards the $J = 2^-$ peak and at high temperatures a broad maximum due to breaking of the pairs for which $\hbar\omega > 2\Delta(T)$, as the transition temperature is approached. In the central region (marked at 1.82Δ) a resonance is seen to increase and broaden as the magnetic field $\mathbf{B} \perp \mathbf{q}$ increases from 15 to 80 mT. At first it was thought that this resonance might be evidence of *f*-wave pairing ($L = 3$) in some of the correlated pairs producing a collective mode $J = 4^-$ near the pair breaking edge, $\hbar\omega = 2\Delta(T_p)$. However, Tewordt and Schopohl have shown that a more likely explanation is that the imaginary $J = 1^-$ mode is split by the magnetic field, producing an attenuation peak from the $J_z = -1$ component, no acoustical coupling to the $J_z = 0$ component and a possible "antipeak" from the $J_z = +1$ component. The decrease in the pair breaking attenuation with field near T_c may be evidence of this "antipeak".

Since the energy gap is isotropic in ^{3}He-B in zero-magnetic field, the pair breaking edge should be a rapid fall of the attenuation at T_p. A spectroscopic study of $T_p(p)$ over a range of pressures would thus provide direct evidence of the temperature dependence of the energy gap, $\Delta(T)/\Delta(0)$. Deviations of this experimental curve from that predicted theoretically in a weak coupling limit would be further evidence of strong coupling effects in superfluid ^{3}He. Although the data in Fig. 5 show this would be difficult, the North-Western University group have shown that the phase velocity c_ϕ does change abruptly at T_p and so measurements of $c_\phi(\omega)$ should establish $\Delta(T)$ accurately.

A Phase Spectroscopy

The presence of nodes in the anisotropic energy gap of the A phase, Fig. 1, means that a background of attenuation due to pair-breaking is present at all temperatures $T < T_c$. However, a small cusp at $\hbar\omega = 2\Delta_m(T)$, where Δ_m is the maximum value of $\Delta_k(T)$, has been predicted in the attenuation and may be observable in a magnetic field, when it is predicted to be split through gap distortion. A further complication of the anisotropy is the necessity of fixing the ℓ vector relative to the sound vector **q** for meaningful data. A decade ago J.W. Serene at Cornell showed that the attenuation α in the A phase could only be described in terms of the angle θ_q between ℓ and **q**, and attenuation components α_c, $\alpha_\perp$ and $\alpha_\parallel$. Experimentally D.N. Paulson, M. Krusius and J.C. Wheatley showed that by applying a small magnetic field $\mathbf{B}$($<$ 1 mT) the attenuation was reproducible for all angles between **B** and **q** except 90°. It turns out that when **B** is parallel to **q** one has $\ell \perp \mathbf{q}$ and θ_q is well defined, while when **B** is $\perp$ **q** the ℓ vectors lie in the same plane as **q**, and θ_q is indeterminate.

Measurements for θ_q at three angles (for example 30°, 60° and 90°) enabled the attenuation components to be separated and revealed the attenuation peaks due to two collective modes in the A phase, Fig. 6. The resonance in $\alpha_\perp$ is due to an oscillation of the angle between $\hat{n}$ and $\hat{m}$ in the plane normal to ℓ and so is an oscillatory distortion of the order parameter. On the other hand the resonance in α_c is an oscillation of the undistorted order parameter in which the $\hat{n}$ and $\hat{m}$ vectors "flap" up and down with ℓ, which therefore also oscillates. P. Wölfle has calculated the positions of these resonances in zero field and L. Tewordt and N. Schopohl have shown that in high magnetic fields (0.6 T) the splitting of both modes should be observable: that in α_c at low temperatures and that in $\alpha_\perp$ near T_c. To date such splittings have not been seen, although the North-Western group have recently claimed to have observed a third collective mode using their acoustic impedance technique. This is a surprising result as it was calculated to have a line width $> \omega$ at all temperatures.

The spectroscopy of the *A* phase is even less well known than that of the *B* phase and much remains to be measured and calculated in this exciting new field.

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Fig. 6 — Acoustic resonances in ^{3}He-A associated with the clapping ($\alpha_\perp$) and flapping (α_c) pair vibration modes, observed with 15 MHz sound at 2410 kPa ($T_c \cong$ 2.6 mK). (Diagram courtesy of Journ. Low Temp. Phys. *after D.N. Paulson, M. Krusius and J.C. Wheatley,* ***26** (1977) 73.)*