Relativistic Electrons and Coherent Radiation

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A review of the different ways of obtaining coherent radiation in the vacuum ultraviolet, soft X-rays and even X-rays, all based on relativistic electrons circulating in a storage ring and traversing a magnetic device called an undulator.

When an electron (or a positron) is accelerated in a magnetic field there is emission of radiation at a wavelength that depends essentially on the energy E of the electrons. For E of the order of few GeV hard X-rays are obtained, whereas for $E \sim 500$ MeV the spectrum is concentrated in the vacuum ultraviolet and the soft X-rays. This emission is called synchrotron radiation and is emitted in any storage ring.

Over the past ten years, synchrotron radiation has become the most powerful source of X-rays and vacuum ultraviolet because of its intensity and tunability. However, the radiation is not coherent and this limits some of the possible applications. Here we describe work done at Orsay to obtain coherent radiation with relativistic electrons circulating in a storage ring, using a device called an undulator.

Storage Ring

A schematic of a storage ring (ACO) is shown in Fig. 1. The ring comprises eight bending magnets, "optics" to keep the beam to a reasonable size (quadrupoles and hexapoles) and a radio frequency cavity to keep constant the energy of the electrons and control the electron bunch length ($\sim$ 30 cm or 1ns). Whilst ACO has a single bunch operation, some storage rings work with many bunches (up to a few hundred).

The position of a circulating electron and the angle of its trajectory at any moment relative to the equilibrium closed orbit are correlated. Assuming a gaussian distribution the size of the beam in the x (or z) plane is characterized by σ_x (σ_z), the standard deviation of the displacements, and its divergence by σ'_x (σ'_z), the standard deviation of the direction angle. At a focus, the product ($\sigma_x \times \sigma'_x$) is equal to the horizontal (vertical) emittance ε_x (ε_z).

The horizontal emittance is the result of a balance between the oscillations induced by the quantum emission of radiation and the damping which is created by the average loss of energy. In an ideal storage ring there is no radiation in the vertical plane and the vertical emittance is very small. However in a real machine, imperfections cause coupling of the motions between the two planes.

At relativistic energies, the dipole pattern radiated by the electrons is sharply peaked in the direction of motion of the electrons with a typical half-angle opening of the order of γ^{-1} where $\gamma = E/mc^2 = 1.96\,E$ (MeV). Synchrotron radiation has, therefore, at least in the vertical plane, a very good spatial (or transverse) coherence.

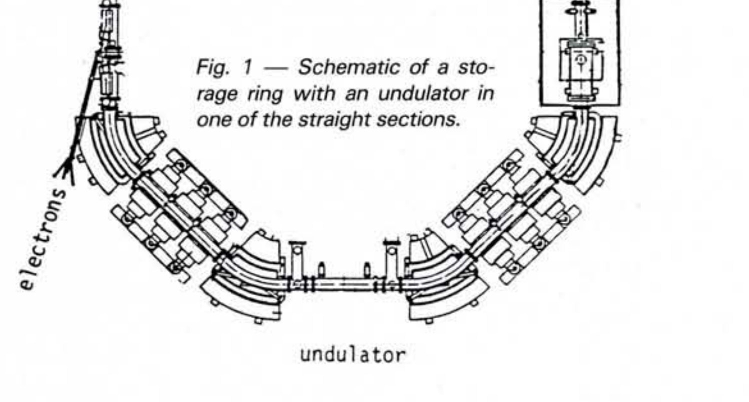


Fig. 1 — Schematic of a storage ring with an undulator in one of the straight sections.

The term undulator (Fig. 2) is used for a periodic transverse magnetic field (quite often quasi-sinusoidal) with many periods (10 to 100) intended to produce a spectrum which is composed of one or several narrow lines (harmonics) produced by the interference between electromagnetic fields emitted by the same electron at different points in its trajectory. The main properties can be summarised as follows:

a) The wave-length of the lines (along the axis) is given by

$$\lambda_n = \lambda_o\,(1+K^2/2)\,/\,2\,\gamma^2 n \qquad (1)$$

where λ_o is the magnet period, $\gamma = E/mc^2$, K is a parameter which defines the field strength ($K = 93.4\,B\,\lambda_o$, SI units), n the harmonic.

b) If the emittance of the storage ring is small enough, the increase in spectral brightness, compared with a bending magnet, is N^2 where N is the number of periods.

c) The bandwith of the lines is $\sim 1/nN$.

d) The power in the n^{th} harmonic which passes through a pinhole selecting a band $\triangle\nu/\nu$ is given by:

$$P_n = 109 E^2 IN\,[\triangle\nu/\nu]\,F_n(K)\,\lambda_o$$

where I is the current and $F_n(K)$ a combination of Bessel Functions.

As an example, with $E = 5$ GeV it is possible to obtain easily $P_1 \cong 30$ W around 1 or 2 Å !

First Method of Obtaining Coherency

We have complete coherence for an optical beam if we have spatial (or transverse) coherence and temporal (or longitudinal) coherence. The spatial coherence is related to the divergence of the source, the temporal to its monochromaticity. The undulator is the only source apart from the laser with potentially very high spatial coherence, of the order of $\gamma^{-1} \cong 0.1$ mrad for $E = 5$ GeV, so the divergence of the source is important. If we are limited by the diffraction we must have

$$\sigma_x\sigma'_x\sigma_z\sigma'_z = \varepsilon_x\varepsilon_z < \lambda^2$$

This condition shows that with increasing photon energy, smaller emittances are required. As example, the design for the European Synchrotron Radiation Facility has adopted the parameters:

$E = 5$ GeV

$\varepsilon_x = 6 \times 10^{-9}$ m rad

$\varepsilon_z = 6 \times 10^{-10}$ m rad

For $\lambda > 19$ Å the source has a total spatial coherence. It is possible to go

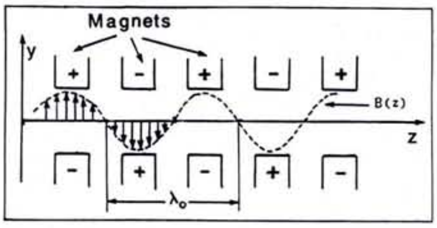


Fig. 2 — Permanent magnet configuration used to obtain a quasi sinusoidal field.

down to 1 or 2 Å by inserting pinholes with a loss of intensity proportional to the reduction in aperture which means that powers of the order of 1 W with total spatial coherence should be available around 1 or 2 Å in the near future and are already possible on existing rings down to 10 Å. At the exit of the undulator, the optical beam has a poor temporal coherence, but this can be corrected easily by adding a monochromator. Around 10 keV, a resolution of about 1 meV is already possible which gives a coherent length $\ell_{coh} = \lambda^2/\Delta\lambda \cong$ 1 or 2 mm.

It is clear that with the appearance of storage rings of very low emittance such as Bessy (Berlin), Super ACO (Paris), ALS (Berkeley) or the European project there will, in the next few years, be a lot of excitement in the field of microscopy, holography and probably crystallography.

Free Electron Laser (FEL)

There are basically two classes of Free Electron Laser. One type operates in the "collective" regime where the electronic density and Coulomb interaction are very strong, while the electron energy is rather low (a few MeV) and the radiation is emitted in the far IR. The other class operates in the low density or "Compton" regime, where Coulomb interaction is negligible, the energy is high (10-10^3 MeV) and the emission is at short wavelength (IR, visible, UV). Our experiment belongs to the second category as does the FEL studied by Madey and his group at Stanford where the first laser action was achieved in 1977 in the IR [1]).

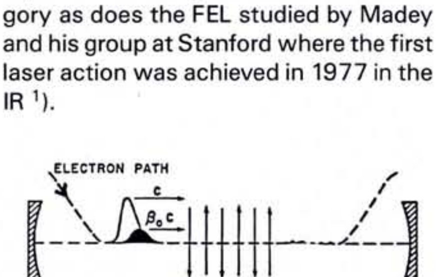


Fig. 3 — Principle of the amplification: an electron bunch enters the undulator and emits a photon pulse which will be reflected by the exit mirror (R = 99.5%) and the entrance mirror (R = 100%). The optical cavity is chosen in a such way that this photon pulse is again in coincidence, with the next electron bunch.

Compton free electron lasers can be:

(i) Single pass, where the electron beam is lost after having interacted with the light in the undulator, the remaining beam energy being recycled or not. These FEL are driven by linear accelerators, microtrons or other low energy machines. The expected wavelength is the far and near IR and possibly the visible and near UV.

(ii) Storage ring, where the electron bunch interacts with the light stored in an optical cavity at each pass through the undulator (typically 10^6/s).

A free electron laser on a storage ring is, in principle, a simple device comprising merely an undulator and an optical cavity (see Fig. 3). In essence the main features are set out below.

An electron travelling in a storage ring has mainly longitudinal velocity so it cannot couple to an electromagnetic field that is transverse. The purpose of the undulator is to give a transverse velocity to the electron to allow this coupling to take place and the transfer of kinetic energy to the photon. By adding an optical cavity on each side of the undulator a photon pulse can be amplified.

When an electron bunch enters the undulator there is emission of radiation in the direction of the beam. This photon pulse will be reflected by the exit mirror (R = 99,5%) and then by the entrance mirror (R = 100%) and if the length of the optical cavity is suitably chosen the photon pulse will again be in coincidence with the next electron bunch (Fig. 3). Amplification can continue for a few hundred or few thousand reflections and if the gain is larger than the mirror losses, laser oscillation results.

First laser oscillation was obtained at Orsay in June 1983 [2]) with an emission around 6508 Å. Typical spectra are given in Fig. 4 with the optical cavity detuned (a) and tuned (b).

The laser oscillates at three wavelengths at maximum gain, the gain per pass G being given by $G = \alpha\lambda^{3/2}N^3 I_p/\Sigma$. I_p is the peak current and Σ the transverse dimension of the beam. The dependence of G on the cube of the number of periods is the reason why on ACO it is only possible to have a laser in the range 4000-6000 Å. Gain optimization demands long undulators (3 to 5 m instead of the 1.2 m available on ACO). The $\lambda^{3/2}$ dependence gives also an idea of the lower limit to the wavelength range ($\sim$ 500 Å). The average power in the laser is of the order of few percent of the total synchrotron radiation emitted by the storage ring.

It was generally expected that the laser would reproduce the electron

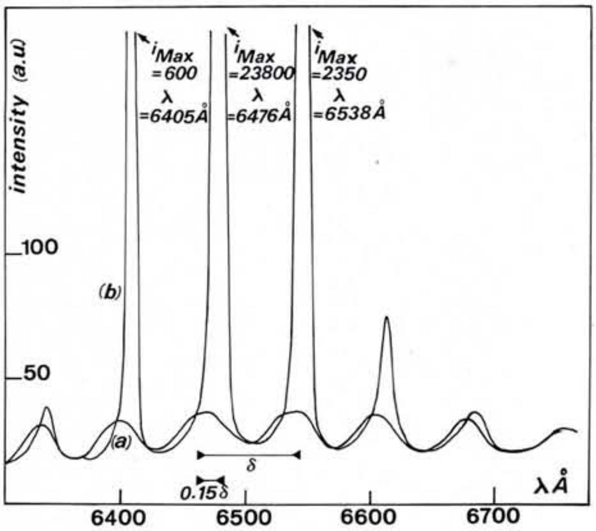


Fig. 4 — Laser oscillation obtained at 166 MeV.
a) the spectrum is recorded without amplification (the optical cavity is detuned);
b) laser oscillation when the cavity is tuned.

TABLE 1

a) **Expected Super ACO electron beam characteristics at 400 MeV**

Emittance	:	2.8×10^{-8} m rad
Energy spread	:	2.6×10^{-4}
I	:	50 A
RF freq.	:	500 MHz
Injection	:	e^+
Bunch length	:	2.7 mm

b) **Optical Klystron**

Length	:	3.3 m
Magnetic period	:	10 cm
Number of periods N	:	15 (per undulator)
N_d	:	0 to 300
Expected wavelength range	:	140-700 nm
Gain at λ = 150 nm	:	20%
Maximum K	:	6

bunch time structure (pulses of 1ns with a repetition period of 37ns) but, in practice, the laser exhibits a pulsed macrostructure with a pseudo-period around 10-20 ms and a pulse duration of about 2-3 ms. Fig. 5a shows this structure which is due to the competition between the rise time of the laser ($\sim$ 50 µs) and the damping time of the storage ring ($\sim$ 200 ms). The laser oscillation induces an energy spread in the beam and the gain drops below the threshold, only to recover and become again higher than the losses — a detuning of the RF cavity equivalent to ± 5 µm mirror displacement stops the laser. A pulsed Q switch laser can thus be obtained by modulating slightly the radiofrequency or by translating periodically the electron beam. For the same average power, the peak power can be increased by a factor 10^2-10^3, with a very regular structure (Fig. 5b).

With modern storage rings such as Bessy (in operation) or Super ACO (under construction) which are not specially optimized for free electron laser experiments but have (Bessy) or should have (Super ACO) small emittances one can expect the characteristics given in Table I. The lower limit of the expected wavelength range is determined mainly by the mirror reflectivity which drops rapidly below 120 nm. The output power of the laser is limited to a few % of the synchrotron power emitted all around the ring so for Super ACO the expected laser power is of the order of a few watts in the visible and the UV.

We should mention that a 1 GeV storage ring specially designed for a free electron laser (with 25 m undulator) has been proposed by J. Madey and is under construction at Stanford. With this machine, a rather high gain should be obtained and should allow radiation at lower wavelength.

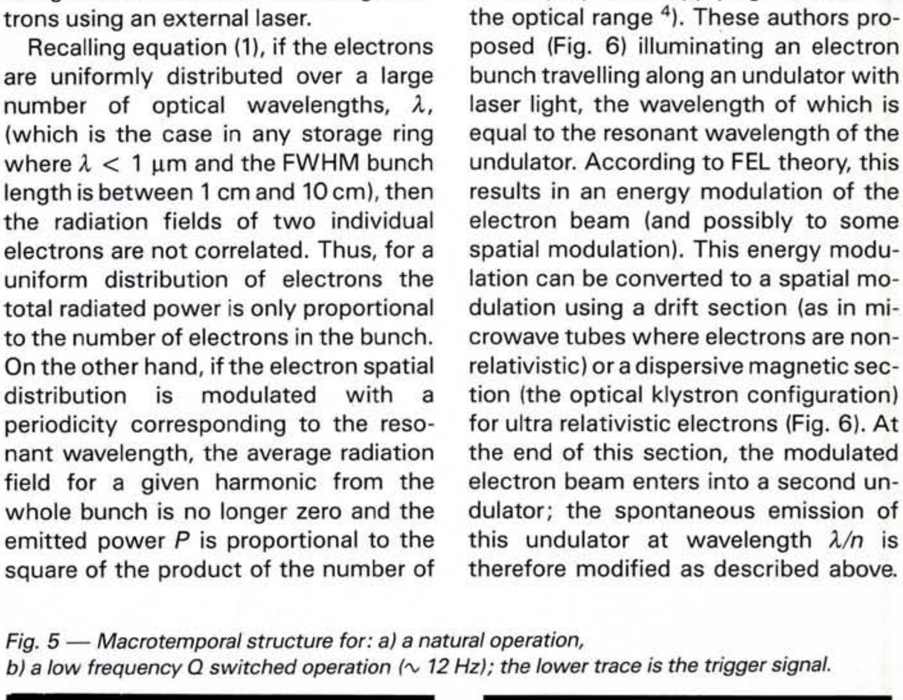
Ultraviolet Generation from an Optical Klystron

We have seen that it is not easy to make a free electron laser for $\lambda < 1200$ Å although by using an undulator and a monochromator it is possible to obtain coherent radiation with a good average power. However, for those interested in peak power a different technique is being studied based on bunching electrons using an external laser.

Recalling equation (1), if the electrons are uniformly distributed over a large number of optical wavelengths, λ, (which is the case in any storage ring where $\lambda < 1$ µm and the FWHM bunch length is between 1 cm and 10 cm), then the radiation fields of two individual electrons are not correlated. Thus, for a uniform distribution of electrons the total radiated power is only proportional to the number of electrons in the bunch. On the other hand, if the electron spatial distribution is modulated with a periodicity corresponding to the resonant wavelength, the average radiation field for a given harmonic from the whole bunch is no longer zero and the emitted power P is proportional to the square of the product of the number of electrons and the Fourier coefficient for this harmonic. In this case, the spontaneous emission of the undulator is strongly enhanced, and, in addition, the coherence properties of the radiation are modified. This effect in the microwave range was studied several years ago and led to the development of klystrons.

Several authors have published theoretical proposals, applying this idea to the optical range [4]). These authors proposed (Fig. 6) illuminating an electron bunch travelling along an undulator with laser light, the wavelength of which is equal to the resonant wavelength of the undulator. According to FEL theory, this results in an energy modulation of the electron beam (and possibly to some spatial modulation). This energy modulation can be converted to a spatial modulation using a drift section (as in microwave tubes where electrons are nonrelativistic) or a dispersive magnetic section (the optical klystron configuration) for ultra relativistic electrons (Fig. 6). At the end of this section, the modulated electron beam enters into a second undulator; the spontaneous emission of this undulator at wavelength λ/n is therefore modified as described above.

Fig. 5 — Macrotemporal structure for: a) a natural operation,
b) a low frequency Q switched operation ($\sim$ 12 Hz); the lower trace is the trigger signal.



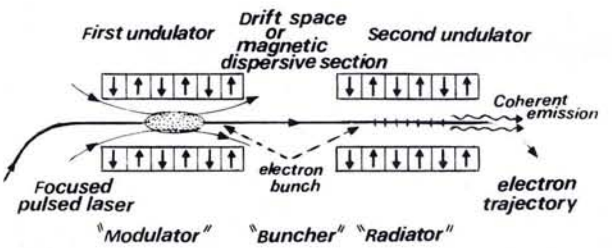


Fig. 6 — In the optical klystron an external laser is focussed on the electron bunch in the first undulator producing a velocity modulation which the dispersive section converts to a spatial modulation at the laser wavelength. Coherent radiation is produced in the last undulator.

This technique avoids the use of mirrors to produce UV light and with most of the existing storage rings should be efficient in producing light of wavelength between about 100 and 2000 Å starting with a commercially available visible or UV laser. Although this process is often called "multiplication" or "up-conversion", it is different from the usual harmonic production since the coherent output power is taken from the electron energy and not from the pumping laser.

The ratio, R_n, of the coherent over the incoherent (spontaneous) emission, for the harmonic n of the laser frequency, for a given laser power and within the bandwidth of the coherent emission, is given by:

$$R_n \sim N \times I \times f_n^2, \qquad (2)$$

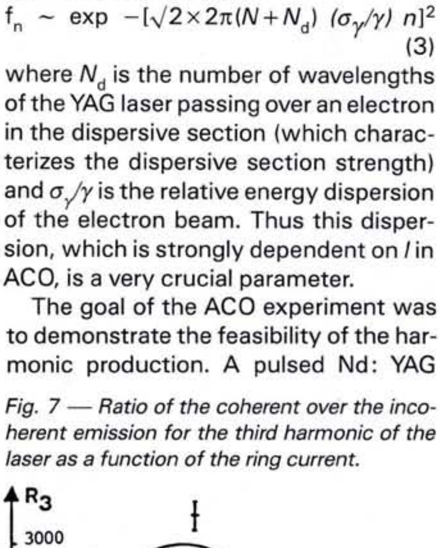
where N is the number of periods of the radiator, I the ring current and f_n the spontaneous emission modulation rate, resulting from the interference of the two undulators at wavelength λ_L/n. This interference is driven by the strength of the dispersive section and the energy spread of the electrons:

$$f_n \sim \exp -[\sqrt{2} \times 2\pi (N+N_d)\ (\sigma_\gamma/\gamma)\ n]^2 \qquad (3)$$

where N_d is the number of wavelengths of the YAG laser passing over an electron in the dispersive section (which characterizes the dispersive section strength) and σ_γ/γ is the relative energy dispersion of the electron beam. Thus this dispersion, which is strongly dependent on I in ACO, is a very crucial parameter.

The goal of the ACO experiment was to demonstrate the feasibility of the harmonic production. A pulsed Nd: YAG laser with a fundamental line of 1.06 μm was focussed into an optical klystron on the storage ring ACO working at 166 MeV. The 3[rd] harmonic was observed at 3550 Å. At this energy, the modulation rate f_3, is much smaller than one owing to the anomalous bunch lengthening on ACO which makes the energy spread much larger than the nominal energy spread at 166 MeV (1.4×10^{-4} for $I <$ 0.01 mA). Also there is an additional energy spread due to the interaction with the YAG pulse, since the ring energy damping time is 180 ms at 166 MeV and the YAG repetition rate is 20 Hz.

Fig. 7 — Ratio of the coherent over the incoherent emission for the third harmonic of the laser as a function of the ring current.



Since the spontaneous emission is very broad ($\triangle\lambda \cong 200$ Å), the measured values of R_3 depend linearly on the spectral resolution used and an absolute value can be set only by assuming a given value for the coherent emission spectral width. By using a monochromator of spectral resolution $\triangle\lambda \cong 0.3$ Å the value of R_3 (corresponding to a small solid angle), has been measured (Fig. 7). The theoretical curve corresponding to formula (2) is also drawn on Fig. 7. It can be seen that the variations of R_3 with the ring current are qualitatively well explained by two opposite effects: the increase in the number of electrons and the stong decrease of the modulation rate ($f_3 \cong 10^{-3}$ for $\sigma_\gamma/\gamma = 12 \times 10^{-4}$ at $I \cong 10$ mA). The combination of these two factors produces the maximum observed at about 1 mA with R_3 approximately 3×10^3 (for $\triangle\lambda = 0.1$ Å).

Future Experiments

On ACO we expect to use a shorter pulse (3 ns) Nd: YAG laser as a pump when the available power, for about the same input energy, will be higher. By working between 240 and 350 MeV the numerical calculations show that the region 10-20 eV can be reached and that $10^7 - 10^8$ coherent photon/pulse will be produced for an input power of 50 MW.

Although the working energy of Super ACO will be 800 MeV, the machine has been designed to be able to run at lower energies. We have calculated the number of photons produced at 400 MeV (where the energy spread σ_γ/γ is only 2.5×10^{-4}) by taking reasonable figures for the parameters ($\rho_e = 1.3 \times 10^{12}$ electron/cm^3, $I = 7$ mA/bunch, bunch length = 0.7 cm). Figure 8 shows the results obtained for an undulator optimized for FEL studies rather than for this experiment. However, one can see that typically 10^{10} photon/pulse can be obtained down to at least 500 Å, corresponding to coherent peak powers close to 1 kW. These high peak powers should allow multiphoton excitation, studies of non linear processes in the VUV range, time resolved (30 ps) angular resolved photoemission, production of excited states in atoms and molecules. We should mention that this is not the only way to obtain pulsed coherent radiation in the VUV. Harmonic generation from a pulsed gas jet has already been achieved by different groups. However, in our case the advantage is to be able to couple ordinary synchrotron radiation with this coherent beam.

REFERENCES

1. Deacon D.A.G., Elias L.R., Madey J.M.J., Ramian G.J., Shewettman M.A. and Smith J.I., *Phys. Rev. Lett.* **38** (1977) 892.
2. Billardon M., Elleaume P., Ortega J.M., Bazin C., Bergher M., Velche M., Petroff Y., Deacon D., Robinson Ke and Madey J.M., *Phys. Rev. Lett.* **51** (1983) 1652.
3. Elleaume P. and Farge Y., *Europhysics News* **13** (1982) 4.
4. Csonka P.L. "Enhancement of synchroton radiation by beam modulation", *Particle Accel.* **8** (1978) 225;
Vinokurov N.A. and Skrinskii A.N., "Limit power of an optical klystron on a linear electron accelerator", *Preprint IMP 78-88*, Novosibirsk (1978);
Coison R. and de Martini F., "Free electron coherent relativistic scatterer for U.V. generation", *Phys. of Quant. Electr.* **9** (Addison Wesley) 1982, 939-960.
5. Girard B., Lapierre Y., Ortega J.M., Bazin C., Billardon M., Elleaume P., Bergher M., Velghe M. and Petroff Y., "Optical frequency multiplication by an optical klystron on the A.C.O. storage ring" *Phys. Rev. Lett.* **53** (1984) 2405.

Fig. 8 — Expected number of photons on the ring Super ACO for various harmonics for a pump laser at λ = 2260Å.