Vortices in Rotating Superfluid ^{3}He

G.E. Volovik, Moscow

(L.D. Landau Institute for Theoretical Physics)

One decade ago, three superfluid phases (A, A_1, B) of the lighter helium isotope ^{3}He were discovered only a millidegree away from absolute zero. In addition to the broken gauge symmetry, which is responsible for the frictionless superflow through narrow channels, it was found that other symmetries were broken. The states of the new superfluids are not invariant under mechanical rotation and their anisotropy makes them the first superfluid liquid crystals in Nature. Moreover, invariance under spin rotation is broken: the nuclear spins of the ^{3}He atoms are coherently ordered, as in antiferromagnets. This magnetic behaviour of the superfluid phases of ^{3}He can be probed by NMR techniques, which have proved to be very powerful.

Coupling between different broken symmetries results in striking effects that were completely unexpected a decade ago. Most of the new features have been observed during the investigation of quantized vortices, in a unique rotating cryostat, built by a joint Finnish-Soviet research group at the Helsinki University of Technology. NMR experiments showed that the rotating A phase possesses continuous vorticity, which is forbidden in "classical" superfluids such as superfluid ^{4}He and the electron liquid in metallic superconductors. In those systems, superflow is necessarily vortex free; the generation and motion of singular quantized vortices is accompanied by dissipation and signals the destruction of the pure superfluid state.

Continuous vorticity is the most interesting manifestation of the intrinsic coupling between different broken symmetries in the A phase. This coupling also results in the unusual topological conservation laws governing the transformation of continuous into singular vortices and vice-versa. The analogous intrinsic coupling in the B phase gives rise to a magnetic moment concentrated inside the cores of the quantized vortices. This magnetization, though extremely small (10^{-11} nuclear magnetons per atom of the liquid), was detected by the Finnish-Soviet team thanks to a peculiar gyromagnetic effect. Quantized vortices themselves produce a further reduction of symmetry in the A and B phases. In addition to the translational symmetry broken by the formation of a periodic vortex lattice in a rotating sample, discrete symmetries are also broken inside the vortex cores. This is manifested in a new transition observed in the B phase. Moreover, parity violation inside the vortex core can produce a spontaneous electric polarization of the vortices in both the A and B phases.

All these properties make quantized vortices in ^{3}He one of the most interesting objects in condensed matter physics.

Quantized Vortices in "Classical" Superfluids

Both stable helium isotopes, ^{3}He and ^{4}He, form quantum liquids at low temperatures and do not crystallize at ordinary pressures even at absolute zero. The large zero-point motion of these light atoms, together with their inert electronic structure, prevents their localization in a periodic lattice. Crystallization occurs only at high pressures: at 25 bar for ^{4}He and at 34 bar for ^{3}He.

The principal difference between these chemically indistinguishable twins is revealed only at temperatures low enough for the quantum character of the motion to become evident. The different quantum statistics of an ensemble of ^{3}He and ^{4}He atoms determines their vastly different properties and the huge difference in the value of their superfluid transition temperature T_c: whereas for liquid ^{4}He $T_c = 2.2$ K at the saturated vapour pressure, for ^{3}He it is only 1.0 mK.

This striking difference stems from the fact that a ^{4}He atom contains an even number of fermions with half-integer spin and so constitutes a quantum-statistical ensemble of bosons, while the ^{3}He atom has one neutron less and so an uncompensated nuclear spin 1/2. Consequently, liquid ^{3}He obeys Fermi statistics.

The fundamental property of a boson system is to form a highly coherent state, a Bose condensate, at low enough temperatures. A macroscopic fraction of the atoms are then occupying the same quantum state, and their motion is highly correlated and can be described by a single wavefunction ψ. This new type of correlated, frictionless motion in liquid ^{4}He below $T_c = 2.2$ K is at the origin of the superfluidity. Outside the condensate the atoms form a normal component, which has the properties of ordinary viscous liquid. The density of the superfluid component which is zero at T_c, gradually increases with decreasing temperature, and at $T = 0$, when the normal motion is finally frozen out, all the fluid consists of the superfluid component.

The coherence of the superfluid motion is expressed explicitly by a simple equation, relating the superflow velocity $\mathbf{v}_s$ with the phase Φ of the Bose condensate wavefunction

$$\psi = |\psi| \exp(i\phi) : \mathbf{v}_s = (\hbar/m_4)\, \nabla \phi$$

m_4 being the mass of the ^{4}He atom. The Landau condition $\nabla \times \mathbf{v}_s = 0$ is thus fulfilled for the superfluid velocity which means that the motion of the superfluid component is always vortex free (i.e. pure potential flow). The superfluid component cannot therefore take part in the rotation of the liquid. From a formal view-point, in such a superfluid state, invariance with respect to shifts in the origin of ϕ (gauge symmetry) is broken.

Pure potential flow is not the only property particular to superfluid motion. From the expression for the superfluid velocity, one may conclude that the superflow should be quantized. Indeed, the circulation of $\mathbf{v}_s$ along a closed path is expressed in terms of the change of the phase ϕ along the path:

$$\oint \mathbf{v}_s \cdot d\mathbf{r} = (\hbar/m_4)\, (\phi_{final} - \phi_{initial}).$$

Only if the phase changes by $2\pi N$, where N is an integer, will the condensate wavefunction ψ coincide with its initial value. Thus the circulation of superfluid velocity may assume finite values, but quantized in units of the circulation quantum $\hbar/m_4$.

In a normal liquid, non-zero circulation implies the existence of vorticity in the liquid. The same holds in a superfluid, only the vorticity is quantized and is therefore stable. The superfluid velocity around the axis of a vortex increases with decreasing distance r from the axis as $v_s = N\hbar/m_4 r$, tending to infinity at the vortex axis. In order to avoid a divergence in the kinetic energy of the superflow, the superfluid density must go to zero at the vortex axis.

Quantized vortices in superfluid ^{4}He were first introduced by Lars Onsager and then invoked by Richard Feynman for an explanation of the experiments which showed that the superfluid component is involved in rotation just as the ordinary viscous liquid. Feynman explained this paradox in terms of quantized vortices threading the rotating fluid and forming a regular structure. In equilibrium, the area density n of vortices is proportional to the angular velocity of rotation: $n = (m_4/\pi\hbar)\,\Omega$. Then the averaged superflow around the vortices closely mimics the normal rotation of a viscous liquid. The same situation arises in superconductors, where the quantized vortices are called Abrikosov vortices.

Until the mid 1970's, the same behaviour was predicted for any superfluid system and only after investigating the superfluid properties of liquid ^{3}He was it realised that the situation can in fact be much more interesting.

Continuous Vortices in the Superfluid Liquid Crystal ^{3}He-A

^{3}He atoms cannot form a Bose condensate as not more than one fermion can occupy a given quantum state. The coherent superfluid state of liquid ^{3}He arises only if the fermions are coupled in pairs; the pairs can then form a Bose condensate. In superconductors, where the pairing occurs between electrons, these entities are known as Cooper pairs.

In liquid ^{3}He, the formation of Cooper pairs only occurs below $T_c = 0.001$ K. While in all superconductors examined so far, electrons are paired with opposite spins, producing magnetically neutral pairs with zero spin ($S = 0$), ^{3}He atoms form magnetic pairs with the (nuclear) spin $S = 1$. The coherence of the magnetic moments in the Bose condensate results in an ordered magnetic structure, i.e.: the new phases of ^{3}He are liquid magnets. Moreover, as distinct from the Cooper pairs in superconductors, the shape (wavefunction) of the Cooper pairs in ^{3}He is not spherically symmetric which in the highly coherent Bose condensate produces an anisotropy of the whole liquid as in a liquid crystal. The anisotropy is the result of a non-zero quantum number L, characterizing the relative orbital motion of the ^{3}He atoms in the pairs. In ^{3}He, $L = 1$ while the motion of electrons in the Cooper pairs in superconductors is isotropic with $L = 0$.

S and L are not the only quantum numbers describing the internal motion; in addition, there are the projections of the spin μ and orbital momentum m in certain directions. Consequently different superfluid phases can exist, the three states that are now known being (i) the A phase with $\mu = 0$ and $m = 1$, (ii) the A_1 phase, which exists only in an applied magnetic field with $\mu = m = 1$, and
(iii) the B phase, with total angular momentum $J = 0$, where $\mathbf{J} = \mathbf{L} + \mathbf{S}$. The B phase may also be considered as an equal mixture of three substates with ($\mu = 1$, $m = -1$), ($\mu = -1$, $m = 1$), and ($\mu = 0$, $m = 0$). States with other sets of quantum numbers may exist in the cores of vortices.

The A phase has many features common to ordinary liquid crystals, resembling one with uniaxial anisotropy. The direction of the anisotropy axis ℓ coincides with the axis of quantization of the orbital motion of the ^{3}He atoms in the Cooper pairs. The similarity is reinforced by the variety of observed textures, characterized by the non-uniform distributions of ℓ in space. Textures arise due to the competition of different orientating effects: boundaries tend to orient ℓ along the surface normal, whereas superflow orients ℓ along the superfluid velocity. This anisotropy of superflow is an example of the mechanical mixture of superfluid and liquid-crystal properties.

However, the most striking property of the A phase is the non-trivial conjugation of superfluid and liquid-crystal behaviour: an ℓ-texture produces continuous vortex superflow in the A phase! In the presence of a non-uniform ℓ-texture, the Landau criterion is violated and the circulation of the superflow is not quantized. Although the gauge and rotational symmetries are broken separately, the combined gauge-orbital symmetry is retained.

This gives rise to many interesting phenomena. One of them has been discussed by David Mermin: there is a spontaneous superflow in a closed vessel, which is the result of the texture produced by the competition between different parts of the boundary. Another is the behaviour of the A phase in a rotating vessel. The A phase can support arbitrary vorticity so could become completely involved in the rotation. However, this would be costly in the energy of the texture. Instead, an intermediate situation arises in which the superfluid velocity $\mathbf{v}_s$ and the ℓ-texture are everywhere continuous and form a periodic structure, which imitates ordinary rotation on the average. The circulation of $\mathbf{v}_s$ along the boundary of a primitive hexagonal lattice cell of the periodic texture is $2\hbar/M$ where $M = 2m_3$ is the mass of a Cooper pair. Thus each cell constitutes a vortex with two quanta of circulation, without any singularity on the vortex axis. An isolated non-singular vortex with two quanta of circulation was first described by Philip Anderson and Gerard Toulouse and independently by Vladimir Chechetkin.

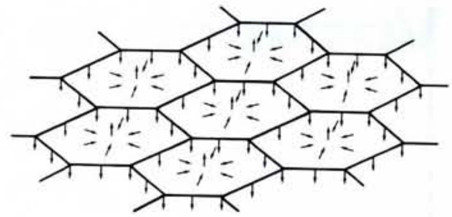


Fig. 1 — The periodic hexagonal ℓ-texture (arrows) in the rotating A phase. Each cell contains a continuous vortex with two quanta of circulation of the superfluid velocity along the boundary (solid lines).

Singly quantized vortices can also exist in the A phase with a core the size of the superfluid coherence length $\xi \sim 10^{-6} - 10^{-5}$ cm. Inside the core, the other superfluid phase exists with the quantum numbers $\mu = 0$ and $m = 0$. As in ^{4}He, these vortices can form a singular vortex lattice under rotation, which competes with the continuous one. Which is preferred depends on external conditions.

The singularity of the vortex with unit quantum of circulation and the continuity of the vortex with two quanta are the manifestation of the peculiar topology laws of the A phase, which in turn are the consequence of the specific symmetry breaking. Topological methods of investigation introduced in condensed matter physics by Toulouse, Maurice Kleman, Vladimir Mineev and the author, were inspired by the complicated textures encountered in superfluid ^{3}He. It follows that besides the usual quantum numbers and charges, such as spin, orbital quantum number, electric charge, etc., such structures are characterized by integer topological charges. The conservation of these charges explains the stability of different intrinsic defects in condensed matter: dislocations in solid crystals, disclinations in liquid crystals, domain boundaries in ferromagnets, and many others.

The topological charge N in the A phase has quite unusual properties. It assumes only two different values: 0 and 1 with the summation laws: $0 + 1 = 1$ and $1 + 1 = 0$. A singular vortex has the charge $N = 1$, while a continuous vortex has $N = 0$. The summation law $1 + 1 = 0$ means that any two singular vortices will coalesce into a non-singular one after collision.

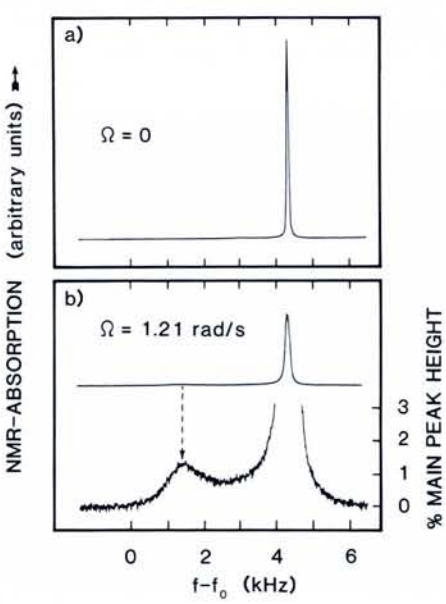
The investigation of the unique properties of vortices in the A phase was one of the tasks of the joint Finnish-Soviet research project ROTA based on a rotating millikelvin cryostat (Fig. 2) which began operation in 1981. NMR allows one to measure with great precision the frequency of magnetic excitations which in antiferromagnetic superfluid ^{3}He, are collective magnetic modes known as spin waves some of which may be trapped by textures producing localized collective excitations with discrete frequencies below the continuous spectrum. The vortex texture in the rotating A phase should produce a satellite absorption peak caused by the excitation of spin waves localized on the vortex textures, the frequency of which should not depend on the angular velocity of rotation Ω, in contrast to the intensity which should be proportional to the density of the vortices and so to Ω.

In August 1981, Pertti Hakonen and the author observed the first reproducible effect of rotation on ^{3}He-A: a broadening of the main peak roughly proportional to Ω. It is caused, as was explained by Igor Fomin and Slava Kamensky, by spin-wave scattering on the vortex lattice. Half a year later, Hakonen, Olli Ikkala and Islander resolved the vortex satellite peak in the NMR signal, and from a comparison with the theoretical calculations made by Harri Seppälä and the author, it could be concluded that continuous vortices had been discovered.

Magnetic Vortices in the Quasi-isotropic ^{3}He-B

Of the three superfluid phases of liquid ^{3}He, ^{3}He-B was believed to be the least interesting, because it lacks the intrinsic coupling between the superfluid and liquid-crystal properties. The quantized vortices in the B phase were considered to have the same properties as the "classical" vortices in superfluid ^{3}He. However, the size of the core, hundreds of times larger than in ^{4}He, and the peculiar intrinsic coupling between liquid-crystal and magnetic properties makes the B phase vortices intriguing.

Fig. 3 — The satellite absorption peak in the NMR signal caused by the excitation of spin waves trapped by the continuous vortex texture. The additional broadening of the main peak caused by rotation is also shown.



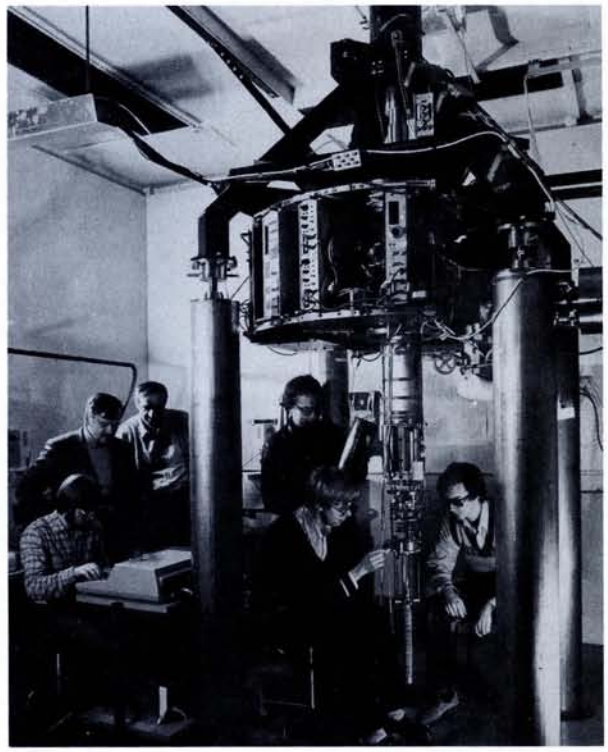


Fig. 2 — The ROTA cryostat at the Low Temperature Laboratory, Helskinki University of Technology for which Seppo Islander was responsible. This device has produced the first data on vortices in rotating superfluid ^{3}He, which have been identified as quite new and unexpected structures. The ROTA collaboration includes both the Institute for Physical Problems and the Landau Institute for Theoretical Physics in Moscow.

At first glance, a Cooper pair in the B phase is completely isotropic, since it has $J = 0$. However, in contrast to a superconductor where the orbital and spin motions are decoupled ($L = 0$ and $S = 0$), the ^{3}He-B Cooper pair is isotropic only under combined rotations in the spin and orbital spaces, but is anisotropic under separate rotations. This is the essence of the so-called broken relative spin-orbital symmetry, which produces the intercoupling of magnetic and liquid-crystal behaviour. This broken symmetry results in a peculiar degeneracy of the B states, described by the orthogonal matrix $R_{\alpha i}$. The state labelled by the matrix $R_{\alpha i}$ differs from the state with $J = 0$ by a rotation of the spins of Cooper pairs given by the matrix $R_{\alpha i}$. It is characterized by the quantum number $J' = 0$, where $J_i' = L_i + S_\alpha R_{\alpha i}$.

The weak spin-orbital interaction partially lifts this degeneracy: in the states with minimal spin-orbital energy, the spins make a fixed angle with respect to the common axis **n** of spin and orbital anisotropy whose orientation defines the remaining degeneracy. This angle $\Theta_0 = \arccos(-\frac{1}{4}) \cong 104°$ is seen in many different NMR experiments, and is known as the magic angle. The anisotropy, produced by the weak spin-orbital interaction, is rather small; for example, the difference between the longitudinal and transverse magnetic susceptibilities is $\chi_\parallel - \chi_\perp \sim 10^{-5}\,\chi_\parallel$. All orientating effects on the **n** vector are thus tiny, five orders of magnitude less than the orientating effects on the ℓ vector in the A phase: the B phase is quasi-isotropic. Nevertheless, NMR absorption is sufficiently sensitive to detect it.

Alik Gongadze, Givi Gurgenishvili and Gogi Kharadze proposed looking for quantized vortices in ^{3}He-B through their orientating effect on the **n** texture caused by the small anisotropy of superflow around the vortices. The effect was seen by Hakonen, Ikkala, Islander and Yuri Bunkov at the beginning of 1982. Applying an axial magnetic field they observed a series of peaks in the transverse NMR signal, corresponding to spin waves localized on the **n** texture, which is always present, because perturbations created by the boundaries penetrate into the bulk liquid. The splitting $\Delta\nu$ between these peaks increased appreciably upon rotation.

The most surprising result of this experiment was in the temperature dependence of $\Delta\nu$: it exhibited a discontinuity at $T = 0.6\,T_c$ (at a pressure of 29.3 bar), manifesting the features of a first-order phase transition. The transition was seen only under rotation, but the transition temperature did not depend on the angular velocity of rotation Ω!

Such behaviour is consistent only with a transition inside the vortex core: this has non-trivial structure that is different below and above the transition. New experiments were therefore conducted by Bunkov, Hakonen, and Matti Krusius with an external magnetic field tilted with respect to the axis of rotation, as according to the hypothesis, the transition temperature should not depend on the tilting angle. In practice, the effect of rotation proved to be stronger, the main peak shifting substantially. However, the temperature dependence jumped at $T = 0.6\ T_c$ for all angles!

The next series of experiments, which resulted in the discovery of the magnetic moment of the vortices in ^{3}He-B, was conceived by Mineev and the author and was based on the rigidity of an ordered state. In a solid crystal, because of rigidity, a small external perturbation cannot excite an isolated atom, and excites instead the collective motion of phonons. The same holds for an ordered magnet, where spin waves are excited in NMR.

Anthony Leggett found that because of rigidity in superfluid ^{3}He-B: the quantum state of any Cooper pair with the quantum number $J' = 0$ cannot be changed by a weak external perturbation, e.g. by an applied magnetic field, because the states of all the pairs are correlated. But a magnetic field induces a magnetization $\mathbf{M} = \chi\mathbf{H}$ and therefore also a spin density $\mathbf{S} = \chi\mathbf{H}/\gamma$, where γ is the gyromagnetic ratio for a ^{3}He nucleus. In order to conserve $J'_i = L_i + S_\alpha R_{\alpha i} = 0$, an orbital angular momentum $\mathbf{L}$ of the pairs should arise and the Cooper pairs should begin to rotate under the influence of a magnetic field. We proposed to measure this effect in a rotating vessel, where the orbital momentum of the Cooper pairs tends to orient itself along Ω. The corresponding orientational energy

$$-\Omega\cdot\mathbf{L} = \chi H_\alpha R_{\alpha i}\Omega_i/\gamma$$

and therefore the NMR-signal should change sign if one reverses the sense of rotation or the magnetic field.

Such a gyromagnetic effect was discovered in the NMR experiments by Hakonen, Krusius, Juha Simola and Bunkov. Quite unexpectedly they found also that the gyromagnetism undergoes a jump at $T = 0.6\ T_c$. It was immediately understood that only the magnetization, which is concentrated inside the vortex core and which changes in magnitude at the core transition, can produce such an effect. The magnetic moment of the ^{3}He-B vortices arises because the superflow around the vortex induces an internal rotation of the Cooper pairs inside the core with the orbital momentum $\mathbf{L}$ directed along the vortex axis Ω. Due to the rigidity of their common quantum state this produces a spin $S_\alpha = -R_{\alpha i}L_i$ and therefore a magnetization inside the vortex core, $M_\alpha = \gamma S_\alpha \sim -R_{\alpha i}\,\Omega_i$. The interaction of the magnetic moments of the vortices with an external magnetic field gives rise to the additional gyromagnetic energy $-\mathbf{M}\cdot\mathbf{H} \sim H_\alpha R_{\alpha i}\Omega_i$, which essentially depends on the vortex core structure.

Possible core structures have recently been classified by Martti Salomaa and the author using symmetry arguments. It was found that an axially symmetric vortex in ^{3}He-B with unit quantum of circulation may be in five different states, distinguished by discrete symmetries. These states are labelled o, u, v, w, and uvw; in all of them the vortex possesses a magnetic moment.

The most symmetric o vortex has a normal core with superfluidity destroyed on the vortex axis, while in the u, v, and w vortices the discrete symmetry is partially destroyed, and in the uvw vortex it is completely broken.

However, the complete identification of the new vortices will require a large amount of further experimental and theoretical work and no doubt new puzzles and unexpected phenomena will continue to arise during the investigation of quantized vortices in rotating superfluid ^{3}He.

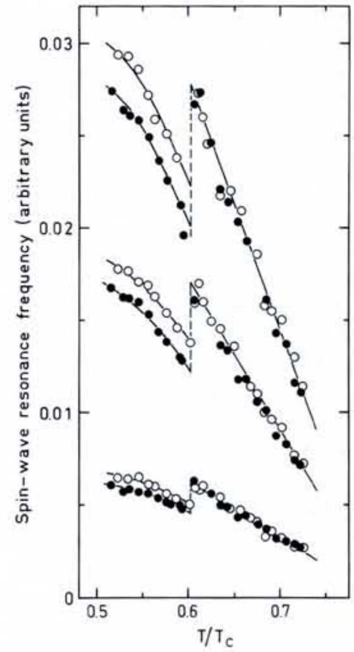


Fig. 4 — The gyromagnetic effect in ^{3}He-B: the resonance frequency measured during rotation with Ω parallel to the magnetic field **H** *(open circles) differs from that with Ω antiparallel (closed circles). The effect is large at $T < 0.6\ T_c$ and is hardly seen at all for $T > 0.6\ T_c$; the vortices below and above the transition temperature have different core structure and therefore also different magnitudes of intrinsic magnetic moment.*

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