The energy of a nucleus in an intrinsic configuration, ν with associated energy E_ν and moment of inertia J is

$$E = (\hbar^2/2J)\,R^2 + E_\nu$$
$$= (\hbar^2/2J)\,(I-i)^2 + E_\nu \qquad (2)$$

Here the total angular momentum I is assumed to be composed of two components (Fig. 1): that generated by collective rotations about an axis perpendicular to the nuclear symmetry axis, labelled **R**, and the angular momentum of the excited nucleons, **j**. The total component of **j** aligned with the rotational axis x is designated by $i = \Sigma j_x$. The first derivative of the rotational energy with respect to spin defines a kinematic moment of inertia, $J^{(1)}$:

$$dE/dI = (\hbar^2/J^{(1)})\,(I-i) \qquad (3)$$

A moment of inertia, $J^{(2)}$ can also be defined in terms of the second derivative:

$$d^2E/dI^2 = \hbar^2/J^{(2)} \qquad (4)$$

This dynamical moment of inertia is related to the difference in transition energies between neighbouring transitions in a rotational sequence. Therefore, it is the $J^{(2)}$ that is measured by the width of the central valley in the $E_{\gamma 1} - E_{\gamma 2}$ correlation plots.

The width of the valley in Fig. 2 is nearly constant over a large frequency range, $0.35 < \hbar\omega < 0.6$ MeV, indicating a nearly constant moment of inertia. The observed value is only slightly smaller than the moment of inertia of a rigid prolate ellipsoid with deformation $\beta = 0.3$. Similarly the narrow ridges imply that the spread in the values of $J^{(2)}$ for the various configurations rotating at these frequencies is small. Such large nearly constant values of the moment of inertia for $^{167,\,168}$Yb suggest that at large frequencies where the pairing correlations are expected to be small, these nuclei behave much like macroscopic rotors.

Perspectives

As discrete gamma-ray spectroscopy approaches the regime where pairing correlations appear to become less important, it is interesting to speculate on the future of such studies. Much of the recent discrete line studies have addressed the question of the frequency and configuration dependence of pairing correlations. In the region where such correlations are minimal, more specific information on other spectroscopic quantities, e.g. nuclear shapes and the details of the nuclear potential, is already emerging.

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Supersymmetry in Nuclei

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Symmetry considerations are playing an increasingly important role in all fields of physics. In particular, in the last few years, the concept of *dynamic symmetry* has emerged as a powerful tool in the study of complex systems. This concept has been further extended to include a new, more elaborate type of symmetry, called supersymmetry. Applications of dynamic symmetry considerations to nuclear spectra have provided (i) a deeper understanding of properties of nuclei, (ii) an elegant and very concise classification scheme of these properties and finally (iii) led to the discovery of experimental examples of dynamic supersymmetries in physics.

Dynamic Symmetries

The property that makes a dynamic symmetry a powerful tool in analyzing complex systems is that, when such a symmetry exists, the energy levels (and other properties) can be written in closed form. These closed forms, containing only the quantum numbers labelling the states, can be easily checked by experiment. In addition to providing concise and elegant classification schemes, dynamic symmetries give clues to the underlying dynamics and almost invariably lead to a fundamental understanding of the problem.

The oldest and most familiar example of a dynamic symmetry is provided by the hydrogen atom. Pauli (1926) and Fock (1935) showed that one could write a closed form for the energy levels

$$E(n,\ell,m_\ell) = -A/n^2, \qquad (1)$$

because of the existence of a dynamic symmetry, O(4). However, dynamic symmetries did not receive much attention until the early 60s, when they were applied to the study of the low-lying spectra of hadrons. As is well known, it was suggested at that time by Gell-Mann and Ne'eman that these spectra could be classified as representations of an SU(3) group. This led to an energy formula (the Gell-Mann-Okubo mass formula)

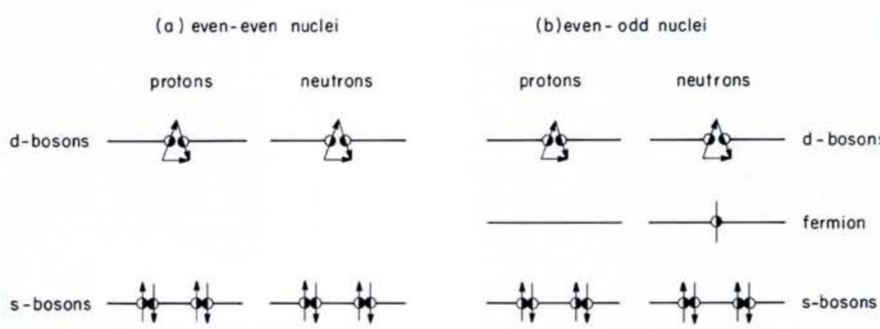
$$E(I,I_3,Y) = a + bY + c[I(I+1) - \frac{Y^2}{4}] \qquad (2)$$

which has SU(2) U(1) dynamic symmetry, and later to an understanding of the underlying dynamics in terms of quarks.

Dynamic Symmetries in Nuclei

In recent years, the concept of dynamic symmetries has been applied to the study of complex nuclei, and has led to major advances in this field. The applications are based on a nuclear model, called the interacting boson model, that I now briefly review. In 1974, Arima and I[1]) suggested that the low-lying states of nuclei with an even number of protons and neutrons could be described by assuming that these nuclei are constructed out of boson-like building blocks. These building blocks were later interpreted as being highly correlated pairs of protons and neutrons[2] similar to the Cooper pairs of the electron gas. Of all the possible values of the total angular momentum, J, of the pairs, only $J=0$ and $J=2$ were retained. The $J=0$ pairs were called s-bosons, while the $J=2$ pairs were called d-bosons, as shown in Fig. 1. Since the single component of the s-boson and the five components of the d-boson span a six-dimensional space, we suggested that the low-lying spectra of even-even nuclei be classified as

Fig. 1 — Schematic representation of the structure of even-even (left) and even-odd (right) nuclei. In the low-lying states of even-even nuclei all protons and neutrons are paired together; in odd-even nuclei complete pairing is impossible and in addition to s- and d-bosons there is an unpaired nucleon (fermion).



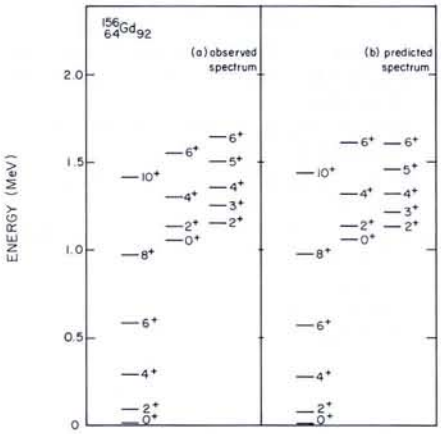


Fig. 2 — The energy spectrum of $^{156}_{64}Gd_{92}$ is shown here as an example of a dynamic symmetry in nuclear physics, specifically SU(3). On the left is the observed spectrum, on the right that predicted by Eq. (3). Each allowed energy level is labelled by its total angular momentum and parity. In both the observed and predicted spectra, there are more energy levels at higher energies, but only levels up to ~ 1.5 MeV are shown.

representations of an SU(6) group. The group SU(6) here plays the same role as Gell-Mann-Ne'eman SU(3) plays in the spectra of hadrons. A study of the group structure of SU(6) then revealed that, within the framework of the interacting boson model, three dynamic symmetries were possible, with dynamic groups SU(5), SU(3) and SO(6), according to all possible ways in which SU(6) can be broken down to the rotation group, SO(3). Each of the three symmetries leads to an energy formula similar to, but more complex than, Eqs. (1) and (2). The SU(3) symmetry, for example, leads to

$$E(N,\lambda,\mu,K,L,M_L) = \alpha[L(L+1)] + \beta[\lambda^2 + \mu^2 + \lambda\mu + 3\lambda + 3\mu], \quad (3)$$

where α and β are related to the strength of the boson-boson interactions and N,λ,μ,K,L,μ_L label the states.

Many examples of all three dynamic symmetries have been found experimentally in the spectra of medium-mass and heavy nuclei. An example is shown in Fig. 2. Although the symmetry is slightly broken, it provides an excellent zeroth-order description of the observed properties. Improvements can be (and have been) made by considering small symmetry breakings. As a result of this (and other) successes, dynamic symmetry considerations are now being widely used in nuclear structure studies.

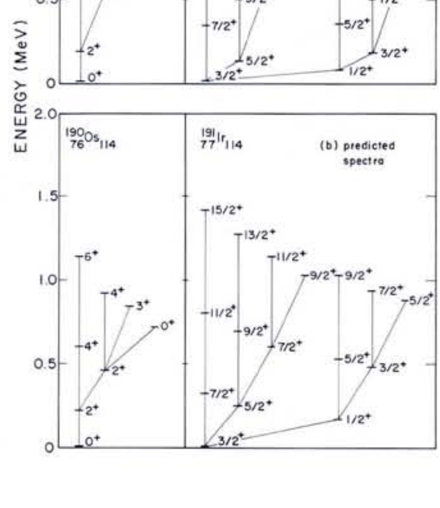
Supersymmetries

The symmetries discussed above apply separately either to a system of bosons or to a system of fermions. In 1973-74 Volkov and Akulov[3]) and Wess and Zumino[4]), while studying problems in elementary particle physics, extended the concept of symmetries to include *symmetries of mixed systems of bosons and fermions.* These symmetries have been called supersymmetries. Their treatment requires the introduction of a new mathematical formalism, known as graded Lie groups, or supergroups. Considerable effort has gone into the search for experimental evidence of supersymmetry in elementary particle physics. Several models with supersymmetric properties have been constructed but no experimental confirmation has yet been found.

Dynamic Supersymmetries in Nuclei

In 1980-81, Balantekin, Bars and I[5]) suggested that supersymmetry ideas could be applied to the study of nuclear spectra. The reason is that, in addition to even-even nuclei, one has also odd-even and odd-odd nuclei (and states in even-even nuclei with two or more unpaired particles). The treatment of these nuclei and states requires the simultaneous introduction of bosonic (correlated pairs) and fermionic (unpaired particles) de-

Fig. 3 — The observed (above) and theoretically predicted (below) energy spectra of $^{190}_{76}Os_{114}$ and $^{191}_{97}Ir_{114}$, are shown as examples of a dynamic supersymmetry in nuclear physics. Each allowed energy level is labelled by its total angular momentum and parity. The lines between levels indicate intense electromagnetic transitions.



grees of freedom, Fig. 1. Dynamic supersymmetries would now lead to energy formulae describing simultaneously sets of nuclei (supermultiplets), comprising both even-even and even-odd nuclei. The first experimental evidence of supersymmetry in nuclei was found in the spectra of the osmium-iridium nuclei, (Fig. 3), and is based on a classification in terms of the supergroup U(6/4). Since then, several other examples have been found in the same and in other regions, corresponding to other supergroups U(6/m). The search for even more examples is, at present, a very active area of research in Europe and throughout the world.

A general feature of the experimental examples found so far is that supersymmetry in nuclei appears to be more broken than normal symmetry. This is not an unexpected result since supersymmetry implies more stringent conditions on the interactions than normal symmetry. For example, it implies particular relations between the boson-boson, fermion-fermion and boson-fermion interactions. Nonetheless, supersymmetry appears still to be a useful tool in the study of spectra of odd-even nuclei. The full power of this tool has not yet been exploited.

Conclusions

Experimental examples of dynamic symmetries and supersymmetries have been found in the spectra of complex nuclei. These symmetries and supersymmetries have provided an elegant and concise classification of many nuclear spectra. In addition, they have given important clues for constructing a microscopic theory of collective states in nuclei, a subject not discussed here. Indeed, by studying this microscopic theory, one has been able to understand why a given nucleus can be described by a particular dynamic symmetry or supersymmetry.

The implications of the experimental discovery of supersymmetry in nuclei to other fields of physics are not yet clear. Supersymmetries observed in nuclei are of a type different from those mostly sought in elementary particle physics, since the bosons of the interacting boson model are actually correlated pairs of fermions (Cooper pairs) rather than fundamental objects. On the other hand, it may be that experimentally realizable supersymmetries are only of the type encountered in nuclear physics, i.e. where the bosons are composite rather than fundamental particles. In this connection, particularly interesting is some recent work of Y. Nambu (Chicago) who has discussed the analogy between the supersymmetry observed in nuclei and the motion of electrons around a vortex in a type II superconductor. Both problems arise in conjunction with spontaneous symmetry breaking of a nontrivial topological nature.

Be as it may, the occurrence of dynamic symmetries and supersymmetries in nuclei, has stimulated renewed interest in nuclear structure studies and led to major new advances in this field.

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Heavy-Ion Reactions

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Up to the mid-sixties, our knowledge of nuclear properties was confined to the individual behaviour of nucleons inside the mean field which is responsible for nuclear stability. However, the evidence for rather large deformations in several regions of the periodic table, and extensive studies of fission made it clear that many nuclear excitations exhibit a strongly collective aspect. Any nuclear model has to account for these collective features, which should ultimately be obtained from the nucleon-nucleon interaction only. At the extreme, nuclear theory introduced the concept of "nuclear matter" where any finite-size effect was neglected.

Heavy-ion collisions offer the opportunity of making large pieces of nuclear matter interact, and observing a host of extremely different collective phenomena. Each of them is a challenge to our theories of nuclear models and reactions. The extreme variety of the experimental situations makes heavy-ion reaction studies a fascinating and rich field of investigation.

The acceleration of heavy-ions was made possible during the sixties, by a technical breakthrough in ion sources. To begin with, beams of carbon, nitrogen and oxygen ions were accelerated, but recently, uranium beams have been produced with energies ranging from 10 MeV to 0.96 GeV per nucleon (u). The available energy range (in MeV/u) and the spectrum of ions to be accelerated are now used to characterize the machines in use and under construction Fig. 1.

Probably, the main interest initially found in heavy-ion collisions relied upon kinematical arguments: by an appropriate choice of the target-projectile pair

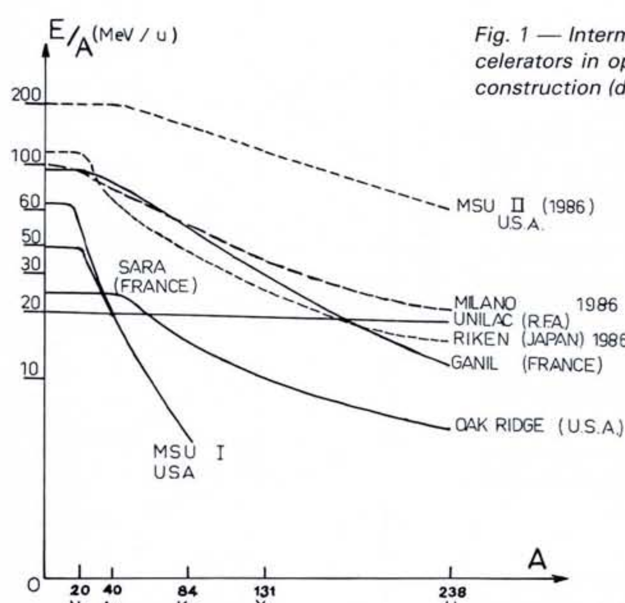

Fig. 1 — Intermediate-energy heavy ion accelerators in operation (full lines) or under construction (dashed).