Leonhard Euler 1707-1783

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Two hundred years after the death of Euler, research into his prolific writings continues to reveal his extraordinary prescience

In honour of by far the most productive mathematician and physicist that has ever lived — Leonhard Euler — a memorial edition of his life and work[1]), referred to here as *EGB*, has been commissioned by his native city of Basle. Twenty nine specialists from ten nations and four continents have combined to produce this carefully blended synthesis of his scientific and literary work.

The *Opera omnia* of Euler, well-known to every mathematician and historian of science (their covers at least) have been appearing since 1911 through an international collaboration under the auspices of the Swiss Academy of Sciences. They occupy more than 70 quarto volumes and are being supplemented by a fourth series *(Series quarta)* of 14 volumes containing his scientific correspondence (Series IV A, 8 volumes) and note and day books (Series IV B, 6 volumes). Two volumes of these series have been published since 1975: volume IV A, 1 (1975) offers a global survey of the roughly 3000 known letters to and from Euler, and volume IV A, 5 (1980) contains the correspondence of Euler with Clairaut, d'Alembert and Lagrange[2]).

The figures alone show Leonhard Euler to have been one of the greatest scholars of all times. A cosmopolitan in the true sense of the word — he lived his first twenty years in Basle, was active altogether more than thirty years in Petersburg (now Leningrad) and a quarter of a century in Berlin (then in Prussia) — Euler attained a celebrity and popularity with which but few scholars, as e.g. Galileo, Newton and Einstein can compare.

In the 18th century, mathematics and physics were not yet separate disciplines and in the case of Euler it is particularly difficult to define his field of activity: his complete publications count "only" about 30 volumes (in modern terms) of pure mathematics; the majority of the remainder are spread over physics, astronomy, techniques (i.e. engineering sciences), philosophy, theology and music theory. Many of his manuscripts which are in Leningrad have not yet been studied thoroughly, nor have they been edited. We have still to see what lies in store for present day researchers, but it can be expected that not only historians will find them rewarding. In the following we shall try to summarise the main work of Euler in physics. For mathematics and disciplines not dealt with here, as well as Euler's biography see *EGB* which contains a bibliography of more than 700 titles of publications *about* Euler *(Burckhardt - Verzeichnis).*

Table 1 — Main Periods in the Life of Euler

1707 - 1727	Basle (Switzerland)
1727 - 1741	St. Petersburg (Russia)
1741 - 1766	Berlin (Prussia)
1766 - 1783	St. Petersburg

Mechanics

In the introduction to his two-volume *Mechanica* (Petersburg 1736) Euler presents an extensive revue of this science with, as main feature, the systematic application of analysis to the then current as well as new problems in mechanics. The predecessors of Euler had adopted a largely synthetic-geometric approach, Newton's immortal *Principia mathematica* serving as a pregnant example. Euler's approach here — as later also in optics — was fully analytical, applying systematic analytical methods that were to lead to clear and direct descriptions and then solutions of the relevant problems. The full title of the book expressed the essential theme: Mechanics or the Science of Motion, analytically presented.

Euler starts with the kinematics and the dynamics of a point mass in vacuum and in a resistant medium. The section dealing with the motion of a point mass under the influence of a force directed towards a fixed point is a brilliant analytical reformulation of the corresponding chapter in Newton's *Principia*. In the second volume he examines the forced motion of a point mass and solves a number of differential-geometric problems of surface theory and the theory of geodetic lines. Almost thirty years later, Euler gave in the *Theoria motus* (1765) a new presentation of point mechanics in three dimensions, resolving according to the model of Maclaurin (1742), the force vectors on to a fixed, orthogonal system of coordinates. In addition, in his analyses of rotating bodies, he established the differential equations relating the dynamics to the main inertial axis, which characterizes the motion. He also formulated the law, expressed as an elliptic integral, describing the motion of a rigid body round a fixed point, "Euler's angle", to which he was led when studying the precession of the equinoxes and the nutation of the Earth's axis. Other special cases of the theory of gyration, where the differential equations can be solved, were discovered and dealt with by Lagrange (1788) and by the Weierstrass student S.V. Kovalevskaja (1888).

In an appendix to his *Methodus inveniendi* ... (1744), his detailed exposition on the calculus of variations, Euler suggests a formulation for the famous-infamous "principle of least action" for the case of the motion of a point mass under the influence of a central force: the relevant trajectory minimizes the integral $\int mv\, ds$, whereas Maupertuis, the then President of the Academy of Berlin, at almost the same time, propounded the principle in a much more special way.

In the second appendix of the *Methodus inveniendi,...*, following a suggestion of Daniel Bernoullis — Euler applied variation calculus to the theory of beam bending and arrived via the relation

$$\int ds/R^2 = C \int y''^2\, dx \,/\, (1 + y'^2)^{5/2}$$

where R is the radius of curvature and C is a constant, at the really spectacular "Euler bending formula" without which engineering sciences would be unthinkable even today

$$P = \pi^2 Ek^2/4f^2$$

in which Ek^2 is the "absolute elasticity" (rigidity) and $2f$ the length of a bar supported at both ends. Besides this first calculation of an elastostatic eigen value, Euler was also the first to calculate the elasto-kinetic property of the eigen frequencies of a transversally oscillating beam.

Hydromechanics

In the domain of hydromechanics, the first big work of Euler was his extensive opus on vessels, the *Scientia navalis* (1749). This work represents after the *Mechanica* ... the second milestone in the development of rational mechanics and it has lost nothing of its importance up to the present day. Not only, for the first time, were the principles of hydrostatics set out in splendid clarity and based on it the scientific fundamentals of the theory of shipbuilding, but the subjects covered gave an overview of almost all relevant lines of development in mechanics during the 18th century (W. Habicht in *EGB* p. 243).

In the first volume Euler deals with the general theory of the equilibrium of floating bodies — at that time a "novum" — problems of stability as well as small oscillations in the neighbourhood of the state of equilibrium. In this connexion he defined, via the pressure of liquids (independent of the direction), the "ideal liquid", which, unquestionably provided Cauchy later on with the pattern for his definition of the strain tensor. The second volume gives the applications of the general theory to the special case of a ship. With the *Scientia navalis*, Euler established a new science and influenced markedly the development of navigation and marine engineering. Only a few specialists are aware of the fact that it is due to none other than Leonhard Euler that we first understood the technically realisable principles of the impeller drive and the propeller. The bold projects of Euler's time for its application were, of course, condemned to remain theoretical, since the necessary propulsion energies were not yet available.

Other work that is well known in the history of techniques, are Euler's experiments on the Segner hydraulic power machine and the associated theory of the water turbine. Just 40 years ago, J. Ackeret had such a turbine constructed according to the specifications of Euler and determined that its efficiency lay above 71%. This is a sensational result bearing in mind that today, employing the most up to date techniques, the efficiency of a turbine of comparable dimensions lies slightly over 80%.

The formulation of some truly classic papers on the analytical theory of fluid mechanics, for which Euler developed a system of fundamental formulae on hydrostatics and hydrodynamics, came in the early fifties of the 18th century. Among them one finds the continuity equation for fluids of constant density, the equation for velocity potential (normally named after Laplace) and the general "Euler equations" for the motion of ideal (i.e. frictionless flowing) compressible and non-compressible liquids. Characterising this area of work is the application of certain partial differential equations to the relevant problems. Of these, Euler was especially proud as we know from his own statements — and he had the right to be.

Optics

Euler was occupied with questions of optics during the whole of his life, and nowhere does one see more evidently than in this science the contrast with the school of Newton. In his first papers on optics (*Nova theoria lucis et colorum*, 1746) Euler opposed the corpuscular theory of Newton with a wave theory akin to Huygen's. However, in England, opposition to the emission theory was long in coming: with the exception of Robert Hooke, the first British physicist of high calibre openly to stand up against it was Thomas Young in his Bakerian Lecture — it is true, with the most important argument which Euler had not yet at his disposal: the theory of interference.

Subsequent to the utilization of the refractor telescope by Galileo and Harriot at the beginning of the 17th century, the then unavoidable colour rings in the picture field proved to be very disturbing. Because of this, David Gregory and Newton turned to the (in this respect) better reflector telescope. Only on the basis of Newton's examination of the dispersion of light in a prism could the possibility of eliminating the chromatic errors be envisaged. Newton himself initially considered it impossible to reach achromasy by employing materials of different refractive index. That was true also at the beginning for the London optician John Dollond, until he was successful (after having followed several misleading paths) in building an achromatic assembly from a combination of lenses of crown- and flintglass. The contribution of Euler to this famous discovery was considerable, although one is not allowed to say that Euler had discovered the achromatic telescope (*cf. EGB* p. 310). Dollond was decisively influenced by a publication (1749) of Euler as well as a paper by the Swedish Samuel Klingenstjerna, which was stimulated directly by the study of Euler.

All that has been said concerning methods in mechanics is no less true for the three-volume giant work *Dioptrica* (1769/71), which was for a long time the textbook on geometric optics and was Euler's own Synopsis. In contrast to his predecessors, who almost exclusively proceeded synthetically, Euler treated optics analytically. Certainly he restricted himself in his theory always to points on the axis, but in this case he treated aperture errors and chromatic errors thoroughly and completely as nobody else; so at least the theory of the astronomical telescope was brought to a tentative conclusion. But Euler was subject to a fundamental and fatal error in assuming that the aberration effects at axis-oblique light-incidence (aplanasy and koma errors) could be neglected with respect to aperture errors (spherical aberration). This is, in practice not the case since all errors are of the same order of magnitude. Nevertheless the findings of Euler are astonishing, even if one simply regards — in comparison with the famous optics of Gauss — his theory established in 1765 *Théorie générale de la dioptrique* (*cf.* W. Habicht in *EGB* p. 283).

One should not forget that one frequently finds in Euler's works remarks which are off the central theme. These have often fertilized or even anticipated the subsequent work of other scientists. In this respect one can cite the difference between light-power and illumination-power, as one meets later on in Lambert's *Photometria*. William Herschel, in a certain sense, was continuing Euler's dioptric studies when making calculations for duplets and telescope eyepieces.

The penetration and richness of Euler in all his work including optics remain astonishing — *Optica* occupy seven quarto volumes — and what is almost unbelievable, is that his main work on light was the work of a blind man.

REFERENCES

1. *Leonhard Euler 1707-1783, Beiträge zu Leben und Werk, Gedenkband des Kantons Basel-Stadt* (Birkhäuser Verlag, Basle) 1983, 555 pp.
2. *Op. cit.* pp. 489-509 and the prospectus *Leonhard Euler - Opera omnia* (Birkhäuser Verlag, Basle) 1982.

Table 2 — Main Works of Leonhard Euler

Shortened titles in chronological order according to year of publication

1736	*Mechanica* (2 volumes)
1738 1740	*Rechenkunst* (2 volumes) (Arithmetics)
1739	*Tentamen novae theoriae musicae* (Music theory)
1744	*Methodus inveniendi* (Calculus of variations)
1744	*Theoria motuum planetarum et cometarum* (Mechanics of celestial bodies)
1745	*Neue Grundsätze der Artillerie* (Ballistics)
1747	*Rettung der göttlichen Offenbarung gegen die Einwürfe der Freygeister* (Theology)
1748	*Introductio in analysin infinitorum* (Introduction, 2 volumes)
1749	*Scientia navalis* (Theory of vessels, 2 volumes)
1753	*Theoria motus lunae* (First lunar theory)
1755	*Institutiones calculi differentialis* (Differential calculus, 2 volumes)
1762	*Constructio lentium objectivarum* (Achromatic lenses)
1765	*Theoria motus corporum* (Second mechanics)
1766	*Théorie générale de la dioptrique* (Theory of lenses)
1768	*Lettres à une Princesse d'Allemagne* (Philosophical letters, 2 volumes)
1768	*Institutiones calculi integralis* (Integral calculus, 3 volumes through to 1770)
1769	*Dioptrica* (Optics, 3 volumes through to 1771)
1770	*Vollständige Anleitung zur Algebra* (Algebra, 2 volumes, 1768 preprint of a Russian translation)
1772	*Theoria motuum lunae* (Second lunar theory)
1773	*Théorie complète de la construction et de la manœuvre des vaisseaux* (Second theory of vessels)