Kenneth G. Wilson

Winner of the 1982 Nobel Prize in Physics for his contribution to the theory of phase transitions

Per Bak, Copenhagen

(H.C. Ørsted Institute)

Wilson's work is a remarkable example of ideas being developed in one area of physics — field theory of particle physics — being applied successfully to a seemingly completely different field — critical properties of materials undergoing phase transitions. Wilson's work has had a tremendous impact on both particle physics and solid state physics and the reward of the Nobel Prize has been warmly applauded by the physics community — we had been expecting it for a long time.

Wilson is generally regarded as one of the founders (together, in particular, with M.E. Fisher at Cornell and L.P. Kadanoff at Chicago) of the modern renormalization group theory of phase transitions. He was not the first to realize the similarity between the analysis of problems in relativistic quantum field theory and the unsolved problems of phase transitions, but he was the first actually to develop general methods for calculating the so-called critical indices, which characterize phase transitions. In particular, Wilson's theories provide insight into the concept of universality from the fact that the critical indices usually depend on the dimensionality and the symmetry only, not upon the microscopic details of the interactions of the system. It does not matter whether we are dealing with a magnetic, structural, or liquid-gas transition near the critical point.

A phase transition is usually described in terms of an order-parameter which is nonzero only on one side of the transition, for instance when the temperature is lower than a critical temperature T_c. For a magnetic system, the order parameter is the spontaneous magnetization, M. According to the theory of scaling developed by Kadanoff, and by Patashinsky and Pokrovsky in the sixties, the magnetization goes to zero as:

$$M \sim (T_c - T)^{\beta}$$

at the phase transition. The critical index β was found experimentally to have a value between 0.3 and 0.4 for all ferromagnets. The magnetic susceptibility χ is characterized by another index γ

$$\chi \sim (T - T_c)^{-\gamma}$$

and the specific heat by the exponent α

$$c_p \sim (T - T_c)^{-\alpha} .$$

Scaling theory gives relations between these exponents (such as $\beta = (2\text{-}\alpha\text{-}\gamma)/2$), but it does not yield a method for calculating them. The exponents can be calculated using high-temperature expansion techniques, and although such calculations give support to the idea of universality, they do not provide any insight into its origin. Even an exact solution of the 2d Ising model of a ferromagnet by Onsager in 1944 does not throw any light on this problem. While the simplest theory of phase transitions — Landau's phenomenological theory of continuous transitions — gives the same indices, $\beta = \frac{1}{2}$, $\gamma = 1$ etc. for all transitions, the figures are in disagreement with experiment.

Ken Wilson was born in 1936 and received his Ph.D. from Caltech in 1961 since when he has been Professor of Physics at Cornell University.

How is it possible for the critical behaviour not to depend on the details of the system? Assuming that one of the irrelevant details is the lattice constant, Kadanoff[1]) developed a block spin transformation theory (grouping a number of spins together to form one spin) thereby reducing the independent spin degrees of freedom, but retaining the essential features governing the properties near the phase transition. At each step of a recursive procedure, the length scale was increased by a factor two, and scaling relations were derived.

A similar problem occurs in quantum field theory. When calculating physical quantities such as masses and charges from an initial Lagrangian, infinities occur. To remove these infinities the momentum (over which the Lagrangian is integrated) is arbitrarily cut-off at a value Λ. For the theory to be useful (renormalizable) the physical properties must not depend on the value of Λ, so a change of scale where Λ is replaced by 2Λ should not affect the physics. Gell-Man and Low[2]) wrote down the equations, called the renormalization group (RG) equations, expressing the invariance under a scale transformation.

These were applied by Wilson to problems in field theory. It is quite remarkable that during the sixties, Wilson published almost nothing; neither, fortunately, did he perish. In 1966, Michael Fisher came to Cornell, and Wilson's interest in the problems in phase transition probably comes from his presence. In order to make progress in field theory, Wilson transformed a problem defined in continuous space to one defined on a lattice, and in the theory of phase transitions, he transformed the Ising model (and more general n-vector models), which are defined on a lattice, into a continuum field theory. In practice, Wilson's Hamiltonian was the same as the Ginzburg-Landau Hamiltonian introduced already in the classical theory of phase transitions.

Near the phase transition, the correlation length ξ diverges ($\xi \sim (T_c - T)^{-\nu}$) so there is no basic length scale at T_c. Hence, if you study the system through a microscope, you should see essentially the *same* picture (except for a simple spin-rescaling factor) as you gradually reduce the magnification when you are at the critical temperature. Thus, if the system behaves in the same way at different length scales, its Hamiltonian should also be the same. The problem, therefore, is first to construct a transformation — the RG transformation R — which carries the Hamiltonian at a given length scale into one defined with a larger length scale, H', and then to find a Hamiltonian H^* — the fixed point Hamiltonian — which is invariant under successive applications of the transformation R. One of the parameters in H must necessarily be the temperature, and the value of this parameter for which $H \rightarrow H^*$ under the transformations defines the critical temperature. Once the fixed point Hamiltonian has been found, the various critical indices can be found by studying the linearized behaviour of appropriate operators (such as temperature and field) around the fixed point.

In 1971 Wilson published two papers dealing with an approximate RG calculation in three dimensions[3]) and he was able to estimate the critical index γ. At this point it was well known that the RG equations could be solved for systems in a d-dimensional space, for d larger than the upper critical dimension 4. Already in 1969, Larkin and Khmel'nitskii[4]) had realized that for a special system — a uniaxial dipolar ferromagnet — with an upper critical dimension of 3, the RG equations could be solved to yield classical critical exponents with logarithmic corrections. The values have been confirmed experimentally by Als-Nielsen[5]).

The real break-through came in 1972 when Wilson and Fisher[6]) discovered a systematic way of calculating the exponents near 4 dimensions. Defining $\varepsilon = 4\text{-}d$ the exponents were expanded in a power series (for instance $\beta = \frac{1}{2} + b_1\varepsilon + b_2\varepsilon^2 + \ldots$) through the application of the RG techniques, and it thus became possible to calculate systematically the exponents in three dimensions by setting $\varepsilon = 1$. This approach (which may seem abominable at first sight) is no more suspicious than the high temperature series which is in fact an expansion around $T = \infty$. Best of all, it works! By systematically applying the ε-expansion technique one has now calculated critical exponents with an accuracy similar to that of the high temperature series — and the results agree.

What is more important is that the ε-expansion explicitly shows how the dimensionality and the symmetry of the system affect the critical indices. The RG theory provides an understanding of the concept of universality, and it also constitutes an independent derivation of the scaling hypothesis. All that one needs to calculate critical indices is information on the *symmetry* of the system, as expressed for instance in Landau's phenomenological free energy expansion.

The RG theories were quickly absorbed by the physics community, and probably more theoretical physicists have worked on these ideas during the seventies than on any other.

Since his remarkable discoveries concerning phase transitions for which he has now been awarded the Nobel Prize, Wilson has given important contributions towards the solution of the famous Kondo problem concerning the conductivity of a metal with a single impurity spin — again using RG techniques. He has also continued his work in particle physics, introducing the "Wilson loop" concept.

In many ways, Wilson is different from the mainstream physicist. He is very stubborn in pursuing his idea, and he does not want to waste his time on problems which are more accessible and could lead to some "easy" publications, but which according to his judgment are not important; most physicists cannot afford this attitude — they would probably end up with nothing. Recently, he has been doing numerical work using Monte-Carlo RG methods developed with R. Swendsen. Such calculations provide a direct confirmation of the RG ideas in three dimensions. Eventually, the method will be applied to problems in field theory (QCD). Wilson seems now to put all his efforts into computing technology, advocating bigger and more powerful computers. Most colleagues disagree with this philosophy, but this does not bother him, — and who knows, maybe some day he will make an important contribution also to this field.

Wilson has had a remarkable impact on theoretical physics. Before Wilson, solid state theorists and field theorists were working in two different worlds. Many solid state theorists have now been working with field theoretical ideas, and the methods have been developed to an extent where there is now a feed-back into particle physics — not only of RG ideas but also of other methods such as Monte-Carlo techniques and mean-field theories which are well known in solid state physics. This development is largely due to Wilson.

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