Synergetics

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While science is developing at an enormous pace, it more and more splits into specialized disciplines. It appears that these disciplines do not only speak their own languages, but are governed by completely different phenomena and principles. Synergetics is a new field of interdisciplinary research which, rather amazingly, has succeeded in revealing essential *common features* of seemingly diverging disciplines. How synergetics proceeds and what it has achieved so far, is the subject of this article. Indeed, there exist profound analogies between certain classes of phenomena in physics, astrophysics, chemistry, biology, ecology, electrical and mechanical engineering, sociology and other disciplines. These analogies become apparent when we adopt a certain level of abstraction and if we focus our attention on the — in general — most interesting phenomena, namely when dramatic qualitative changes of structures (in the widest sense of this word) occur. To elaborate on this, let us be somewhat more specific.

When scientists try to explain the properties of objects, they often decompose the objects into subsystems, for example into atoms, molecules, cells, plants, animals, human beings etc. In many cases scientists then discover that the subsystems cooperate in a well regulated manner, which often seems to be purposeful, even in the unanimated world. This cooperation leads to macroscopic spatial or temporal structures or well defined processes. Furthermore, when certain external parameters are changed, the macroscopic structure or function may change dramatically. These statements are not new to physicists who have been concerned with phase transitions. Here the mechanical, thermal, electrical, magnetic or other macroscopic properties change drastically at a certain critical temperature. Just think of the onset of ferromagnetism or superconductivity. Furthermore, phase transitions can be grouped into classes, e.g. discontinuous or continuous transitions. A transition is accompanied by a whole bunch of interrelated phenomena. A discontinuous transition (first order) is connected with latent heat, hysteresis etc. A continuous transition (second order) is connected with a symmetry breaking instability, a soft mode, critical undamping, critical fluctuations etc.

It came as a surprise to many scientists when it turned out, that completely analogous features are shared by quite other types of systems. Among these systems are physical systems far from thermal equilibrium or even numerous nonphysical systems. Again the transitions can be grouped into classes. While some of them exactly correspond to phase transitions or phase transition-like phenomena in thermal equilibrium, new classes occur which can be considered as a natural extension of the old classes. In the following we want to elucidate the essential points either by specific examples or general arguments. We hope that in this way also new light is shed on the question of whether there are general principles or mechanisms governing selforganization of systems and the evolving macroscopic structures or functions.

Some Typical Examples (Fig. 1)

Let us begin with the laser which in a way plays a similar role in nonequilibrium systems as the Ising model does in equilibrium phase transition theory. The laser basically consists of a rod of laser active material. The laser is kept working by an energy flux through it. When the energy flux from the outside is only small, the laser acts as a common lamp, emitting short incoherent light tracks each of a few metres in length. Beyond a certain power input, however, a light track of say 300 000 km is emitted. This indicates that the internal state of the laser has completely changed. The formerly randomly oscillating atomic antennae are now oscillating in phase in a completely selforganized manner.

A seemingly quite different system is a fluid layer heated from below. When the temperature gradient between its lower and upper surfaces is small, heat is transported by conduction and no macroscopic motion shows up. However, above a certain critical temperature gradient, a macroscopic pattern of fluid motion appears. The velocity field of the fluid has now the form of regular rolls or hexagons.

Our third example is taken from chemistry. Here, for example, in the Belousov-Zhabotinsky reaction, spatial-temporal patterns occur on a macroscopic level, for instance oscillations of colour, changing from red to blue, or coloured ring patterns.

In mechanical engineering, the deformation (buckling) of plates or thin shells of cooling towers etc. under load is studied. A typical phenomenon is the following : beyond a certain critical (but homogeneous) load, the for-

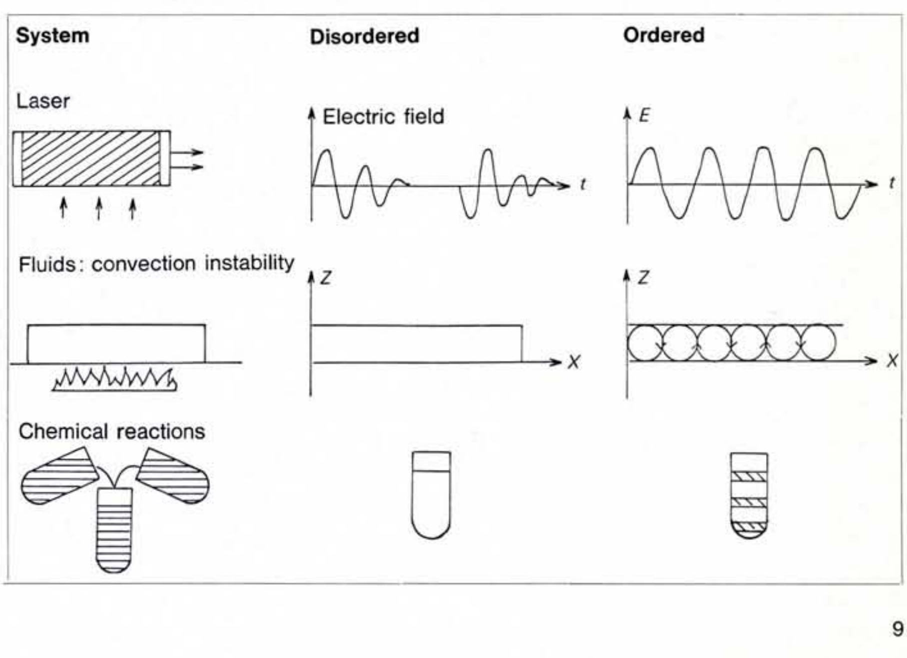


Fig. 1. Structures of continuous systems.

merly homogeneous shell shows buckling with hexagonal cells.

To conclude these examples: in models of evolution it is shown how out of a soup of different self-reproducing molecules, only one kind survives. In a way, a higher degree of order has been achieved. In all these systems and in numerous others, mysterious forces or « demons » seem to tell the subsystems how to behave so as to produce regular patterns on a macroscopic scale. We now want to give an outline of the concepts to cope with these problems.

The Order Parameter

The first question is how we can describe disorder and order (or different states of order). Here the concept of the order parameter, originally introduced by Landau to treat structural phase transitions, has proved extremely useful. To mention a well known example: in the ferromagnet, the total magnetization may be taken as order parameter. When the spins point in random directions and thus are disordered, the order parameter is zero, while it acquires its maximum value when all spins point in the same direction and thus are ordered. The properties of the total system are described by one or several order parameters, i.e. very few degrees of freedom replace the enormous number of degrees of freedom of the subsystems. In a good deal of explicit cases this reduction of the number of degrees of freedom can be traced back to the « adiabatic elimination principle » which we explain by means of two variables, one symbolizing the order parameter u, and the other s, a subsystem. Incidentally, our following example will show that "subsystem" has a very broad meaning.

In the spirit of synergetics we are free to choose the discipline from which we take the equations for u and s. Let us take chemistry and identify u and s with concentrations of molecules called U and S respectively. We consider the following processes: U is created by an autocatalytic reaction, i.e. its generation rate is proportional to u, and it decays by an autocatalytic reaction involving molecules S. Denoting the corresponding rate constants by λ_u and "1", we obtain

$$\dot{u} = \lambda_u u - s u \qquad (1)$$

Assuming a decay of molecules S proportional to their concentration and a production from U by a bimolecular process, we obtain

$$\dot{s} = -\lambda_s s + u^2, \quad \lambda_s > 0 \qquad (2)$$

Now let us assume that the order parameter, u, relaxes much more slowly than the subsystem which implies $|\lambda_u| << \lambda_s$. Then in many cases we can neglect the time derivative in (2) and solve (2) by

$$s = u^2/\lambda_s \qquad (3)$$

The stable, damped mode of the subsystem must follow the order parameter immediately or, in other words, the subsystem is slaved by the order parameter. This suggests that we shall find order parameters by looking for those modes which decay the slowest. When we change external parameters (the pump of the laser, the temperature gradient of the convection instability etc.) λ_u may become positive. As long as we neglect the terms s, u in (1), the order parameter u grows exponentially; i.e. an *instability* occurs.

We now pass over to realistic systems consisting of many subsystems each of which is described by a variable q_j or a set of variables. Examples for such variables are the electric field strength and atomic dipole moments in a laser, the velocity field of a fluid, concentrations of molecules, numbers of animals etc. Typical equations are

$$q_j = F_j(\mathbf{q}) \qquad (4)$$

Let us assume that we have found a certain set of stationary stable solutions q°_j (this problem can be formulated in a much more general way, but we just want to illustrate the main points here). When we alter certain external parameters the stationary state q°_j might become unstable. To check this, one usually superimposes on q°_j small deviations δq_j which obey linearized equations determining the collective modes of the system. The system will become unstable, if any one of the modes becomes undamped. The undamped modes serve as candidates for order parameters. In many practical cases, only very few modes become unstable so that we achieve an enormous reduction in the really important degrees of freedom via the "adiabatic elimination principle". While several criteria have been developed, for example in irreversible thermodynamics, to find instabilities, the crucial task remains to determine the new structure which arises out of the old structure. This is made possible by studying the resulting order parameter equations.

The simplest example for the resulting order parameter equation is obtained when we insert (3) into (1):

$$\dot{u} = \lambda_s u - C u^3.$$

Apparently in the case of an undamped situation, ($\lambda_u > 0$) we get a stable solution: u is not equal to zero. (In the terminology of our chemical example: a macrospic concentration of molecules U is produced). Thus the formerly unstable modes are stabilized via stable modes which now play the role of "subsystems". Of course, in practice we deal with much more complex systems (in mathematics, problems of this kind are dealt with in bifurcation theory, though further development is needed here). However, in many explicit cases the *resulting* order parameter equations have certain simple structures. Such a structure may be the same for originally completely different systems (laser, chemical reaction, etc.). This allows us to define instability classes. Quite similarly to classifications in conventional phase transition theory, each class incorporates a set of typical static and dynamic phenomena. A first class consists of (order parameter) equations of the type

$$\dot{q}_j = \partial V(\mathbf{q})/\partial q_j, \quad j = 1, ..., N$$

i.e. there exists a potential function $V(q)$. When we look for stationary states, $\dot{q}_j = 0$, we just have to seek the extrema of V. One of the simplest examples of V is exhibited in Fig. 2. When we change an external parameter, the minimum at $q = 0$ may split and shift to another value corresponding to a new structure on a macroscopic level. Small parameter changes cause dramatic changes of the macroscopic structure.

The change of the minima as a function of external parameters has been studied by several authors, in particular by Thom who called his theory "catastrophe theory". He gave a classification of such transitions for one or two unstable modes and up to four external parameters. For which systems do such gradient equations apply? They apply for instance, in mechanics when dealing with the construction of bridges or cooling towers, etc. It is not surprising that mechanical engineers have arrived at the same classification as Thom.

Are such equations also applicable to dynamical, pumped systems? We have studied many examples which show that the original total systems do not satisfy gradient conditions. Indeed open systems, through which energy is pumped, do not obey the laws of reversibility. However, in many important cases, by virtue of the adiabatic elimination principle, the effective *order parameters* obey equations which can be expressed as gradient equations.

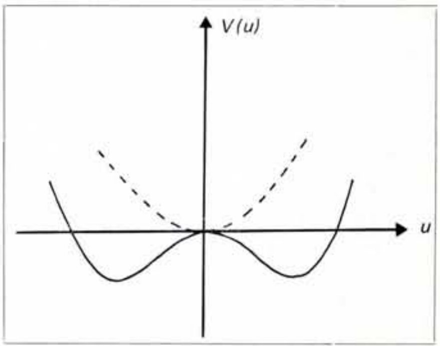
A very interesting situation arises when a system has several order parameters, i.e. modes which become unstable or nearly unstable under the same conditions. In physics this may be the case in two or three dimensions on account of symmetry. Each order parameter governs a certain microscopic configuration giving rise to a macroscopic pattern. Thus to each order parameter at its instability point, an "embryonic state" of the system is attached. Which structure will eventually emerge is determined by the equations of the order parameters, by initial conditions and by fluctuations. Roughly speaking, the equations describe the cooperation or competition of order parameters. A typical example for their cooperation can be found in fluids at the convection instability in certain circumstances. Three order parameters each governing a plane wave stabilize each other and give rise to hexagon cells. When the temperature gradient is enhanced, this cooperation breaks down, and only one order parameter survives. The hexagons disappear and rolls of the velocity field are formed instead. Examples for competition between order parameters which lead to a severe selection of one mode can also be found in lasers or in biological processes, for example in evolution theory. It is quite in the spirit of synergetics that the corresponding equations are practically the same.

Further classes of equations deal with spatially slowly varying order parameters. In their simplest form, these equations are identical with the time-dependent Ginzburg-Landau equation known from superconductivity. Other equations, which play a central role in synergetics, are certain generalizations of those Ginzburg-Landau equations. A very interesting class represents undamped macroscopic oscillations, for instance of chemical reactions.

Hierarchy of Instabilities

When we further change the external parameters, the newly formed structure can again become unstable. Quite amazingly, lasers and fluids can show exactly the same type of instability, namely pulse formation. The Gunn instability of semiconductors falls under the same class. In many systems we find an ever increasing number of examples in the form of new spatial or temporal or functional patterns. Among other things, this allows us to store or process more and more information. Sometimes this sequence of instabilities seems to come to an end, when chaotic (turbulent) states are reached. Again in completely different systems including biological and eco-systems, the same type of chaotic states, i.e. irregular oscillations, are found.

Fig. 2. Simple example of V.



Fluctuations

So far we have talked as if we dealt with completely deterministic processes. However, when we eliminate the subsystems, the "underworld" of subsystems is still active causing fluctuations of the order parameters. Furthermore, the surrounding exerts fluctuations on the system. Fluctuations play a decisive role in many respects. For instance we have mentioned above that a variety of new states may occur. For example, the two minima of Fig. 2 correspond to two completely different macroscopic states. A system driven by fluctuations does not stay in one state for ever but it can explore new states. Since most of the systems of interest here are finite, fluctuations are decisive with respect to reliability, adaptability and switchability of devices using these kinds of systems. It transpires that these considerations shed new light on the interplay between "chance" and "necessity" in the theory of evolution.

In the foregoing, we have stressed analogies with phase transitions. It should be noted, however, that in some respect our emphasis is being shifted. In traditional phase transition theory we deal with infinite systems and focus our attention on singularities of the free energy. In this new field we mainly deal with finite systems. Thus the phenomena are more analogous to those found, for example, in one dimensional superconductors. Here then the agreement is perfect.

Summary and Outlook

While some phase transition analogies for special systems have independently been discovered by several authors (particularly by Landauer, by myself and Graham, by Scully and others) it now falls to synergetics to study these problems in a systematic manner. Synergetics analyses and compares the behaviour of systems composed of many subsystems in situations where the macroscopic structure changes qualitatively. So far, the results are rather amazing. Though subsystems may be quite different, the mechanisms by which an old structure is replaced by new ones are the same. Certain collective variables, called order parameters, become unstable, grow, and, by competition or cooperation among each other, establish a new structure. The subsystems then have to obey those configurations of order parameters which have survived the competition. Even the dynamics of quite different systems essentially obeys the same laws. The advantage of analogies is obvious. Once a problem is solved in one field, its solution can be transferred to another field. A system can be used as analogue computer for a different one. For instance, electronic computer elements can be translated into chemical computer elements, which has been done, for example, by Rössler and others. Here is a small list of systems which have been analysed by various authors and which can be grouped into classes : tunnel diodes, lasers, Gunn oscillators, convection instability in fluids, formation of cloud strata, shock waves, nonequilibrium transitions in polymers, deformation of rods, plates and thin shells, change of shape or state of stars, oscillatory phenomena in lasers, fluids, vacuum tube oscillators, spatial and temporal patterns of real or model chemical processes. In biology, population dynamics, certain models of evolution, morphogenesis, neuron networks, and perception. Some of these concepts are applicable to ecology, economic processes, and sociological models, and presumably to psychology.

Depending on the individual disciplines, the meaning of the approach and classification, discussed above, acquires different shades. For instance, in physics, the phase transition-like behaviour of certain non-equilibrium systems has been quantitatively checked or is presently checked by experiments, whereas in other fields, say sociology, such considerations permit, at least, a qualitative understanding.

In conclusion let us discuss some aspects of the future development of synergetics. It is very young discipline and an enormous amount of work is still ahead of us. Besides a systematic search for further instabilities and instability hierarchies, mathematical methods dealing with, for example, stochastic nonlinear partial differential equations must be further developed. While such equations may describe macroscopic patterns, they may simultaneously represent continuously distributed logical elements. In this way, new insights into the relation between micro- and macrosystems can be expected. Possibly, also new insights into morphogenesis can be gained. The considerations discussed above suggest that nature might use very clever tricks to exploit instabilities by steering the competition of order parameters by means of small parameters stemming from a lower morphogenetic level (for instance genoms). Here and elsewhere a very profound question will presumably show up again and again : to what depth will the human brain be able to unearth the right "order parameters" ?

Bibliography

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Conference Reports

Computing in Plasma Physics and Astrophysics

The 2nd. European Conference on Computational Physics, organized by the Computational Physics Group of EPS, took place on April 27-30, at the Max-Planck-Institut für Plasmaphysic, Garching, FRG. It had become apparent since the 1st. Conference in April 72 that a meeting on computational physics would be particularly interesting if restricted to a small number of fields in physics where similar numerical models are used and similar numerical problems arise. For several reasons a natural choice for this 2nd. Conference was the combination of plasma physics and astrophysics.

The scientific programme included about 50 contributions spread over eight plenary sessions and one session for the presentation of papers which arrived after the deadline. In addition, there were ten survey talks, essentially one introducing each session. The main topics were stellar evolution and pulsars, particle simulation, MHD equilibrium, stability and nonlinear dynamics, transport in stars and laboratory plasmas; computational physics and numerical analysis provided the formal link between the two fields of plasma physics and astrophysics. Plasma physics had a somewhat larger share, as had been anticipated, because the computational efforts in fusion oriented plasma physics are particularly strong at the moment. Nevertheless a reasonable balance between the two fields was reached in the conference programme.

MHD computations have reached a high degree of sophistication and play an important role in the study of fusion plasmas as well as astrophysical plasmas. Powerful methods have been developed to compute MHD equilibria in two dimensions and to investigate their (linear) stability. First results on the problem of general (3-dimensional) equilibria were presented. Multidimensional MHD motions are being computed in various approximations, including different transport effects. The relativistic MHD models for pulsar magnetospheres have been refined, as well as the essentially hydrostatic models for protostar and stellar evolution. Numerical computations of astrophysical convection have produced very interesting results, although the range of Rayleigh and Prandtl numbers accessible to computation is still somewhat limited. Transport codes for laboratory plasmas, especially in Tokamaks, are constantly being refined and updated to take account of increasing observational material; new important processes such as neutral injection are incorporated. Particle simulation of quasicollisionless plasmas has been extended a) to 3-dimensional fully electromagnetic and relativistic descriptions, b) to model self-consistent non-radiating magnetic effects, c) to describe low-frequency instabilities. In the field of computational physics, more accurate treatment of convective terms and the advantages of finite element methods in hydrodynamics have attracted particular interest.

The general impression was that the conference was successful from its conception as well as its actual scientific programme. Only one comment was made that, also in the topical sessions of plasma physics or astrophysics, the purely numerical aspects could have been emphasized more strongly.

The Board of the Computational Physics Group convening immediately after the meeting agreed to continue the series of CPG organized conferences on computational physics every two years, covering a different field of physics (or combination of two fields) each time. Several possibilities for the next Conference have been proposed and a decision will be taken this autumn.

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