

Mag(net)ic Liquid Mountains

When exposed to a transversal magnetic field, a layer of ferrofluid may exhibit surprising surface structures. Their topography can now be measured precisely by means of X-rays. This allows new insights in the laws of pattern formation for these static deformations.

Some 650 years ago magnetic mountains were reported to attract and capture bypassing ships which contained too many nails. Today the interest has shifted from the nail to the nano-scale.

Magnetic liquids or Ferrofluids are colloidal dispersions of magnetic nano-particles in a carrier liquid like water or silicon oil [1]. The particles have a diameter around 10 nm and are prevented from sticking together by a cover of tensides, or by electric charges. In this way magnetic liquids with susceptibilities from one to ten are synthesized. In comparison, a natural paramagnetic fluid - liquid oxygen - has only a susceptibility of $3 \cdot 10^3$. Therefore these fluids are said to be *superparamagnetic*.

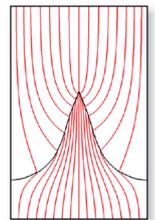
Like other paramagnetic materials, ferrofluids are drawn towards regions of higher field strength. Due to its large susceptibility, the fluid interface can take shapes that are unstable for ordinary fluids. This is utilized, e.g., in rotary feedthroughs, which seal most hard disc drives.

Magnetic forces are absent in a homogeneous magnetic field, oriented vertically to a ferrofluid layer. Should there be a small disturbance of the flat layer, then the magnetic induction is increased at a wave crest, whereas it is reduced in a trough, as sketched in Fig. 2. When the externally applied field exceeds a critical value, the self-generated field inhomogeneities are sufficient to attract more ferrofluid against gravitation and surface tension. The liquid crests grow further, which increases the field gradient even more. Eventually magnetic liquid mountains emerge (c.f. Fig. 1). This pattern-forming instability has been reported by Cowley & Rosensweig [2] soon after the synthesis of the first ferrofluids.

Besides the elegance of the Rosensweig pattern there are more down-to-earth reasons for its study. Pattern formation was mostly investigated in systems with a continuous energy input [3], like Rayleigh-Bénard convection in a liquid layer heated from below. However, patterns evolve as well in systems without permanent energy dissipation, like in elastic shells, which buckle under a load. Those systems excel, because they have an energy functional. This is also true for the Rosensweig instability, which can nicely be controlled by the external field. However, for long the clifty and gloomy crests delivered a fierce resistance against quantitative analysis. This was overcome by radioscopy (see box).

Whereas the onset of the instability and the growing pattern of small amplitude are amenable to a linear

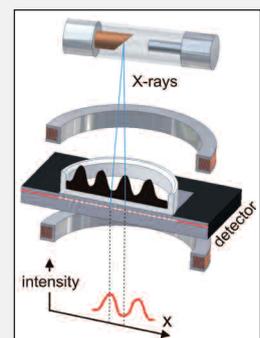
▲ FIG. 1: Hexagonal arrangement of liquid crests of ferrofluid in the succession of the Rosensweig instability. The diameter of the vessel is 12 cm.



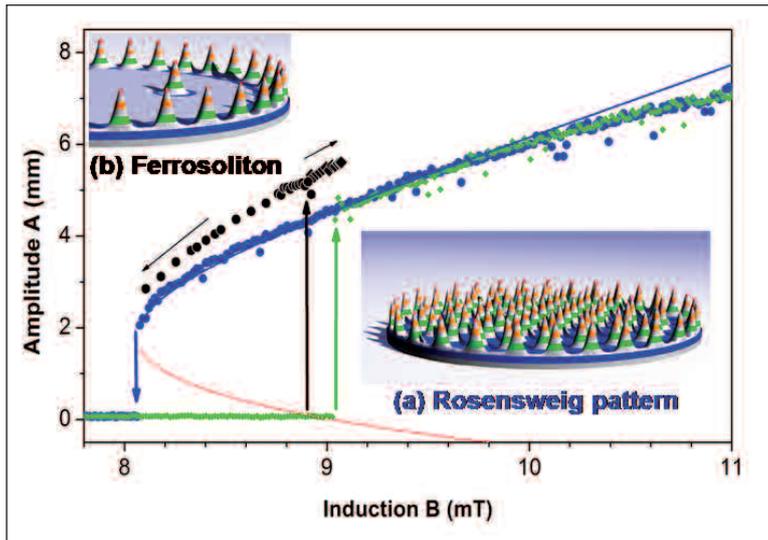
▲ FIG. 2: The magnetic field lines are focused in a peak of ferrofluid (courtesy of A. Boudouvis, Athens).

▶ Measuring the topography by means of X-rays

X-rays which are penetrating spikes are more attenuated than those trespassing troughs of the pattern. After calibration the full surface topography can be reconstructed with micrometer precision [4]. Examples are shown in Fig. 4 and 7.



▶ FIG. 3: Sketch of the setup.



▲ FIG. 4: Amplitude of the Rosensweig pattern for increasing (green) and decreasing (blue) induction. The black symbols mark the height of a localized state [8].

description [5] the fully developed spike patterns remain a challenge - both for experiments and non-linear models. Below we will summarize recent measurements of hexagonal, solitary and square spike configurations.

Hexagons

For supercritical magnetic induction a surface perturbation will grow until the gain in magnetic field energy is compensated by the losses due to hydrostatic and surface energy. The three terms make up a free-energy functional. Its minimization predicts a hexagonal array of spikes to be the first stable pattern, which appears due to a transcritical bifurcation [6]. We have checked the predicted scaling [6,7] by means of radioscopy. The green and blue data points in Fig.4 mark the hysteretic evolution of the pattern amplitude for increasing and decreasing induction, respectively [8]. The solid lines give a fit to the solution of the amplitude equations [6]. Solving the Young-Laplace and Maxwell equations the full topography for the actual nonlinear magnetization has been calculated [9]. Figure 5 compares numerical and experimental results.

Localized states

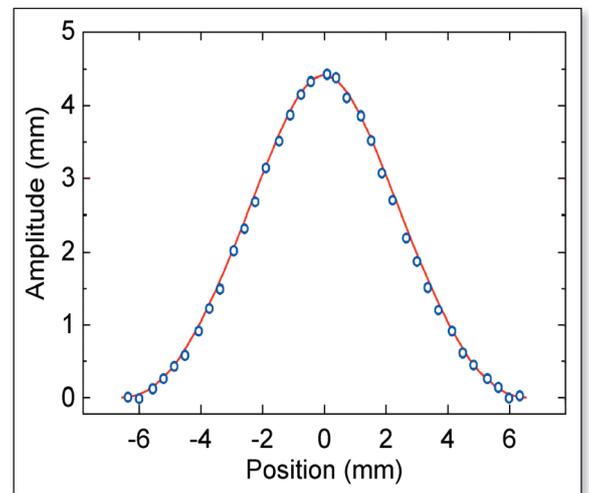
In addition to the hexagonal pattern we can generate, in the bistable regime (cf. Fig.4), a localized state, as shown in Fig.6. This "ferrosoliton" is initiated by a

short, local perturbation of the liquid surface or the magnetic induction [8]. Here the question arises: what hinders such a perturbation from propagating into the whole layer? Why does not a

full Rosensweig pattern appear? As suggested by Yves Pomeau [10], the propagation of the wave front is locked by the periodicity of the pattern itself. This effect is camouflaged in the microscopic regime, e.g., at

the solidification of crystals out of a melt, by thermal activation. However, in our macroscopic system $k_B T$ is not sufficient to initiate a new row of spikes. Thus the wave front locks. The amplitude scaling of a ferrosoliton, as shown in Fig.4, was recovered also numerically [11].

Moreover we recently found an alternative path to ferrosolitons. After preparing a full yet unstable pattern in the bistable regime, the pattern decays and often ferrosolitons appear alone or in patches [12]. This occurs in the neighbourhood of the unstable branch of the bifurcation diagram, as marked in Fig.4 by the red line. In models this range has attracted much attention because here a complex intertwined structure (homoclinic snaking) gives rise to localized states [13].

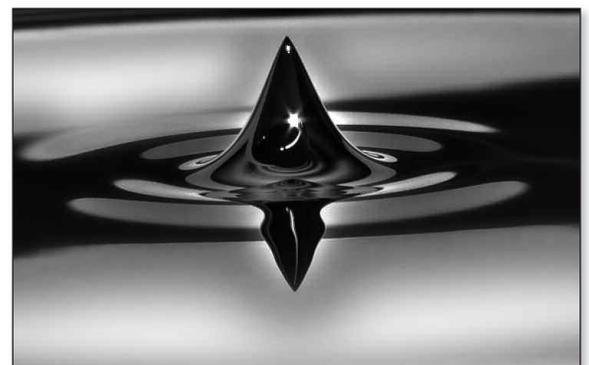


▲ FIG. 5: Experimental (blue circles) and numerical (red line) results for the spike profile of the hexagonal Rosensweig pattern [9].

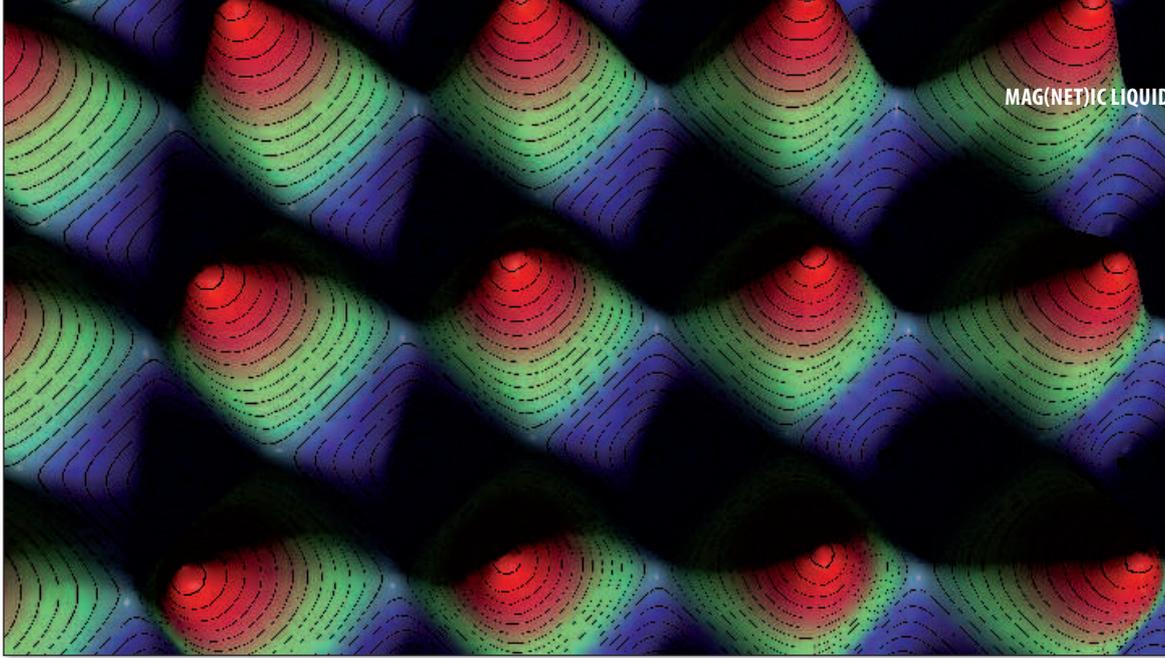
Squares

For higher inductions a transition from hexagonal to square symmetry is predicted [6,7] and found experimentally [14]. Figure 7 shows the final state. To illustrate the transition, Fig.8 shows structures (or 'tessellations') arising at four inductions. The black dots mark the center of the cusps. The 'Voronoi tessellation' is defined by the nearest neighbours around each center. The color of each polygonal cell is coded from the maximum

▼ FIG. 6: A localized state in the bistable regime of the Rosensweig instability.



Why does the wave front not propagate? ”



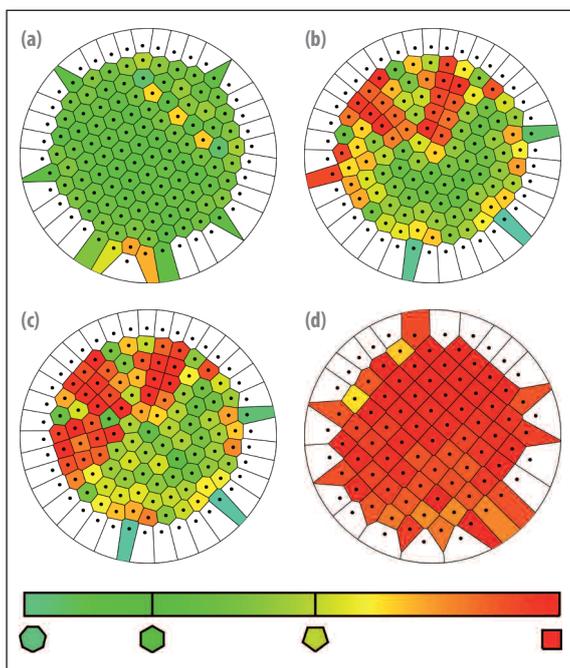
◀ FIG. 7: Measurement of a square lattice of spikes, resembling an egg carton.

angle enclosed by the rays emanating from the center to the corners of the cell. For a regular hexagon this angle is 60° and for a square 90° . With this coding we unveil that the squares appear in patches. This block-wise transformation may be again a consequence of a wave front locking [14].

Conclusion

We have surveyed only some of the many aspects of surface instabilities in ferrofluids. Adding, *e.g.*, a horizontal field component gives rise to ridges and stretched hexagons, whereas a temporal modulation may result in spatio-temporal complexity. All this and a more quantitative treatment is found in the reviews [1,15]. Warning: when approaching “magnetic liquid mountains” more closely, the reader may be captured like the ancient ships. ■

▼ FIG. 8: Transition from a hexagonal to a square lattice for increasing induction. The bar indicates the colour coding derived from the maximum central angle of each Voronoi-cell [14].



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Reinhard Richter studied physics in Erlangen and Tübingen. He received his Ph.D. in 1994 for experiments on pattern formation in semiconductors. After two years at the Weizmann Institute he went to Magdeburg, focusing on ferrofluids. Since 1999 he works in Bayreuth, where he received in 2002 the Emil-Warburg-research-prize, followed in 2010 by his habilitation.

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