

# QUANTUM ENTANGLEMENT AND LOCALITY

\* Miriam Blaauboer,

\* Kavli Institute of Nanoscience, Delft University of Technology, Lorentzweg 1, 2628 CJ Delft, The Netherlands

\* E-mail: m.blaauboer@tudelft.nl

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*Can a measurement at location A depend on events taking place at a distant location B, too far away for the information to be transmitted between the two? It does seem too mysterious to be true. Even Einstein did not believe so. But experiments seem to prove him wrong.*

It is well-known that until the end of his life Einstein was dissatisfied with quantum mechanics as a fundamental theory [1]. He strongly believed that the real world should consist of “systems” (particles, fields) that possess objective properties, *i.e.*, properties that do not depend on measurements by external observers. A second essential aspect of these objective properties is that the outcome of a measurement at location A cannot depend on events taking place at another location B, if B is sufficiently far away from A that information about the event

at B travelling at the speed of light cannot possibly reach A before the measurement at A has been performed. Theories satisfying both of these criteria are called ‘local’ and ‘realistic’. Quantum mechanics, in contrast with earlier developed classical theories, does not satisfy either of these criteria: properties of a quantum-mechanical system do depend on the experimental conditions under which they are measured, and a measurement at A can be influenced *instantaneously* by an event happening at B via the mechanism called entanglement (see ■■■).

▲ John Bell thinking - ahead of his time - about Bell tests with electrons in nanomaterials

box). Einstein believed quantum mechanics to be mathematically consistent, but insisted that in spite of this it should be a local realistic theory. Based on this conviction, he came to the conclusion that present-day quantum mechanics is an incomplete theory, and that

there should be a deeper underlying theoretical framework that does enable us to establish an objective description of quantum-mechanical phenomena.

In 1935 Einstein, together with Boris Podolsky and Nathan Rosen, wrote a by now famous article in which

## Entanglement and Bell's inequality

An entangled state of two or more particles is characterized by the fact that it is not possible to write the state of the full wavefunction as a product of the wavefunctions of the individual particles. A prototype entangled state is the spin singlet  $|\psi\rangle = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$ , which we use here to illustrate the locality paradox as mentioned in the main text. This paradox consists of the following: image two experimentalists in two distant laboratories who each receive one of the two spins in the singlet state. If both of them measure the direction of their spin using identically positioned polarizers, they will always find opposite polarizations, *i.e.* the outcomes of the measurements are one hundred percent correlated with one another. Moreover, this correlation is formed instantaneously and without communication between the two experimentalists and thus it appears as if information has been exchanged at a speed faster than the speed of light. The solution of this apparent paradox lies in the fact that the entanglement of the two spins already existed prior to the measurements. In addition to traditional entangled states such as the spin singlet, more "exotic" types of entanglement exist such as bound entanglement (see the article following this one by Horodecki *et al.*).

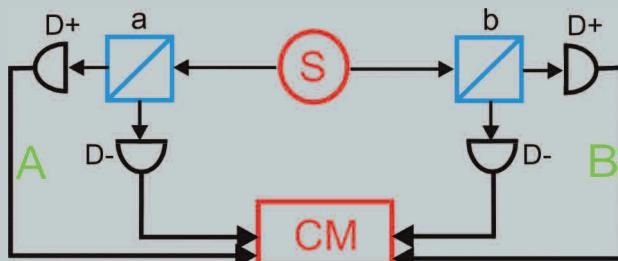
To illustrate Bell's inequality, we use the formulation given by Clauser, Horne, Shimony and Holt [3] which was used in the photon experiment performed by Aspect, and also forms the basis of the proposals for a Bell test with electrons. Consider a source that emits pairs of correlated particles – photons or spin-1/2 particles – such that one particle is sent to polarizer a and the other to polarizer b, see Figure 1. The polarizers measure the polarization (or spin) components of the particles in a chosen direction and yield outcome + or -. Depending on this outcome the particle is then registered by detector D+ or D-. We consider the situation in which 4 settings of the polarizers are used, namely  $\varphi_A$  and  $\varphi'_A$  for polarizer a and  $\varphi_B$  en  $\varphi'_B$  for polarizer b. In a local realistic description both particles have fixed intrinsic polarisations for each of the 4 settings. Each pair of particles can then be symbolically described by the combination  $(\sigma_A \sigma_{A'} \sigma_B \sigma_{B'}; \tau_A \tau_{A'} \tau_B \tau_{B'})$ , where  $\sigma_A = \pm$  and  $\tau_A = \pm$  denote the outcomes of the polarisation measurements of, respectively, particle 1 and particle 2 in the direction  $\varphi_A$ , etc. Assuming the particles to be in the singlet state (for other entangled states a similar reasoning can be made) we have that  $\tau_X = -\sigma_X$ , for  $X=A, A', B$  and  $B'$ . Each pair of particles is then fully characterized by the combination  $(\sigma_A \sigma_{A'} \sigma_B \sigma_{B'})$ . Let  $f(\sigma_A \sigma_{A'} \sigma_B \sigma_{B'})$  be the fraction of the total amount of pairs produced by the source which yield outcome  $(\sigma_A \sigma_{A'} \sigma_B \sigma_{B'})$ . We now consider the parameter S, defined as

$$S = |E(\varphi_A, \varphi_B) - (f(\sigma_A, \sigma_{A'}, \sigma_B, \sigma_{B'}) - f(\sigma_{A'}, \sigma_B, \sigma_{B'}, \sigma_A))|, \quad (1)$$

in which

$$E(\varphi_A, \varphi_B) \equiv \sum_{\sigma A', \sigma B'} [f(+, \sigma_{A'}, -, \sigma_{B'}) - f(+, \sigma_{A'}, +, \sigma_{B'}) - f(+, \sigma_{A'}, -, \sigma_{B'}) - f(-, \sigma_{A'}, -, \sigma_{B'}) + f(-, \sigma_{A'}, +, \sigma_{B'})] \quad (2)$$

By substituting (2) in (1) and using that  $\sum_{\sigma A, \sigma A', \sigma B, \sigma B'} f(\sigma_A, \sigma_{A'}, \sigma_B, \sigma_{B'}) = 1$  it follows directly that  $S \leq 2$ . This is Bell's inequality.



**▲ FIG. 1:** Schematic illustration of a Bell test: a source S produces pairs of correlated particles, *e.g.*, photons, which leave the source one by one and in opposite directions. The polarisation of particle 1 is measured by polarizer a in the direction  $\varphi_A$  and, depending on the outcome, the particle is registered by detector D+ or D-. Similarly, the polarisation of particle 2 is measured by polarizer b in the direction  $\varphi_B$ . Coincidences are then counted by the coincidence detector CM. Source: Sketch of a two-channel Bell test by Caroline H Thompson.

they advocated this opinion and explained how the notion of objective properties is incompatible with the assumption of quantum mechanics being a complete theory [2]. This 'EPR' paper launched a heavy debate on the question whether quantum mechanics could be modified into a local realistic theory.

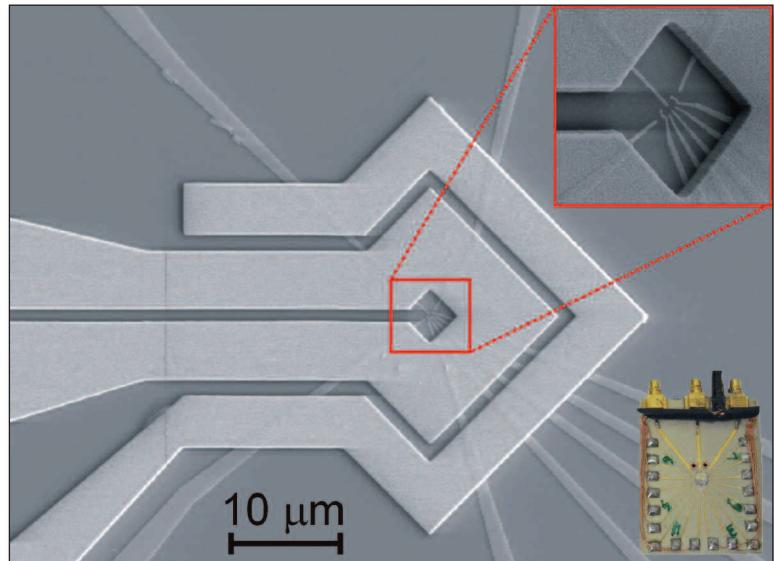
This debate got an interesting turn in 1964, nearly ten years after Einstein's death. In that year John Bell published a quantitative criterion, now known as Bell's inequality, which every local realistic theory should satisfy (see box). This inequality opened the way for experiments that can be used to prove the existence of entanglement and ultimately test whether quantum mechanics is complete or not.

### Bell test with photons

The first experimental tests of Bell's inequality were performed in the 1970s using pairs of photons in a polarization-entangled state. Nearly all of them led to results violating Bell's inequality. In 1982 Alain Aspect and his co-workers in Paris succeeded in violating Bell's inequality by many orders of magnitude. Their experimental set-up is schematically depicted in Fig. 1 [4]. In spite of the fact that these results were in clear agreement with predictions of quantum mechanics, they do not exclude the possibility of a local realistic theory for two reasons. These are known as the detection and locality "loopholes".

The first loophole refers to the technical problem that in practice not all pairs emitted by the source are detected, which is an essential assumption in the derivation of Bell's inequality (see box). In Bell experiments, therefore, great care is taken to ensure that the detected pairs form an accurate reflection of all pairs emitted. Nevertheless, in principle the possibility remains that the latter is not the case, and that the results would satisfy Bell's inequality if all particles were detected. Bell's own opinion on this issue was [5]: *"Although there is an escape route there, it is hard for me to believe that quantum mechanics works so nicely for inefficient practical set-ups, and is yet going to fail badly when sufficient refinements are made. Of more importance, in my opinion, is the complete absence of the vital time factor in existing experiments."*

The last sentence refers to the other loophole, the locality one, which says that it is essential for a reliable Bell test that the measurements at A and B are completely independent and, in particular, that the polarizers should be set well after the moment that the particles left the source. Only in this way it is possible to test for instantaneous long-distance influence among the particles, that could not have been communicated at the speed of light. In the meantime, some experiments have been performed using polarizers that were controlled by random generators on timescales that are short compared to the travel time of photons [6], and also their outcomes are in disagreement with Bell's inequality.



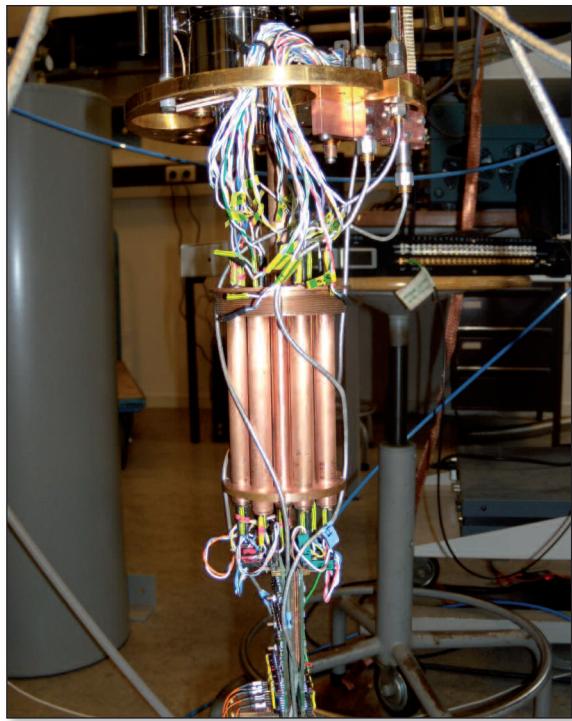
**▲ FIG. 2:** Schematic illustration of a double quantum dot (expanded area) defined by nanoscale metal top gates in a two-dimensional electron gas (2DEG) and covered by a microscale loop antenna. A 2DEG is a two-dimensional layer in a semiconductor structure in which electrons can move freely, i.e. without interactions, while preserving the phase properties of their wavefunction across longer distances (typically a few tens of  $\mu\text{m}$ ). The dots are formed when an electric voltage is applied across the metal top gates which pushes the electrons underneath the gates away. In this way, local islands of electrons surrounded by high potential barriers (the gates) are formed in the 2DEG. The inset shows a photograph of the sample holder. (Source: [www.nano.physik.uni-muenchen.de](http://www.nano.physik.uni-muenchen.de))

### Bell test with electrons?

Apart from Bell measurements with photons, also experiments with entangled pairs of protons, kaons, neutrons, cold atoms and photon-atom pairs have been performed [7] - but not yet with electrons. The main reason for this lies in the difficulty to find or construct a source that produces isolated entangled electron pairs (due to the Coulomb interaction between electrons in solid-state structures) and to preserve the coherence of these pairs, i.e., the phase properties of their quantum state, over distances longer than a few micrometers [8]. Nevertheless, hope exists that proof-of-principle Bell experiments will become possible in the near future due to recent experimental developments in the field of solid-state nanophysics, in particular the progress that is currently being made on coherent manipulation of individual electrons in semiconducting nanostructures [9]. As a result, during the last few years several theoretical proposals have been put forward for testing Bell's inequality in solid-state nanosystems [10-12]. These are for example based on the idea of using a superconductor [10] or tunnel barriers in a two-dimensional electron gas (2DEG, see Fig. 2) [11] as a source of entangled electron pairs. In these schemes the idea is to test Bell's inequality by measuring current-current correlations (noise). This is different from the optical experiments that have been performed, in which photons are detected one by one and thus counted directly. Direct counting of electrons according to their spin direction is more difficult to realize, but recently a technique has been developed to achieve this [13]. ■■■

**► FIG. 3:**

Top view of a cryostat in which low-temperature experiments on quantum dots are done. Source: Quantum Transport group at Delft University of Technology. [Courtesy Lars Schreiber, TU Delft]



This opens the way for a Bell experiment with electrons, analogous to the optical one by Aspect. The basic idea [12] is to use spin-entangled electron pairs in quantum dots, see Fig. 2. Quantum dots are isolated islands of charge in a 2DEG semiconductor structure. Electrons can be transferred one by one onto the islands by manipulating externally controllable gates. Two islands adjacent to each other – a so-called double quantum dot – containing two electrons form the source of entangled electron pairs in this electronic Aspect scheme. To begin with, the gate in between the dots is open and the electrons naturally form a spin-singlet state, which is the ground-state of this system. Then this gate is closed, leaving one electron on each dot (due to Coulomb interaction) while their entanglement remains intact. After opening the two "exit gates" (Fig. 2) the electrons leave the dots and travel through electronic quantum channels to the "polarizers". These polarizers also consist of quantum dots, in which the two electrons are subsequently confined. By switching on a local magnetic field in each dot, the spin of the electrons is coherently rotated via an electron-spin resonance process. The time during which the magnetic field is applied determines the angle of rotation of the spin. Finally, the spin of each electron is measured upon leaving the dots [13]. By repeating this experiment for many electron pairs the probability of detection of each of the four possible outcomes (both spins up, spin 1 up and spin 2 down, etc.) is determined and this is used to test Bell's inequality.

### Loopholes

How about the loopholes in this electronic Bell scheme? In principle, only by detecting every entangled pair the detection loophole can be firmly closed. Until more sensitive

detectors become available we thus depend, as in the photon experiments, on the assumption that the pairs detected form an accurate reflection of all pairs emitted. A more serious problem is the fact that the time required for rotating the spins is much longer than the time required for the electrons to travel from the double quantum dot (the source) to the single quantum dots (the polarizers). This can not be remedied by increasing the distance between the dots, since in that case the travel time of the electrons is so long that the chances of decoherence of their (entangled) quantum state would become rather large. Faster spin rotation times, on which steady progress is being made, would enable the closing of this locality loophole. But the first goal in solid-state Bell nanophysics is more modest and does not require closing of the loopholes: to prove that the detected electron pairs were entangled. This has not yet been achieved and will be an important step forward in the field of solid-state quantum information processing. ■

### About the author

Miriam Blaauboer holds MSc degrees in Theoretical Physics and Mathematics from Leiden University and a PhD in Theoretical Physics from the Free University in Amsterdam. After spending postdoctoral periods at the Weizmann Institute in Israel and at Harvard, she is currently an assistant professor at Delft University of Technology in The Netherlands. Her research focuses on transport and quantum information processing in solid-state nanostructures. She is also Editor-in-chief of the monthly magazine of the Dutch Physical Society.

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